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A SIMPLIFIED APPROACH FOR SHEET FORMING PROCESSES DESIGN

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Abstract—A new approach is developed to improve sheet forming parts and sheet forming processes design. In order to apply classical optimization methods, an energetic characterization of the global transformation of an elasto-plastic structure is proposed. After a brief review of thermostatics in elastoplastic laws, an energetic criterion is defined. With a geometrical definition of sheet forming processes, it is shown how to use this criterion to find the optimal displacement path between a flat blank and the part shape. Then, the classical assumptions of rigid-plastic behaviour and of thin shell geometry are introduced. Eventually, the criterion is used to define a formability criterion, for use at the preliminary design stage of parts. This paper concludes with numerical applications of the approach and analysis of formability criterion applications.

1. INTRODUCTION

Models for sheet forming processes primarily aim at supplying engineers with numerical results that represent the characteristics of a part in its final stage of the process. As a first objective, one likes to be able to simulate the metal flow of the blank for a given geometry of the tools. This is generally performed by using incremental methods taking into account an elasto-plastic behaviour of materials, unilateral contact conditions between the tools and the sheet, friction laws, large displacements and large strains assumptions and springback [1]. It is also required that instability phenomena, e.g. necking, wrinkling and fracture, are predicted in order to evaluate the quality and reliability of the simulated process [2, 3]. Many works have been devoted to building such methods, and even though all problems are not closed, significant progress has been made [4, 5].

However, it is not possible to use these kinds of methods at every stage of the sheet processes design. In the first place, these procedures are time consuming and it is a strong limit to their intensive use. A second limitation lies in the necessity to define the geometry of the tools involved in the simulation. Therefore, high performance methods are restricted to the ultimate stage of the analysis of a sheet forming process, as a technique to control the accuracy of the designed tools.

Yet it is interesting to have some simple techniques for use at the preliminary design stage, in order to perform a rapid estimation of some important characteristics (thickness variations, strain distribution, . . .) of a part at its final stage. One can obtain an early evaluation of the formability of the part, which is of important help for the sheet metal forming parts design, and then use this information at the forming tools design stage. Interest for this kind of approach has grown in the last few years. Some researchers have developed a one stage formulation without taking into account the incremental process of material laws between the initial flat blank and the final workpiece [6]. Others suggested a geometrical approach to estimate the strains with any kind of law for modelling the material behaviour [7, 8].

Following this approach, we have developed a new method to study sheet forming processes at the first design stage [9]. The basic idea of our work is to propose a global criterion to evaluate the “cost” of a known part. To define this criterion, we start from the mechanical formalism used to build the elastoplastic law and we perform a simple energetic analysis. With this criterion at hand, it is possible to define an optimal path between the flat blank and the final part, so that this path minimizes the cost of the transformation. With

this approach, it is possible to state the usual assumptions (shell geometry, membrane state of stress, radial path) of sheet forming processes modelling to simplify numerical calculations. We show in this paper that our method can perform preliminary stage design evaluation of strain, but also helps to find the optimal strain path, in the sense of the criterion. To this purpose, we must define sheet forming as a simple geometrical transformation between a flat part and a non-developable one. Throughout this paper, we choose to look at sheet metal forming processes with a geometrical point of view only, as defined by the following definition: "We call stamping any process that transforms a flat part into a non-developable one." This definition implies that we do not consider technological constraints such as those imposed by the use of rigid tools, e.g., and we regard every path between the two states of the part as admissible.

Eventually, let us explain why we choose to start from the general elasto-plastic problem formulation to present our method. First of all, it enables us to define problems which could be of great interest for sheet forming processes design (even though they are not solved in this paper). Furthermore, the simplified problems 5 and 6, solved below, could be obtained through a geometrical approach. As we did not follow this way to define these problems, it seems necessary to recall how one can reach these results through a complete mechanical approach.

Let us now present the optimal criterion.

2. OPTIMAL CRITERION

As stated above, we chose to perform an energetic analysis of the material behaviour, which implies that we estimate the intensity of a local transformation by the value of an energy, and to generalize this estimation to the whole structure transformation, we integrate this energy over the whole flat. If we can define easily an energy function for elastic materials, it is not quite that simple for elasto-plastic materials because of their dissipative behaviour. Let us first recall some simple results about elasto-plastic model formulations and their applications to structures. We will base our approach on these results.

2.1. Evolution problem for elasto-plastic continua

Let us consider the general problem of the determination of the quasi-static response of an elasto-plastic structure submitted to applied loads and applied displacements under uniform and fixed temperature conditions. We describe the material behaviour by the two potential methods presented in Refs [10–13].

2.1.1. *The two potentials method.* Let $w(\varepsilon, \alpha)$ be the free energy density function of the solid at a material point, $p_{(i)}$, the intrinsic power of internal forces and ϕ_m , the intrinsic dissipation. The first and second laws of thermostatics are then expressed by Eqns (1) and (2), respectively:

$$\phi_m + \dot{w} + p_{(i)} = 0 \quad (1)$$

$$\phi_m = -p_{(i)} - \dot{w} \geq 0, \quad (2)$$

where a quantity surmounted by a dot (·) denotes differentiation with respect to the monotonically increasing time-like parameter.

With strain ε and internal variables α , the associated stress σ and internal force A are given by:

$$\sigma = \frac{\partial w}{\partial \varepsilon} \quad (3)$$

$$A = -\frac{\partial w}{\partial \alpha}, \quad (4)$$

and the two laws of thermostatics can be written in the form:

$$\sigma : \dot{\varepsilon} - p_{(i)} = 0 \quad (5)$$

$$\phi_m = A : \dot{\alpha} \geq 0. \quad (6)$$

The evolution law for the internal variables is then defined by Eqns (3) and (4) and by the classical assumption of normal dissipative system [10, 11] that we now recall.

Let $\varphi(\alpha, \dot{\alpha})$ be a continuous function called the pseudo potential of dissipation which is non-negative, convex with respect to the second argument $\dot{\alpha}$, and equal to zero for $\dot{\alpha} = 0$. The internal force is associated with the rate change of the internal variable $\dot{\alpha}$ under the assumption of normality:

$$A \in \partial_{\dot{\alpha}} \varphi(\alpha, \dot{\alpha}). \quad (7)$$

If we impose the function φ to be positively homogeneous by one degree with respect to $\dot{\alpha}$, we obtain a material behaviour independent of physical time, which is the case of elasto-plastic materials.

2.1.2. General formulation of the evolution problem. Consider now a solid with free energy $w(\varepsilon(u), \alpha)$ per unit reference volume, occupying a volume S . Let u denote displacements with respect to the reference configuration. Let us introduce $\Pi(u, \lambda)$, the potential of the applied forces, such that the work of the applied forces during displacement δu is expressed as

$$-\frac{\partial \Pi}{\partial u}(u, \lambda) \delta u, \text{ where } \lambda \text{ is a control parameter.}$$

We also introduce the solid total potential energy $E(u, \alpha, \lambda)$:

$$E(u, \alpha, \lambda) = \int_S w(\varepsilon(u), \alpha) dS + \Pi(u, \lambda). \quad (8)$$

To simplify the problem, we assume that λ is the control parameter for the prescribed displacement. Under the quasi-static evolution assumption, the equilibrium equation is obtained by making E stationary with respect to u , thus:

$$E_{,u}(u, \alpha, \lambda) \cdot \delta u = \frac{\partial E}{\partial u}(u, \alpha, \lambda) \cdot \delta u = 0. \quad (9)$$

Note that it is possible to define u as a function of (α, λ) from Eqn (9). Assuming at this stage that the displacement u and the irreversible internal variable α are known, one is interested in determining the time rates of change of the fields \dot{u} and $\dot{\alpha}$. As aforementioned, we define an explicit dependence between u and the internal variable α , so we just need to determine $\dot{\alpha}$. It has been shown in Ref. [13] that $\dot{\alpha}$ is the solution of the following equations:

$$\begin{cases} A = -W_{,\alpha}(\alpha, \lambda) & \text{with } W(\alpha, \lambda) = E(u(\alpha, \lambda), \alpha, \lambda) \\ \dot{\alpha} \in \partial_A \varphi^*(\alpha, A) \\ \alpha(0) = \alpha_0 \end{cases}, \quad (10)$$

where:

- $\varphi^*(\alpha, A)$ is the Legendre transform of the pseudo-potential of dissipation, and $\partial_A \varphi^*(\alpha, A)$ the outward normal cone of the convex set of admissible forces C at the point (α, A) ,

$$\dot{\alpha} \in \partial_A \varphi^*(\alpha, A) \Leftrightarrow \dot{\alpha} \cdot (A - \hat{A}) \geq 0 \quad \forall \hat{A} \in C(\alpha). \quad (11)$$

- $\alpha(0)$ is the initial value of the irreversible internal variable field.

Most methods modelling sheet forming processes are based on such an approach, and use finite element codes in order to solve for u and α in Eqns (10) [4, 6, 14, 15].

2.2. Energetic balance of a general transformation

Let us now consider once more the general evolution problem of an elasto-plastic structure. Starting from the rates problem (10), we have recalled that it is possible to determine the response of such a structure for a prescribed loading history (including prescribed displacements and applied forces) during a time interval $[0, T]$. Assume now that the value of the strain variable ε and of the irreversible variable α are known at every

time of the interval $[0, T]$. It is then possible to integrate Eqn (1) over time t . We obtain:

$$W_{(0 \rightarrow T)} + \Phi_{m(0 \rightarrow T)} = -P_{(i)(0 \rightarrow T)}, \quad (12)$$

where:

$$\begin{aligned} W_{(0 \rightarrow T)} &= \int_0^T \left(\int_S \dot{w}(\varepsilon(\tau), \alpha(\tau)) dS \right) d\tau \\ \Phi_{m(0 \rightarrow T)} &= \int_0^T \left(\int_S \phi_m(\varepsilon(\tau), \alpha(\tau)) dS \right) d\tau \\ P_{(i)(0 \rightarrow T)} &= \int_0^T \left(\int_S p_{(i)}(\varepsilon(\tau), \alpha(\tau)) dS \right) d\tau. \end{aligned} \quad (13)$$

As the free energy density is a state function, the total variation of W between time 0 and time T is a function of the state variable fields in the initial and final stages of the particular studied transformation only:

$$W_{(0 \rightarrow T)} = W(\varepsilon(T), \alpha(T)) - W(\varepsilon(0), \alpha(0)), \quad (14)$$

whereas the two other quantities $\Phi_{m(0 \rightarrow T)}$ and $P_{(i)(0 \rightarrow T)}$, which, respectively, represent the total dissipated energy and the total energy received by the system, are path dependent in most cases.

From Eqn (12), we interpret the total variation of free energy as the energy "stored" in the structure during the total transformation considered. Various physical phenomena can contribute to the variations of this energy, and those of most interest here are elastic deformation and modification of micro-states due to strain-hardening.

To illustrate this point, just consider the simple case of an elasto-plastic column submitted to compressive-tensile strain cyclic loading paths such that the largest traction strain is larger than the elastic tensile strain limit (Fig. 1).

We can show [9] that the total free energy variation between the initial and final state could characterize the intensity of the total transformation of the structure. In particular, this quantity does not take into account cyclic paths which do not change state variables. This situation is possible for elasto-plastic materials with kinematic strain-hardening behaviour for example (Fig. 2).

If we consider the case of an isotropic strain-hardening behaviour material for the column, the state variables change between two same strain deformation states which

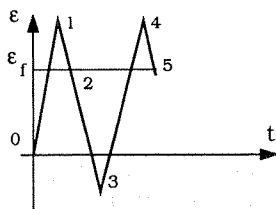


FIG 1. Strain path.

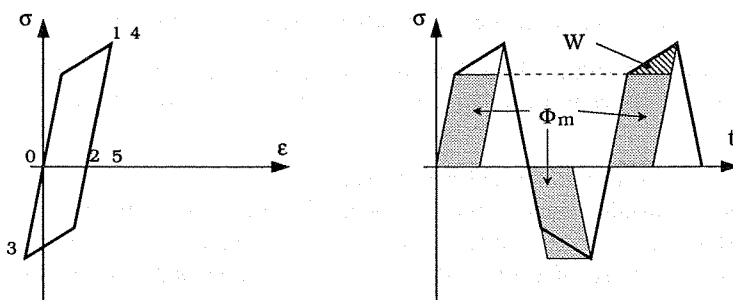


FIG. 2. Kinematic hardening behaviour of an elasto-plastic column.

belong to different cycles. As indicated in Fig. 3, the total free energy variation now takes into account this change.

With this simple example, we see that the total variation of free energy can be a measure of the total transformation of a structure, with respect to the properties of materials behaviour. This is not the case for the total dissipated energy and the total energy received by the system, which are monotonically increasing functions with respect to time for kinematic strain-hardening behaviour (Fig. 2).

So, according to the aforementioned properties of the total free energy variation, we chose to characterize the intensity of the total transformation of any structure by this quantity. We can now propose our optimal design criterion.

2.3. Optimal design criterion

In the context of sheet metal forming process design, we use the characterization of a transformation presented above to propose the following criterion. First, let us introduce some notations.

We denote by $x_{0 \rightarrow T}$ the path of the variable x during the time $[0, T]$. It is a function with respect to time t , and one must be careful not to confuse this function with its particular value at time t , which will be denoted $x(t)$.

Consider now a structure occupying at the initial stage a volume S_0 , whose evolution is described by the displacement field $u_{0 \rightarrow T}$, the strain field $\epsilon_{0 \rightarrow T}$ and the internal variable field $\alpha_{0 \rightarrow T}$. We symbolically denote this set of information as $(S_0, u_{0 \rightarrow T}, \epsilon_{0 \rightarrow T}, \alpha_{0 \rightarrow T})$. We propose:

Definition 1: let T be a set of problems $\{(S_0, u_{0 \rightarrow T}, \epsilon_{0 \rightarrow T}, \alpha_{0 \rightarrow T})^i, i = 1, n\}$, such that the final configuration s_T for each element of T is a given configuration s_F . One element of T is said to be optimal in order to reach the final shape s_F if this transformation minimizes the total free energy variation between the initial and final stage over all the elements of T .

$$W_{0 \rightarrow T}(S_0^{\text{opt}}, u_{0 \rightarrow T}^{\text{opt}}, \epsilon_{0 \rightarrow T}^{\text{opt}}, \alpha_{0 \rightarrow T}^{\text{opt}}) = \text{Min}_T W_{(0 \rightarrow T)}(S_0^i, u_{0 \rightarrow T}^i, \epsilon_{0 \rightarrow T}^i, \alpha_{0 \rightarrow T}^i). \tag{15}$$

The final configuration s_F , will be called the goal configuration. It is the final shape to be obtained and it is given data from the problem for sheet forming process design. In the context of plastic forming processes, each element of the set T is a solution that enables us to reach the final form from an initial shape S_0 which is *a priori* unknown. With the proposed criterion, we define a simple way to choose the 'best' solution in the set T . We now use Definition 1 to perform our simplified approach.

3. GEOMETRICAL APPROACH

As aforementioned, at the sheet forming process design stage, one has to find the best technological process to obtain qualified workpieces. In particular, this process must avoid drawing imperfections (e.g. necking, wrinkling, fracture, . . .) and it must respect geometrical specifications. Indeed, it is a very complex problem to come up with an automatic

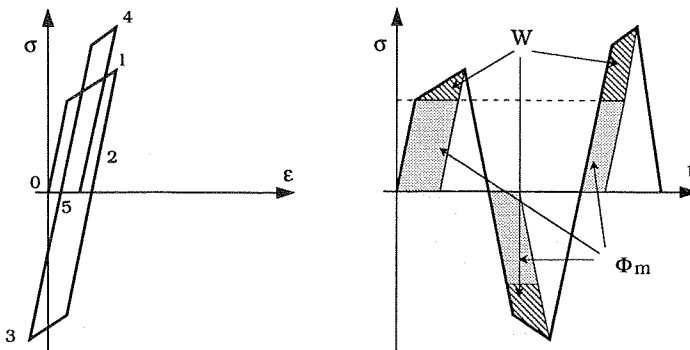


Fig. 3. Isotropic hardening behaviour of an elastic-plastic column

procedure if one wants to take into account all technological constraints. For example, the use of rigid tools does not permit the achievement of any kind of displacement path between the initial configuration and the final shape. The unilateral contact condition is another example of such constraints. For these reasons, we chose to apply our optimal criterion to a simple geometrical approach of sheet forming process.

3.1. *The general problem*

Starting from the geometric definition of sheet forming process, given in the first section, we apply our criterion to define the “optimal” path between an initial flat blank and the given final stage. With a very simple analysis of known and unknown information at the first stage of design, we can formulate this problem in the following way:

Definition 2: the optimal path links a flat blank, of known thickness and unknown boundary, to a part, of known average geometry and unknown thickness, such that this path minimizes the total stored energy at the final stage. Moreover, the final stage is an equilibrium configuration (Fig. 4).

One can define this optimal path as the solution of the following problem:

Problem 3: find $u_{0 \rightarrow T}^{opt}$, defined in $[0, T]$, satisfying:

- the initial value of u^{opt} is zero:

$$u^{opt}(0) = 0;$$

- the final value of u^{opt} is an application from the initial blank to the final part:

$$u^{opt}(T): \mathbb{R}^2 \times] - H/2, H/2[\rightarrow s_F \times \mathbb{R}.$$

- The final stage is an equilibrium state:

$$\forall \delta u, \frac{\partial E}{\partial u}(u^{opt}(T), \alpha^{opt}(T)) \cdot \delta u = 0, \tag{16}$$

such that:

$$W_{0 \rightarrow T}(u_{0 \rightarrow T}^{opt}) = \underset{T}{\text{Min}} W_{0 \rightarrow T}(u_{0 \rightarrow T}), \tag{17}$$

where:

- s_F is a given shape of \mathbb{R}^3 (the average geometry of the final part)
- H is the thickness of the initial flat blank
- T is the set of all the displacement fields linking $\mathbb{R}^2 \times] - H/2, H/2[$ to $s_F \times \mathbb{R}$.

One should not underestimate the difficulties in solving this problem. First, as it is not usual in engineering problems, there are given data and unknown information on the initial and final configurations of the system. Nevertheless, it seems to be logical to start from the final state, for which we know the main information (i.e. the average geometry). Then, two important difficulties have to be solved. The geometry of the final configuration is not completely known and we have to refer to the initial state, in particular to treat the problem

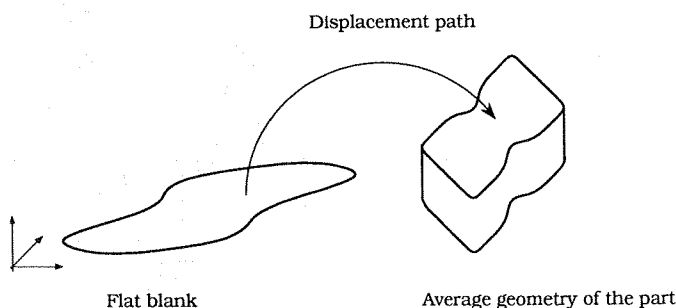


Fig. 4. Geometrical approach of sheet forming process

of the thickness of the structure. In addition, if we start from the final configuration, it is normal to search for the optimal path on an inverse form (as a path from the final configuration s_F to the initial flat blank). Taking into account an incremental law of plasticity in such a way produces some strong difficulties.

For these reasons, we will examine classical assumptions in order to simplify Problem 3.

3.2. Simplified problems

3.2.1. *Shell geometry and membrane strain.* We now present some classical assumptions made in sheet forming process models to simplify the numerical resolution. The first of them is the classical assumption of thin shell for the description of the part geometry. Instead of describing all the geometry of the structure, it is usual to postulate the form of the strain fields throughout the thickness of the structure and then to consider only the average shape of the flat. Thus, we eliminate the aforementioned problem of the unknown through-thickness geometry on the final state of the structure (see Problem 3).

For a very thin structure, one can simplify even more the problem, by using the membrane strain–stress model for shells. This avoids taking into account the distribution of plastic strain along the transversal dimension of the shell.

With this kind of assumption, we can now formulate a simplification for Problem 3, as follows:

Problem 4: find $u_{0 \rightarrow T}^{opt}$, define on $[0, T]$, satisfying:

- the initial value of u^{opt} is zero:

$$u^{opt}(0) = 0;$$

- the final value of u^{opt} is an application from the initial blank to the final part:

$$u^{opt}(T) : R^2 \rightarrow s_F;$$

- the final stage is an equilibrium state:

$$\forall \delta u, \frac{\partial E}{\partial u}(u^{opt}(T), \alpha^{opt}(T)) \cdot \delta u = 0, \tag{18}$$

such as:

$$W_{(0 \rightarrow T)}(u_{0 \rightarrow T}^{opt}) = \text{Min}_T W_{(0 \rightarrow T)}(u_{0 \rightarrow T}), \tag{19}$$

where:

- s_F is a given shape of R^3 (the average geometry of the final part);
- T is the set of all the displacement fields linking R^2 to s_F .

It is clear that for Problem 4 there is no more difficulty in choosing the reference configuration to solve the problem. As mentioned below, the shape of the final configuration s_F is completely defined by its average geometry.

3.2.2. *Rigid plastic behaviour.* One of the most difficult points of solving Problems 3 and 4 is to find displacement fields $u_{0 \rightarrow T}$ satisfying the equilibrium Eqns (16) and (18), and the rate Eqn (10). In fact, a displacement path $u_{0 \rightarrow T}$ belongs to the set T if and only if it is possible to solve the problem of evolution (9), (10) in the following form, well-adapted to our problem.

For a given displacement path $u_{0 \rightarrow T}$:

$$\begin{aligned} \forall t \in [0, T]: \quad & \varepsilon(t) = \varepsilon(u(t)) \\ & W(\alpha(t)) = E(\varepsilon(t), \alpha(t)) \end{aligned} \tag{20}$$

$$\begin{cases} A = -W_{,\alpha}(\alpha) \\ \dot{\alpha} \in \partial_A \varphi^*(\alpha, A) \\ \alpha(0) = \alpha_0. \end{cases} \tag{21}$$

Note the difference between this problem and the evolution problem defined by Eqns (9) and (10). First, the control parameter λ does not appear, because there are no applied forces or imposed displacements as in a classical structure problem. As a logical consequence of this, we do not have to take into account the equilibrium relation, which is replaced by the simple equality (20). For this problem, applied forces will be computed from the internal variable field solution. Such a displacement path will be interesting only if the equilibrium Eqn (18) is satisfied.

On the other hand, it is well-known that in metal forming process modelisation, it is a current assumption not to take into account the elastic behaviour of materials. In the elasto-plastic formalism presented above, we distinguished the measure of the total strains ε from the internal variable field α , which is associated with the irreversible behaviour. In this formalism, the elastic behaviour of materials is associated with variations of ε for α remaining constant. If we decide not to take into account the elastic part of material behaviour, we have to impose a constraint between the two variable fields ε and α . This implies that one does not need to treat variations of ε independently of variations of α . For the elasto-plastic laws treated below, for a given displacement path $u_{0 \rightarrow T}$, the aforementioned relation permits us to compute directly the internal variable path $\alpha_{0 \rightarrow T}$. So, the dissipative variables are simple functions of the strain measure ε or of its time rate of change $\dot{\varepsilon}$. The evolution problem defined by Eqns (20) and (21) is replaced by the simple computation of the internal force A at each time.

3.2.3. Deformation theory of plasticity. This is also an usual assumption in the context of sheet forming process models [7, 15, 16]. This assumption saves a lot of time as it transforms the initial incremental problem to a simple one stage non-linear problem. This approximation gives good results when the real strain path is not really different from the radial path [17].

Making this assumption transforms our optimization incremental approach into a single step problem. We will now show how we can treat this problem in the context of the optimal criterion approach. This assumption will permit us to define what we will call the formability criterion.

4. FORMABILITY CRITERION

Before paying special attention to the single step formulation, let us present two elasto-plastic laws used to perform numerical applications.

4.1. Small strain, finite displacement law of plasticity

We chose to take the Lagrangian configuration as the reference configuration, and we assume complete isotropy for the two materials behaviours presented below. In this case, the strain measure ε is \underline{L} , the Lagrangian strain (where a quantity underlined n times is a tensor of order n):

$$\underline{L}(u) = \frac{1}{2}(\nabla u + {}^t\nabla u + {}^t\nabla u \cdot \nabla u). \quad (22)$$

We now recall for each presented law, simple results by using the formalism of Eqns (3), (4), (7).

4.1.1. Linear kinematic hardening. For the elasto-plastic law with linear kinematic hardening, the internal variable α is \underline{L}^p , the plastic Lagrangian strain. We first recall the two potentials expression:

$$\begin{cases} w(\underline{L}, \underline{L}^p) = \frac{1}{2}(\underline{L} - \underline{L}^p) : \underline{\underline{\Lambda}} : (\underline{L} - \underline{L}^p) + \frac{1}{2} \underline{L}^p : \underline{\underline{H}} : \underline{L}^p \\ \varphi(\underline{L}^p) = k_0 \|\underline{L}^p\| \end{cases} \quad (23)$$

With this choice of the two potentials, we obtain the classical behaviour on the actual configuration [9]:

$$\begin{cases} \underline{D} = \underline{D}^e + \underline{D}^p \\ D^* \underline{\sigma} = \underline{\Lambda} : \underline{D}^e \\ D^* \underline{\beta} = - \underline{H} : \underline{D}^p \\ \underline{D}^p = \lambda \frac{\partial f}{\partial \underline{\sigma}} \end{cases}, \quad (24)$$

where:

- \underline{D} is the symmetric part of the velocity gradient,
- $\underline{\sigma}$ is the Cauchy stress tensor,
- $f = \|\underline{\sigma}' + \underline{\beta}\| - k_0$,
- $\underline{\Lambda}$ and \underline{H} are the elastic modulus tensor and the kinematic hardening modulus tensor,
- $D^* \underline{\sigma}$ is the Jaumann derivative of a second order tensor,

$$D^* \underline{\sigma} = \dot{\underline{\sigma}} - \underline{\Omega} \cdot \underline{\sigma} - \underline{\sigma} \cdot \underline{\Omega},$$

—where $\underline{\Omega}$ is the spin tensor.

4.1.2. *Isotropic hardening.* Here, we describe the internal state of materials by the set of internal variables $\{\underline{L}^p, p\}$, where p is the equivalent plastic strain. If the two potentials are chosen as follows:

$$\begin{cases} w(\underline{L}, \underline{L}^p, p) = \frac{1}{2}(\underline{L} - \underline{L}^p) : \underline{\Lambda} : (\underline{L} - \underline{L}^p) + K(p) \\ \varphi(\underline{L}^p, \dot{p}) = k_0 \|\dot{\underline{L}}^p\| \text{ if } \dot{p} = \|\dot{\underline{L}}^p\| \\ \varphi(\underline{L}^p, \dot{p}) = \infty \text{ if } \dot{p} \neq \|\dot{\underline{L}}^p\| \end{cases} \quad (25)$$

Eqns (3), (4), (7) lead to the following behaviour on the actual configuration:

$$\begin{cases} \underline{D} = \underline{D}^e + \underline{D}^p \\ D^* \underline{\sigma} = \underline{\Lambda}_e : \underline{D}^e \\ \dot{p} = \|\dot{\underline{D}}^p\| \\ \underline{D}^p = \lambda \frac{\partial f}{\partial \underline{\sigma}} \end{cases}, \quad (26)$$

where:

- p is the classical equivalent plastic strain,
- $f = \|\underline{\sigma}'\| + A(p) - k_0$, where $A(p) = -\frac{\partial K}{\partial p}$ defines the isotropic strain-hardening law.

4.2. Radial path assumption

Usually, to perform a single step formulation from an incremental problem, one needs to postulate the form of the stress path and function of the initial and final stage. For example, if the Hencky deformation theory is adopted, the stress tensor at every point is proportional to a tensor independent of time [7, 16]. Here, it is possible to obtain a single step formulation by a simple analysis of the optimization Problem 4. The main difference between the classical approach and our method is that it is possible here to define the radial path as a logical consequence of the optimization problem formulation.

In Problems 3 and 4, the optimal path $u_0^{\text{opt}} \rightarrow_T$ minimizes the total free energy variation. To define a radial path, we now seek to compute a lower bound of the quantity $W_{(0 \rightarrow T)}(u_0^{\text{opt}} \rightarrow_T)$. A simple way to realize this minimization is to proceed in two stages. First, for a given final displacement $u_{0 \rightarrow T}(T)$, search for a lower bound of the quantity $W_{(0 \rightarrow T)}(u_{0 \rightarrow T})$ over all the displacement paths equal to $u_{0 \rightarrow T}(T)$ at the final stage. Then, if we consider now all the final displacement fields that link a flat blank to the final configuration s_F , we can compute the lower bound of the free energy variation. We now perform this analysis for the two material behaviour laws.

4.2.1. *Linear kinematic hardening.* We start this section from Problem 4 with the assumptions of rigid-plastic materials. As aforementioned, we do not have to consider the equilibrium Eqn (18). For elasto-plastic materials with linear kinematic strain-hardening, the rigid-plastic assumption leads to:

$$\underline{L}^p = \underline{L}. \tag{27}$$

It follows from Eqn (27) that for a given final displacement $U(T)$ between a flat surface and the final shape s_F , we can immediately compute the value of the final internal variable field $\alpha(T)$. In this case, it is not necessary to define a real strain path. To find the lower bound W_{\min} we just have to solve the following problem:

Problem 5: find u^{\min} satisfying:

- the displacement field u^{\min} is an application from an unknown initial blank to the final part:

$$u^{\min}: R^2 \rightarrow s_F,$$

such as:

$$W_{\min}(\underline{L}(u^{\min}), \underline{L}(u^{\min})) = \text{Min}_S W(\underline{L}(u), \underline{L}(u)), \tag{28}$$

where:

- s_F is a given shape of R^3 (the average geometry of the final part)
- S is the set of all the displacement fields linking a subset of R^2 to s_F .

4.2.2. *Isotropic hardening.* For this material law, we also have the relation (27). However, because of the second dissipative variable P , it is not possible to proceed as for the kinematic hardening material. The equivalent plastic strain identifies now with the equivalent strain:

$$\|p\| = \|\underline{\dot{L}}^p\| = \|\underline{\dot{L}}\|. \tag{29}$$

If we consider a given displacement field between a flat surface and the final shape, we can not compute the value of P without information about the strain path. Nevertheless, Damamme has shown that it is possible to calculate the minimum of the equivalent strain for all the strain paths which can lead a material element from an initial configuration to a given final configuration [18]. If \underline{F} is the deformation gradient tensor of the considered transformation, we have:

$$P_{\text{dam}} = \sqrt{\frac{1}{4}(\log^2 \lambda + \log^2 \mu + \log^2 \nu)} \tag{30}$$

where λ , μ and ν are the three eigenvalues of the Cauchy strain tensor \underline{C} . The minimizing strain path is achieved when the principal stretch lines are kept constant with respect to the material during deformation and the ratio of principal true strain rates is also kept constant. We can now calculate the lower bound of the free energy, and the associated displacement field, solution of:

Problem 6: find u^{\min} satisfying:

- the displacement field u^{\min} is an application from an unknown initial blank to the final part:

$$u^{\min}: R^2 \rightarrow s_F,$$

such as:

$$W_{\min}(\underline{L}(u^{\min}), \underline{L}(u^{\min}), P_{\text{dam}}(\underline{L}(u^{\min}))) = \text{Min}_S W(\underline{L}(u), \underline{L}(u), P_{\text{dam}}(\underline{L}(u))), \tag{31}$$

where:

- P_{dam} is defined by Eqn (30)
- s_F is a given shape of R^3 (the average geometry of the final part)
- S is the set of all the displacement fields linking a subset of R^2 to s_F .

5. FINITE ELEMENT PROCEDURE

We are now concerned with the numerical resolution of Problems 5 and 6. As aforementioned, we just consider the membrane strain of the part material. On the other hand, the assumed rigid-plastic behaviour leads us to just consider incompressible displacement paths.

5.1. Incompressible shell geometry

For a membrane strain incompressible shell, the Lagrangian strain \underline{L} is:

$$\underline{L} = \frac{1}{2} \begin{pmatrix} \underline{E} & 0 \\ 0 & \frac{\det \underline{A}}{\det \underline{a}} - 1 \end{pmatrix}, \tag{32}$$

where \underline{A} is the metric tensor on the initial flat configuration S_i , \underline{a} is the metric tensor on the final configuration s_F and \underline{E} is the Lagrangian membrane strain [9]:

$$\underline{E} = \frac{1}{2}(\underline{a} - \underline{A}). \tag{33}$$

If we choose as curvilinear coordinates for configuration S_i and configuration s_F , the cartesian coordinates system of the initial flat shape S_i , we have $\underline{A} = \underline{1}$ and:

$$\underline{a} = \begin{pmatrix} \frac{\partial \underline{m}}{\partial X_1} \cdot \frac{\partial \underline{m}}{\partial X_1} & \frac{\partial \underline{m}}{\partial X_1} \cdot \frac{\partial \underline{m}}{\partial X_2} & \frac{\partial \underline{m}}{\partial X_2} \cdot \frac{\partial \underline{m}}{\partial X_1} & \frac{\partial \underline{m}}{\partial X_2} \cdot \frac{\partial \underline{m}}{\partial X_2} \\ \frac{\partial \underline{m}}{\partial X_1} \cdot \frac{\partial \underline{m}}{\partial X_2} & \frac{\partial \underline{m}}{\partial X_2} \cdot \frac{\partial \underline{m}}{\partial X_1} & \frac{\partial \underline{m}}{\partial X_2} \cdot \frac{\partial \underline{m}}{\partial X_2} & \frac{\partial \underline{m}}{\partial X_2} \cdot \frac{\partial \underline{m}}{\partial X_2} \end{pmatrix}, \tag{34}$$

where:

- \underline{m} is a point of the final configuration shape s_F ,
- (X_1, X_2) are cartesian coordinates of the initial flat configuration S_i .

5.2. Energy computations

Then, from the free energy definitions and from Eqns (32)–(34) we can calculate the free energy density.

For linear kinematic hardening law with strain-hardening modulus $\underline{H} = c \cdot \underline{1}$:

$$w = \frac{1}{32} c \left((\underline{a} - \underline{1}) : (\underline{a} - \underline{1}) - \left(\frac{1}{\det \underline{a}} - 1 \right)^2 \right). \tag{35}$$

And for isotropic hardening law:

$$w = K(P_{\text{dam}}), \tag{36}$$

with P_{dam} :

$$P_{\text{dam}} = \sqrt{\frac{1}{4} \left(\log^2 a_I + \log^2 a_{II} + \log^2 \left(\frac{1}{\det \underline{a}} \right) \right)}, \tag{37}$$

where a_I and a_{II} are the two eigenvalues of the metric tensor \underline{a} .

5.3. Global problem resolution

Let S_{ref} denote any flat initial configuration such that one can define a one to one correspondence between S_{ref} and the final shape configuration s_F . We can integrate the free energy density over S_{ref} from Eqns (35) or (36).

$$W = \int_{S_{\text{ref}}} w(\underline{a}(m(X_1, X_2))) \, dS. \tag{38}$$

Let δS be a small variation of the geometric mapping $S_{\text{ref}} \rightarrow s_F$ which leaves s_F unchanged. This small variation can be seen as a small variation of any mapping from S_{ref} to another flat configuration S . We first calculate variation of Eqn (38) with respect to δS :

$$\delta W = \frac{\partial}{\partial X} \left(\int_{S_{\text{ref}}} w(\underline{a}(m(X_1, X_2))) \, dS \right) \cdot \delta S. \tag{39}$$

If the considered mapping is a solution of Problems 5 or 6, then the quantity (39) is zero. Under this form, solutions of Problems 5 and 6 can be searched as a change of variables from S_{ref} to the initial flat configuration S_i .

To discretize these problems, we chose to use linear triangle elements with three nodes and two degrees of freedom by node. The first variation of Eqn (35) is easily computed from Eqns (38) and (39), and the associated minimization problem is solved by a conjugate gradient method [19]. The minimization procedure is started from a given mapping from the initial flat surface S_{ref} and the final configuration s_F . In most cases, we do not need to take into account any boundary condition. If some problems arise during the minimization procedure, it is possible to lock rigid motion of the flat configuration (one rotation and two translations) by imposing three degrees of freedom to be zero. We now present some applications of this method.

6. NUMERICAL RESULTS AND DISCUSSION

To evaluate the accuracy of the method presented here, we solve Problem 5 and 6 for a simple geometry. The final shape is the same square box as studied in Ref. [6]. This box is discretized with 633 nodes and 1216 elements (Fig. 5).

6.1. Kinematic hardening

Kinematic hardening behaviour is considered, with modulus c equal to 1. The computation is carried out in 87 iterations and 520 sec on a SUN 4/670. The initial flat configuration, associated with the optimal solution of Problem 5 is presented in Fig. 6. The distribution of the thickness variation in the final stage is presented in Fig. 7 and the distribution of stored energy in the final stage is presented in Fig. 8.

The thickness distribution in the final configuration associated with the optimal solution is classical (i.e. minimum thickness in the top corner, maximum thickness at the bottom corner of the square box). Note that for this optimal solution, flat parts of the final piece do not store much energy and present small variations of thickness. Conversely, strain and stored energy are localized in the corners of the box. The top of the box undergoes biaxial tensile strain, while the sides and the bottom undergo shear strain.

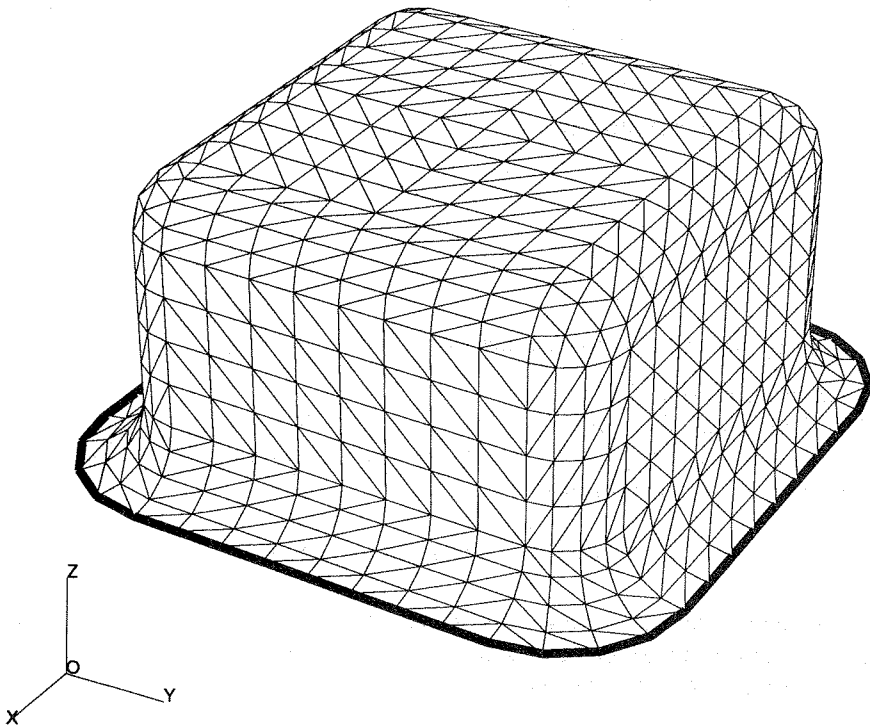


FIG. 5. Mesh of the final shape.

1.241

1.241
1.208
1.174
1.141
1.107
1.074
1.040
1.006
0.9730
0.9394
0.9059
0.8724
0.8388
0.8053
0.7718
0.7382
0.7047
0.6712
0.6376
0.6041

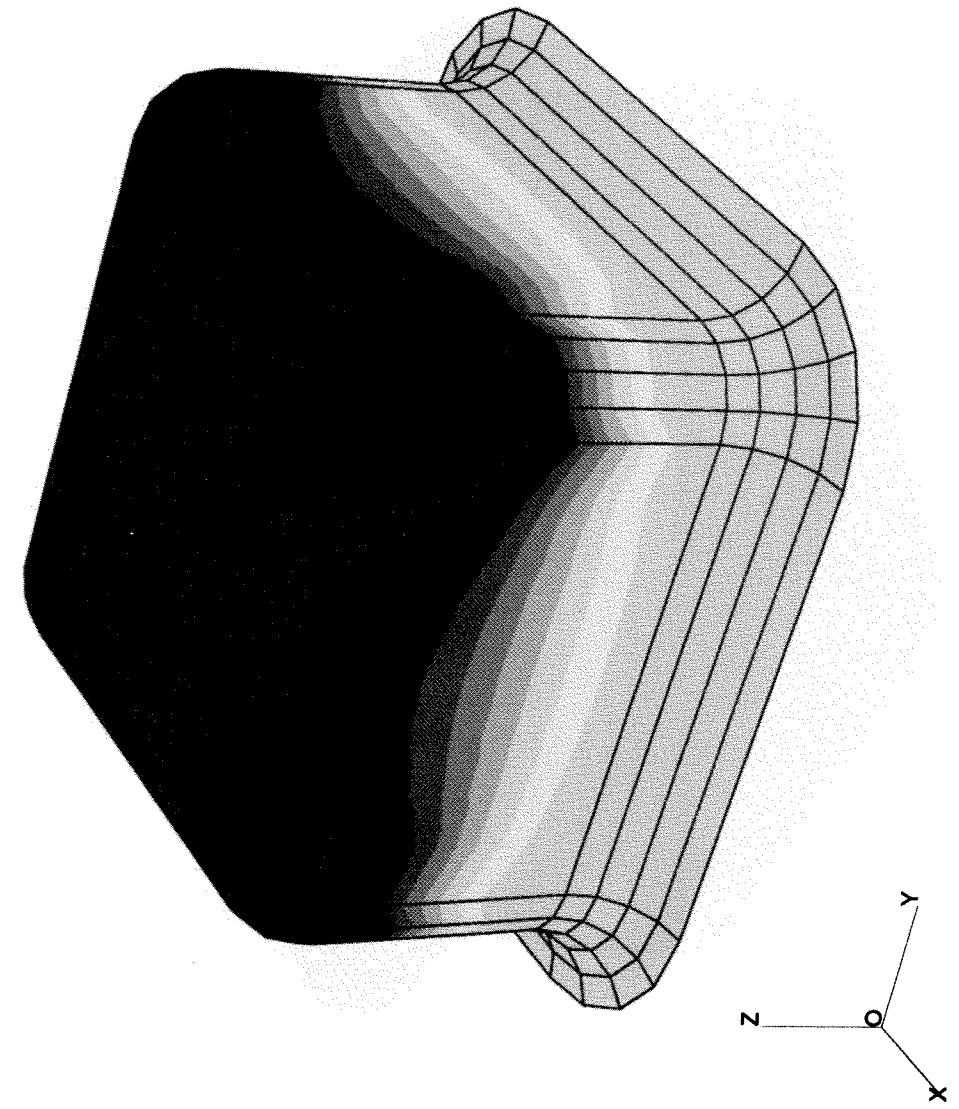
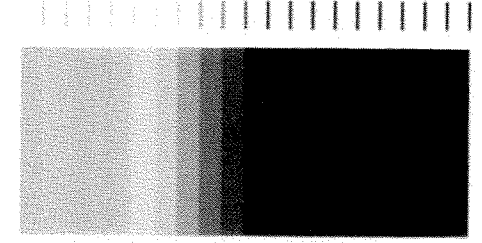


FIG. 7. Thickness variation on the final shape for the optimal solution.

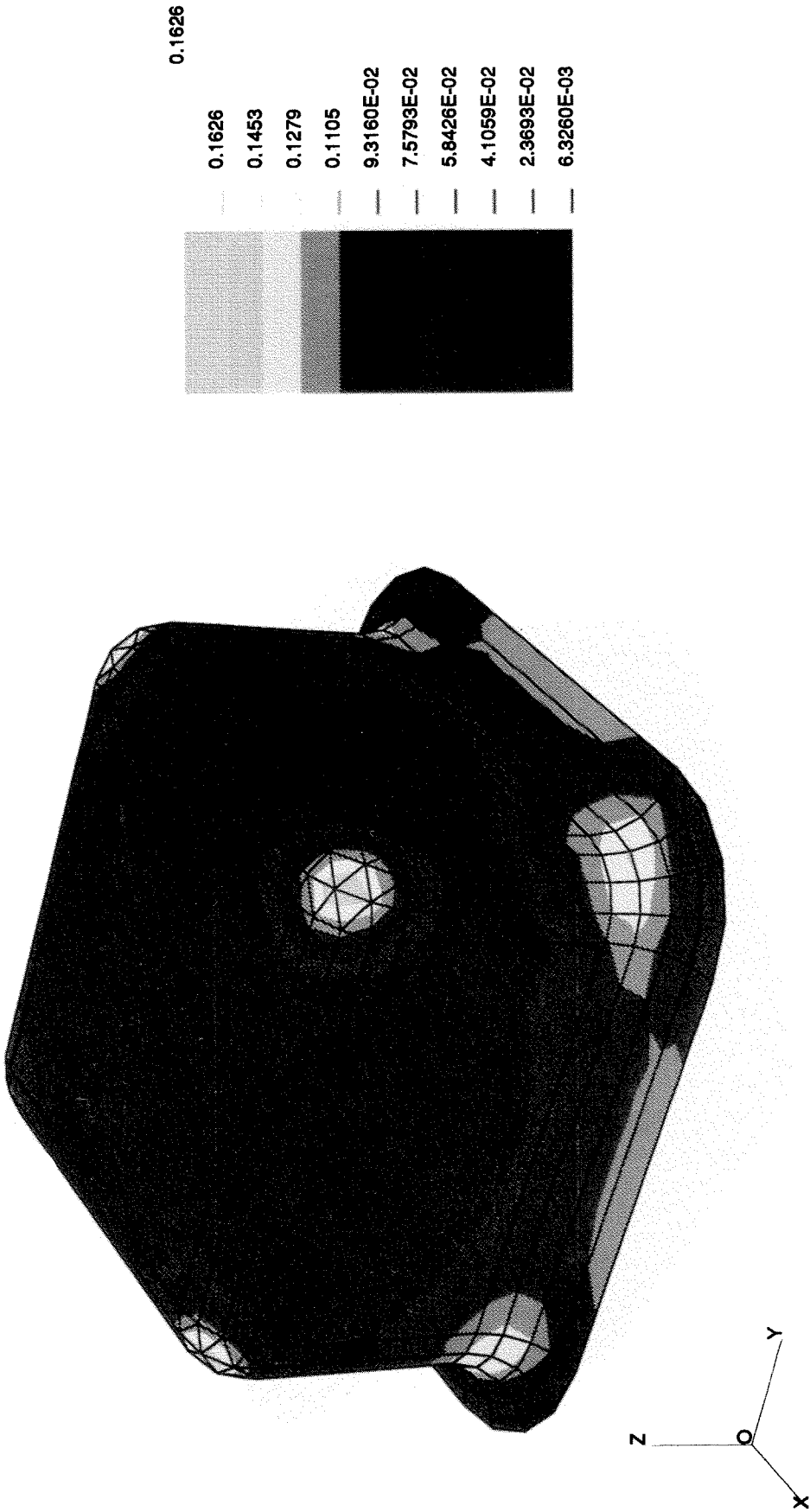


FIG. 8. Energy repartition on the final shape for the optimal solution.

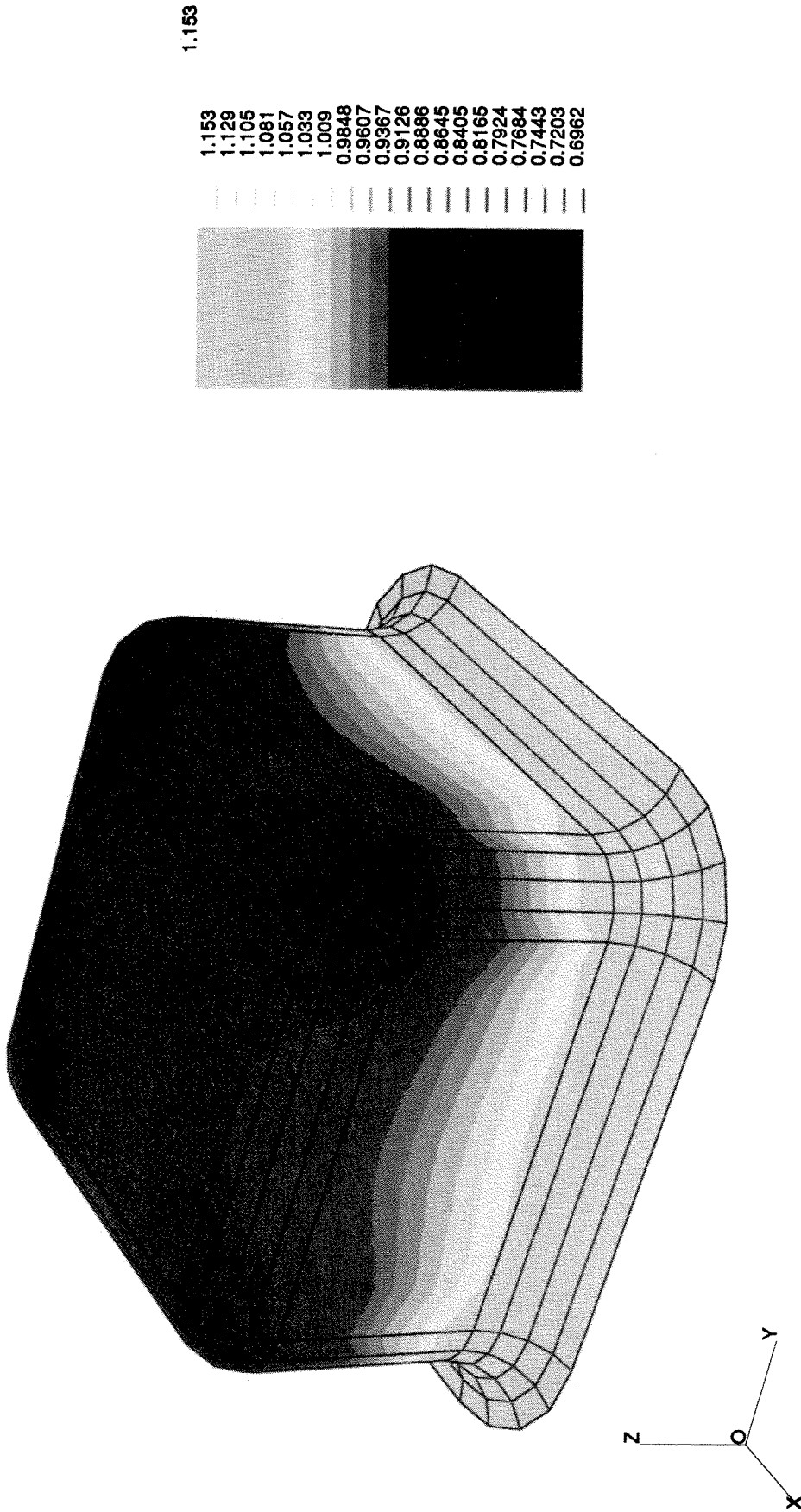


FIG. 10. Thickness variation on the final shape for isotropic hardening.

The maximum eigenvalue of Lagrangian strain is 0.42 and the minimum is -0.31 , corresponding to logarithmic strains in the range of $[-0.49, 0.30]$.

6.2. Isotropic hardening

Isotropic hardening behaviour is then considered with the following expression to define the strain hardening law:

$$K(p) = \frac{1}{2}p^2.$$

Note that this choice of expression for K leads to a constant tangent modulus when logarithmic strain measure is adopted, as in the first calculation performed for the same geometry in Ref. [6]. The computation is carried out in 20 iterations and 230 sec (the optimal solution of Problem 5 is chosen as initial flat configuration to solve this problem). Figure 9 shows the difference between the optimal flat blank obtained in this case and the one obtained when kinematic hardening is considered. The associated distribution in the final stage is presented in Fig. 10.

The thickness distribution and the stored energy distribution are similar, but the magnitudes are different. The thickness variations are smaller in the case of isotropic behaviour. Conversely, the strain magnitude is more important in this case (Lagrangian strain lies between -0.30 and 0.55 , logarithmic strain between -0.48 and 0.37). The maximum stored energy density indicates that the maximum equivalent strain p is 0.59.

In Ref. [6] another method can be found to examine the influence of initial blank by calculation. For the same square box, the authors present three results for three different initial blanks (a square, an octagon and a circle of radius 105 mm). They conclude that the good blank shape for this type of deep drawing is a circle. The optimal initial configurations

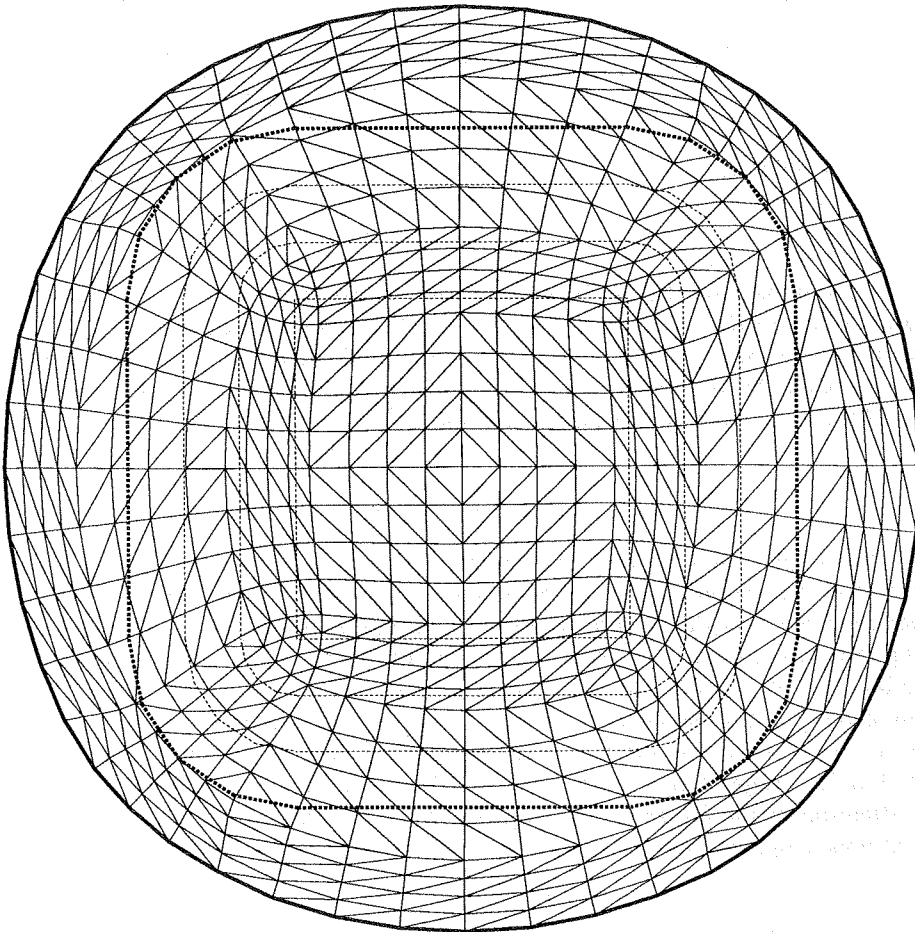


FIG. 6. Optimal flat blank and initial flat configuration of reference (dashed lines).

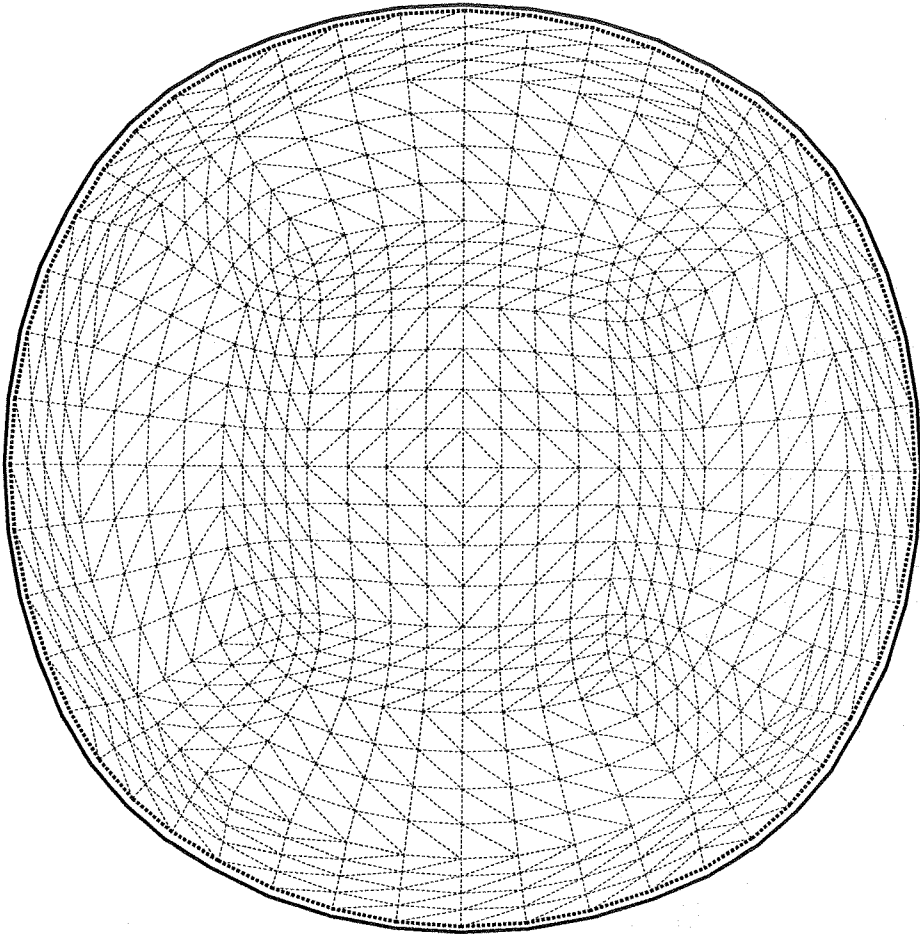


FIG. 9. Optimal flat blank for isotropic hardening (dashed lines) and for kinematic hardening (contour).

found here look like circles (Fig. 9), but the radii are less than 105 mm (83 mm for the first case and 81 mm for the isotropic hardening case). Note that the area of the two optimal flat blanks is approximately equal to the area of the final piece. For the isotropic case, the magnitude of the thickness variation obtained by our method is similar to the one presented in Ref. [6], but the maximum equivalent strain is larger.

7. CONCLUSIONS

A numerical procedure is presented herein to evaluate the formability of sheets obtained by deep drawing operations at the preliminary design stage. The method is based on an energetic approach of the general problem of structural elasto-plastic response. Classical assumptions in the context of sheet forming processes are made in order to simplify the resolution of the general problem and to obtain a single step resolution. The most advantageous point of the method is that it is not necessary to define the initial blank and the geometry of tools to perform calculations. That is why this method could be well adapted to the design of the part. To complete these presented results, we would like to pay more attention in the future to the characterisation of materials behaviour (to take into account anisotropic laws).

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