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Note technique: Throughput Analysis for different power allocation policies with uniform power allocation in a random access channel

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Summary/Introduction:

In this technical note we define a power allocation policy for IoT based user equipment which aims at optimizing the capture effect at the SIC receiver. To benefit from this capture effect we estimate that the power allocation policy is to use a uniform law over a given range of available powers. In this case, we are able to calculate accurate lower bounds for the throughput efficiency in the case of a multiband S-ALOHA based random access channel.

I. Uniform power allocation with SIC capture effect

If we consider the reception of l transmit signals on a given subchannel, we have:

$$y = h_1\sqrt{P_1}s_1 + h_2\sqrt{P_2}s_2 + \dots + h_i\sqrt{P_i}s_i + \dots + h_l\sqrt{P_l}s_l + n$$

If we consider user 1 as the user of interest, we have the corresponding SINR:

$$\text{SINR} = \frac{P_1|h_1|^2}{N_0 + \sum_{j=2}^l P_j|h_j|^2} \quad (1)$$

We will be able to recover packet 1 if the $\log_2(1+\text{SINR})$ is greater than the spectral efficiency of the used modulation format denoted as δ in bits/s/Hz. The probability to decode user 1 is thus equal to:

$$P_{cap}(i, j) = \text{Proba}(\text{SINR} \geq \gamma) = \text{Proba}\left(\frac{P_1|h_1|^2}{N_0 + \sum_{j=2}^l P_j|h_j|^2} \geq \gamma\right) \quad (2)$$

We assume a classical Rayleigh distribution for $|h_i|^2$ that means:

$$p_{|h_i|^2}(x) = e^{-x} \quad (3)$$

In fact, we suppose that real part and imaginary part of h_i follow a Gaussian distribution with mean zero and variance 0.5. In this case, we have:

$$\begin{aligned} & \text{Proba}\left(\frac{P_1|h_1|^2}{N_0 + \sum_{j=2}^l P_j|h_j|^2} \geq \gamma \middle| |h_2|^2, |h_3|^2, \dots, |h_l|^2\right) = \text{Proba}\left(|h_1|^2 \geq \gamma \left(\frac{N_0 + \sum_{j=2}^l P_j|h_j|^2}{P_1}\right) \middle| |h_2|^2, |h_3|^2, \dots, |h_l|^2\right) \\ & = \int_{\frac{N_0 + \sum_{j=2}^l P_j|h_j|^2}{\gamma \left(\frac{N_0 + \sum_{j=2}^l P_j|h_j|^2}{P_1}\right)}}^{+\infty} e^{-x} dx = e^{-\frac{\gamma(N_0 + \sum_{j=2}^l P_j|h_j|^2)}{P_1}} \quad (4) \end{aligned}$$

The most difficult task now is to evaluate the probability mass function or the probability density function of the random variable:

$$R = \sum_{j=2}^l P_j |h_j|^2 \quad (5)$$

To calculate: $f(X) = \text{Proba}(R \leq X)$, we begin with the simplest case $l = 2$, i.e. $R_1 = P|h|^2$, we have :

$$f(X) = \text{Proba}(R_1 \leq X) = \text{Proba}(P|h|^2 \leq X) = \text{Proba}(|h|^2 \leq X/P)$$

We calculate:

$$\text{Proba}(|h|^2 \leq X/P | P) = \int_0^{X/P} e^{-x} dx = [-e^{-x}]_0^{X/P} = 1 - e^{-X/P} \quad (6)$$

Then, we have to average using the p.d.f of P : $p_P(x) = \frac{1}{L}$, we obtain :

$$f(X) = \text{Proba}(|h|^2 \leq X/P) = \int (1 - e^{-XZ}) p_{1/P}(Z) dZ \quad (7)$$

We have to determine the p.d.f: $f(Z) = p_{1/P}(1/P)$ with $Z = 1/P$, we have:

$$p_{1/P}(1/P) \frac{dP}{P^2} = p_P(P) dP \Rightarrow p_{1/P}(1/P) = P^2 p_P(P)$$

$$p_{1/P}(Z) = \frac{1}{Z^2} p_P(1/Z) = \frac{1}{LZ^2}$$

$$f(X) = \frac{1}{1/P_{\min} - 1/P_{\max}} \int_{1/P_{\max}}^{1/P_{\min}} (1 - e^{-XZ}) Z^2 dZ$$

$$f(X) = \frac{1}{L} \int_0^L (1 - e^{-X/u}) du = 1 - \frac{1}{L} \int_0^L e^{-X/u} du$$

We have:

$$\int_0^L e^{-X/u} du = \int_{1/L}^{+\infty} e^{-tX} \frac{dt}{t^2} = X \int_{X/L}^{+\infty} e^{-z} \frac{dz}{z^2} \quad (8)$$

And, this yields:

$$\int_{X/L}^{+\infty} e^{-z} \frac{dz}{z^2} = \left[-\frac{e^{-z}}{z} \right]_{X/L}^{+\infty} - \int_{X/L}^{+\infty} \frac{e^{-z} dz}{z} = L \frac{e^{-X/L}}{X} - E_1(X/L)$$

With: $E_1(X/L) = \int_{X/L}^{+\infty} \frac{e^{-x} dx}{x}$

We obtain eventually:

$$f(X) = \frac{1}{L} \int_0^L (1 - e^{-X/u}) du = 1 - \frac{1}{L} \int_0^L e^{-X/u} du = 1 - \frac{1}{L} \left[X \left(L \frac{e^{-X/L}}{X} - E_1(X/L) \right) \right]$$

$$f(X) = 1 - e^{-X/L} + \frac{X}{L} E_1(X/L)$$

To obtain the p.d.f, of R we have to calculate the derivative function:

$$g(X) = f'(X) = p_R(x) = \frac{1}{L} e^{-X/L} + \frac{X}{L} \left(-\frac{e^{-X/L}}{X/L} \frac{1}{L} \right) + \frac{1}{L} E_1(X/L)$$

$$= \frac{1}{L} e^{-X/L} - \frac{1}{L} e^{-X/L} + \frac{1}{L} E_1(X/L) \quad (9)$$

$$= \frac{1}{L} E_1(X/L)$$

We check that:

$$\int_0^{+\infty} g(X) dX = \frac{1}{L} \int_0^{+\infty} E_1(X/L) dX$$

$$= \frac{1}{L} [X E_1(X/L)]_0^{+\infty} + \frac{1}{L} \int_0^{+\infty} X \frac{e^{-X/L}}{X/L} \frac{dX}{L} \quad (10)$$

$$= \frac{1}{L} \int_0^{+\infty} e^{-X/L} dX = [-e^{-X/L}]_0^{+\infty} = 1$$

To calculate the p.d.f of: $R_2 = P_2 |h_2|^2 + P_3 |h_3|^2$, we have to evaluate the convolutional product:

$$f(X) = \int_{-\infty}^{+\infty} p_R(u) p_R(X-u) du = \frac{1}{L^2} \int_0^X E_1\left(\frac{u}{L}\right) E_1\left(\frac{X-u}{L}\right) du \quad (11)$$

Before developing further (11), we use the following property:

$$\begin{aligned}
 E_1(x-a) &= \int_{x-a}^{+\infty} \frac{e^{-t}}{t} dt = \int_x^{+\infty} \frac{e^{-(u-a)}}{u-a} du = e^a \int_x^{+\infty} \frac{e^{-u}}{u-a} du \\
 E_1(x-a) &= e^a \int_x^{+\infty} \frac{e^{-u}}{u(1-a/u)} du = e^a \int_x^{+\infty} \frac{e^{-u}}{u} \sum_n \frac{a^n}{u^n} du = e^a \sum_n a^n \int_x^{+\infty} \frac{e^{-u}}{u^{n+1}} du
 \end{aligned} \tag{12}$$

Then, we have:

$$\begin{aligned}
 \int_a^{+\infty} \frac{e^{-u}}{u} du &= \left[-\frac{e^{-u}}{u} \right]_a^{+\infty} - \int_a^{+\infty} \frac{e^{-u}}{u^2} du = \frac{e^{-a}}{a} - \int_a^{+\infty} \frac{e^{-u}}{u^2} du \\
 \int_a^{+\infty} \frac{e^{-u}}{u^2} du &= \left[-\frac{e^{-u}}{u^2} \right]_a^{+\infty} - \int_a^{+\infty} \frac{2e^{-u}}{u^3} du = \frac{e^{-a}}{a^2} - 2 \int_a^{+\infty} \frac{e^{-u}}{u^3} du \\
 \int_a^{+\infty} \frac{e^{-u}}{u^3} du &= \left[-\frac{e^{-u}}{u^3} \right]_a^{+\infty} - 3 \int_a^{+\infty} \frac{e^{-u}}{u^4} du = \frac{e^{-a}}{a^3} - 3 \int_a^{+\infty} \frac{e^{-u}}{u^4} du
 \end{aligned}$$

This yields:

$$\begin{aligned}
 \int_a^{+\infty} \frac{e^{-u}}{u} du &= \frac{e^{-a}}{a} - \frac{e^{-a}}{a^2} + 2 \frac{e^{-a}}{a^3} - 3! \int_a^{+\infty} \frac{e^{-u}}{u^4} du = \dots \\
 \dots &= \sum_{n=1}^p \frac{(-1)^{n+1} (n-1)! e^{-a}}{a^n} + (-1)^{p+2} p! \int_a^{+\infty} \frac{e^{-u}}{u^{p+1}} du
 \end{aligned} \tag{13}$$

We eventually obtain:

$$\int_a^{+\infty} \frac{e^{-u}}{u^{p+1}} du = \frac{(-1)^p}{p!} \int_a^{+\infty} \frac{e^{-u}}{u} du + \sum_{n=1}^p \frac{(-1)^{n+p} (n-1)! e^{-a}}{p! a^n} \tag{14}$$

$$E_1(x-a) = e^a \sum_n a^n \int_x^{+\infty} \frac{e^{-u}}{u^{n+1}} du$$

$$E_1(x-a) = e^a \sum_{n=0}^{+\infty} a^n \left[\frac{(-1)^n}{n!} \int_x^{+\infty} \frac{e^{-u}}{u} du + \sum_{m=1}^n \frac{(-1)^{n+m} (m-1)! e^{-x}}{n! x^m} \right]$$

$$E_1(x-a) = e^a \sum_{n=0}^{+\infty} (-1)^n \frac{a^n}{n!} \int_x^{+\infty} \frac{e^{-u}}{u} du + e^a \sum_{n=0}^{+\infty} (-1)^n a^n \sum_{m=1}^n \frac{(-1)^m (m-1)! e^{-x}}{n! x^m}$$

$$E_1(x-a) = e^a \sum_{n=0}^{+\infty} (-1)^n \frac{a^n}{n!} \int_x^{+\infty} \frac{e^{-u}}{u} du + e^{-(x-a)} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \left(\frac{a}{x} \right)^n \sum_{m=1}^n (-1)^m (m-1)!$$

$$E_1(x-a) = E_1(x) + e^{-(x-a)} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \left(\frac{a}{x} \right)^n \sum_{m=1}^n (-1)^m (m-1)! \tag{15}$$

$$E_1(x-a) = E_1(x) + e^{-(x-a)} \sum_{n=0}^{+\infty} \frac{(-1)^n A_n}{n!} \left(\frac{a}{x} \right)^n$$

$$E_1\left(\frac{X-u}{L}\right) = E_1\left(\frac{X}{L}\right) + e^{-\frac{(X-u)}{L}} \sum_{n=0}^{+\infty} (-1)^n \frac{A_n}{n!} \left(\frac{u}{X}\right)^n$$

If we use (11) we obtain: (by denoting: $A_n = \sum_{m=1}^n (-1)^m (m-1)!$)

$$f(X) = \frac{1}{L^2} E_1\left(\frac{X}{L}\right) \int_0^X E_1\left(\frac{u}{L}\right) du + \frac{e^{-\frac{X}{L}}}{L^2} \sum_{n=0}^{+\infty} \frac{(-1)^n A_n}{X^n n!} \int_0^X u^n e^{\frac{u}{L}} E_1\left(\frac{u}{L}\right) du \quad (16)$$

We compute at first:

$$\begin{aligned} \int_0^X E_1\left(\frac{u}{L}\right) du &= L \int_0^{X/L} E_1(t) dt = L [t E_1(t)]_0^{X/L} - L \int_0^{X/L} t \left(-\frac{e^{-t}}{t}\right) dt \\ \int_0^X E_1\left(\frac{u}{L}\right) du &= X E_1\left(\frac{X}{L}\right) + L \int_0^{X/L} e^{-t} dt = X E_1\left(\frac{X}{L}\right) + L(1 - e^{-X/L}) \end{aligned} \quad (17)$$

And then:

$$J_n(X) = \int_0^X u^n e^{\frac{u}{L}} E_1\left(\frac{u}{L}\right) du \quad (18)$$

We have:

$$I_n(X) = \int_0^X u^n e^{\frac{u}{L}} du = L \int_0^{X/L} (Lt)^n e^t dt = L^{n+1} \int_0^{X/L} t^n e^t dt = L^{n+1} K_n(X) \quad (19)$$

And:

$$\begin{aligned} K_n(X) &= \int_0^{X/L} t^n e^t dt = [e^t t^n]_0^{X/L} - n \int_0^{X/L} t^{n-1} e^t dt = \left(\frac{X}{L}\right)^n e^{X/L} - n K_{n-1}(X) \\ K_n(X) &= \left(\frac{X}{L}\right)^n e^{X/L} - n \left[\left(\frac{X}{L}\right)^{n-1} e^{X/L} - (n-1) K_{n-2}(X) \right] \\ K_n(X) &= \left(\frac{X}{L}\right)^n e^{X/L} - n \left(\frac{X}{L}\right)^{n-1} e^{X/L} + n(n-1) K_{n-2}(X) \end{aligned}$$

$$K_1(X) = \int_0^{X/L} te^t dt = [te^t]_0^{X/L} - \int_0^{X/L} e^t dt = \frac{X}{L} e^{\frac{X}{L}} - (e^{\frac{X}{L}} - 1) = \frac{X}{L} e^{\frac{X}{L}} + 1 - e^{\frac{X}{L}}$$

$$\begin{aligned} K_2(X) &= \int_0^{X/L} t^2 e^t dt = [t^2 e^t]_0^{X/L} - 2 \int_0^{X/L} te^t dt = \left(\frac{X}{L}\right)^2 e^{\frac{X}{L}} - 2K_1(X) \\ &= \left(\frac{X}{L}\right)^2 e^{\frac{X}{L}} - 2\left(\frac{X}{L}\right) e^{\frac{X}{L}} - 2\left(1 - e^{\frac{X}{L}}\right) \end{aligned}$$

$$\begin{aligned} K_3(X) &= \int_0^{X/L} t^3 e^t dt = [t^3 e^t]_0^{X/L} - 3 \int_0^{X/L} t^2 e^t dt = \left(\frac{X}{L}\right)^3 e^{\frac{X}{L}} - 3K_2(X) \\ &= \left(\frac{X}{L}\right)^3 e^{\frac{X}{L}} - 3\left[\left(\frac{X}{L}\right)^2 e^{\frac{X}{L}} - 2\left(\frac{X}{L}\right) e^{\frac{X}{L}} - 2\left(1 - e^{\frac{X}{L}}\right)\right] \\ &= \left(\frac{X}{L}\right)^3 e^{\frac{X}{L}} - 3\left(\frac{X}{L}\right)^2 e^{\frac{X}{L}} + 6\left(\frac{X}{L}\right) e^{\frac{X}{L}} + 6\left(1 - e^{\frac{X}{L}}\right) \end{aligned}$$

We deduce:

$$\begin{aligned} K_n(X) &= \left(\frac{X}{L}\right)^n e^{X/L} - n\left(\frac{X}{L}\right)^{n-1} e^{X/L} + n(n-1)\left(\frac{X}{L}\right)^{n-2} e^{X/L} + \\ &\quad \dots + (-1)^k n(n-1)\dots(n-k+1)\left(\frac{X}{L}\right)^{n-k} e^{X/L} + \dots + (-1)^{n+1} n!(1 - e^{X/L}) \end{aligned} \quad (20)$$

$$K_n(X) = \sum_{k=0}^{n-1} (-1)^k n(n-1)\dots(n-k+1) \left(\frac{X}{L}\right)^{n-k} e^{X/L} + (-1)^{n+1} n!(1 - e^{X/L})$$

$$K_n(X) = \sum_{k=0}^{n-1} (-1)^k \frac{n!}{(n-k)!} \left(\frac{X}{L}\right)^{n-k} e^{X/L} + (-1)^{n+1} n!(1 - e^{X/L})$$

To check (20), we verify:

$$K_1(X) = \left(\frac{X}{L}\right) e^{X/L} + 1 - e^{X/L}$$

$$K_2(X) = \left(\frac{X}{L}\right)^2 e^{X/L} - 2\left(\frac{X}{L}\right) e^{X/L} - 2(1 - e^{X/L})$$

$$I_n(X) = L^{n+1} K_n(X) = L^{n+1} \left[\sum_{k=0}^{n-1} (-1)^k \frac{n!}{(n-k)!} \left(\frac{X}{L}\right)^{n-k} e^{X/L} + (-1)^{n+1} n!(1 - e^{X/L}) \right] \quad (21)$$

$$I_n(X) = L \sum_{k=0}^{n-1} (-1)^k \frac{n!}{(n-k)!} X^{n-k} L^k e^{X/L} + (-1)^{n+1} n! L^{n+1} [1 - e^{X/L}]$$

Then, we have to evaluate:

$$J_n(X) = \int_0^X u^n e^{\frac{u}{L}} E_1\left(\frac{u}{L}\right) du = [I_n(u) E_1\left(\frac{u}{L}\right)]_0^X - \int_0^X I_n(u) \left(-\frac{e^{-u/L}}{u} \frac{1}{L} \right) du$$

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + \int_0^X I_n(u) \frac{e^{-u/L}}{u} du$$

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + \int_0^X \left(L \sum_{k=0}^{n-1} (-1)^k \frac{n!}{(n-k)!} u^{n-k} L^k e^{u/L} + (-1)^{n-1} n! L^{n+1} [1 - e^{u/L}] \right) \frac{e^{-u/L}}{u} du$$

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + \sum_{k=0}^{n-1} (-1)^k L^{k+1} \frac{n!}{(n-k)!} \int_0^X u^{n-k-1} du + (-1)^{n-1} n! L^{n+1} \int_0^X \frac{(e^{-u/L} - 1)}{u} du$$

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + \sum_{k=0}^{n-1} (-1)^k L^{k+1} \frac{n!}{(n-k)(n-k)!} X^{n-k} + (-1)^{n-1} n! L^{n+1} \int_0^X \sum_{m=1}^{+\infty} (-1)^m \frac{u^{m-1}}{m! L^m} du$$

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + \sum_{k=0}^{n-1} (-1)^k L^{k+1} \frac{n!}{(n-k)(n-k)!} X^{n-k} + n! L^{n+1} \sum_{m=1}^{+\infty} \frac{(-1)^{n+m-1}}{L^m} \frac{X^m}{m m!}$$

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + \sum_{k=0}^{n-1} (-1)^k L^{k+1} \frac{n!}{(n-k)(n-k)!} X^{n-k} + n! L^{n+1} \sum_{m=1}^{+\infty} \frac{(-1)^{n+m-1}}{L^m} \frac{X^m}{m m!}$$

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + \sum_{m=1}^n (-1)^{n+m} L^{n-m+1} \frac{n!}{m m!} X^m + \sum_{m=1}^{+\infty} (-1)^{n+m-1} L^{n-m+1} \frac{n!}{m m!} X^m$$

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + \sum_{m=n+1}^{+\infty} (-1)^{n+m-1} L^{n-m+1} \frac{n!}{m m!} X^m$$

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + (-1)^n L^n n! \sum_{m=n+1}^{+\infty} (-1)^{m-1} L^{-m+1} \frac{X^m}{m m!} \quad (22)$$

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + (-1)^n L^n n! \sum_{m=n+1}^{+\infty} \frac{(-1)^{m-1}}{L^{m-1}} \frac{X^m}{m m!}$$

It is possible to rewrite it in a more readable manner:

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + (-1)^n L^n n! \sum_{m=n+1}^{+\infty} \frac{(-1)^{m-1}}{L^{m-1}} \frac{X^m}{m m!}$$

$$J_n(X) = I_n(X) E_1\left(\frac{X}{L}\right) + n! X^{n+1} \sum_{k=0}^{+\infty} \frac{(-1)^k}{L^k} \frac{X^k}{(n+1+k)(n+1+k)!} \quad (23)$$

We obtain then:

$$f(X) = \frac{1}{L^2} E_1\left(\frac{X}{L}\right) \int_0^X E_1\left(\frac{u}{L}\right) du + \frac{e^{-\frac{X}{L}}}{L^2} \sum_{n=0}^{+\infty} \frac{(-1)^n A_n}{X^n n!} \int_0^X u^n e^{\frac{u}{L}} E_1\left(\frac{u}{L}\right) du$$

$$f(X) = \frac{1}{L^2} \left[X E_1\left(\frac{X}{L}\right) + L(1 - e^{-X/L}) \right] + \frac{e^{-\frac{X}{L}}}{L^2} \sum_{n=0}^{+\infty} \frac{(-1)^n A_n}{X^n n!} J_n(X)$$

$$f(X) = \frac{1}{L^2} \left[X E_1\left(\frac{X}{L}\right) + L(1 - e^{-X/L}) \right] + \frac{e^{-\frac{X}{L}}}{L^2} \sum_{n=0}^{+\infty} \frac{(-1)^n A_n}{X^n n!} \left[I_n(X) E_1\left(\frac{X}{L}\right) + n! X^{n+1} \sum_{k=0}^{+\infty} \frac{(-1)^k}{L^k} \frac{X^k}{(n+1+k)(n+1+k)!} \right]$$

$$f(X) = \frac{1}{L^2} \left[X E_1\left(\frac{X}{L}\right) + L(1 - e^{-X/L}) \right] + E_1\left(\frac{X}{L}\right) \frac{e^{-\frac{X}{L}}}{L^2} \sum_{n=0}^{+\infty} \frac{(-1)^n A_n I_n(X)}{X^n n!} + \dots$$

$$\dots + \frac{e^{-\frac{X}{L}} X}{L^2} \sum_{n=0}^{+\infty} (-1)^n A_n \sum_{k=0}^{+\infty} \frac{(-1)^k}{L^k} \frac{X^k}{(n+1+k)(n+1+k)!}$$

We have:

$$\begin{aligned} \sum_{n=0}^{+\infty} \frac{(-1)^n A_n I_n(X)}{X^n n!} &= \sum_{n=0}^{+\infty} \frac{(-1)^n A_n}{X^n n!} L \left[\sum_{k=0}^{n-1} (-1)^k \frac{n!}{(n-k)!} X^{n-k} L^k e^{X/L} + (-1)^{n-1} n! L^{n+1} [1 - e^{X/L}] \right] \\ &= \sum_{n=0}^{+\infty} (-1)^n A_n \sum_{k=0}^{n-1} (-1)^k \frac{n!}{(n-k)!} \frac{L^{k+1}}{X^k} e^{X/L} - \sum_{n=0}^{+\infty} \frac{A_n}{X^n} L^{n+2} [1 - e^{X/L}] \end{aligned}$$

And hence:

$$\begin{aligned} f(X) &= \frac{1}{L^2} \left[X E_1\left(\frac{X}{L}\right) + L(1 - e^{-X/L}) \right] + \dots \\ &\dots + E_1\left(\frac{X}{L}\right) \frac{e^{-\frac{X}{L}}}{L^2} \left[\sum_{n=0}^{+\infty} (-1)^n A_n \sum_{k=0}^{n-1} (-1)^k \frac{n!}{(n-k)!} \frac{L^{k+1}}{X^k} e^{X/L} - \sum_{n=0}^{+\infty} \frac{A_n}{X^n} L^{n+2} [1 - e^{X/L}] \right] + \dots \\ &\dots + \frac{e^{-\frac{X}{L}} X}{L^2} \sum_{n=0}^{+\infty} (-1)^n A_n \sum_{k=0}^{+\infty} \frac{(-1)^k}{L^k} \frac{X^k}{(n+1+k)(n+1+k)!} \end{aligned}$$

$$\begin{aligned} f(X) &= \frac{1}{L^2} \left[X E_1\left(\frac{X}{L}\right) + L(1 - e^{-X/L}) \right] + \dots \\ &\dots + E_1\left(\frac{X}{L}\right) \left[\sum_{n=0}^{+\infty} (-1)^n A_n \sum_{k=0}^{n-1} \frac{(-1)^k n! L^{k-1}}{(n-k)! X^k} - \sum_{n=0}^{+\infty} \frac{A_n}{X^n} L^n [e^{-\frac{X}{L}} - 1] \right] + \dots \\ &\dots + \frac{e^{-\frac{X}{L}} X}{L^2} \sum_{n=0}^{+\infty} (-1)^n A_n \sum_{k=0}^{+\infty} \frac{(-1)^k}{L^k} \frac{X^k}{(n+1+k)(n+1+k)!} \end{aligned}$$

We obtain eventually:

$$\begin{aligned}
f(X) &= \frac{1}{L^2} \left[X E_1\left(\frac{X}{L}\right) + L(1 - e^{-X/L}) \right] + \dots \\
&\dots + E_1\left(\frac{X}{L}\right) \left[\sum_{n=0}^{+\infty} (-1)^n A_n \sum_{k=0}^{n-1} \frac{(-1)^k n! L^{k-1}}{(n-k)! X^k} - \sum_{n=0}^{+\infty} \frac{A_n L^n}{X^n} [e^{-\frac{X}{L}} - 1] \right] + \dots \quad (24) \\
&\dots + \frac{e^{-\frac{X}{L}} X}{L^2} \sum_{n=0}^{+\infty} (-1)^n A_n \sum_{k=0}^{+\infty} \frac{(-1)^k}{L^k} \frac{X^k}{(n+1+k)(n+1+k)!}
\end{aligned}$$

With: $A_n = \sum_{m=1}^n (-1)^m (m-1)!$

We will consider here the capture effect taking into account the main following event: if two packets collide over a given frequency resource, we have the following equation:

$$y = h_i \sqrt{P_i} s_i + h_j \sqrt{P_j} s_j + n \quad (25)$$

Considering user i as the user of interest, we will be able to recover packet i if the $\log_2(1+\text{SINR})$ is greater than the spectral efficiency of the used modulation format denoted as δ in bits/s/Hz. The SINR is defined as:

$$\text{SINR} = \frac{P_i |h_i|^2}{N_0 + P_j |h_j|^2} \quad (26)$$

And we have to compare it with the threshold: $\gamma = 2^\delta - 1$. The probability to obtain this capture effect is thus equal to:

$$P_{cap}(i, j) = \text{Proba}(\text{SINR} \geq \gamma) = \text{Proba}\left(\frac{P_i |h_i|^2}{N_0 + P_j |h_j|^2} \geq \gamma\right) \quad (27)$$

We assume a classical Rayleigh distribution for $|h_i|^2$ and $|h_j|^2$ that means:

$$P_{|h_i|^2}(x) = e^{-x} \quad (28)$$

We suppose that real part and imaginary part of h_i follow a Gaussian distribution with mean zero and variance 0.5. In this case, we have:

$$\begin{aligned}
&\text{Proba}\left(\frac{P_i |h_i|^2}{N_0 + P_j |h_j|^2} \geq \gamma \mid P_j |h_j|^2\right) = \text{Proba}(P_i |h_i|^2 \geq \gamma(N_0 + P_j |h_j|^2) \mid P_j |h_j|^2) \\
&= \int_{\gamma(N_0 + P_j |h_j|^2)}^{+\infty} \frac{1}{L} E_1(u/L) du = \int_{\frac{\gamma}{L}(N_0 + P_j |h_j|^2)}^{+\infty} E_1(t) dt = \left[t E_1(t) + 1 - e^{-t} \right]_{\frac{\gamma}{L}(N_0 + P_j |h_j|^2)}^{+\infty} \quad (29) \\
&= e^{-\frac{\gamma}{L}(N_0 + P_j |h_j|^2)} - \frac{\gamma}{L} (N_0 + P_j |h_j|^2) E_1\left(\frac{\gamma}{L} (N_0 + P_j |h_j|^2)\right)
\end{aligned}$$

In fact, the obtained probability is a conditioned probability since we consider a fixed value of $P_j |h_j|^2$ in equation (29). To obtain the complete probability we have to average over the distribution of $P_j |h_j|^2$, this yields:

$$\begin{aligned}
& \text{Proba}\left(\frac{P_i |h_i|^2}{N_0 + P_j |h_j|^2} \geq \gamma\right) \\
&= \int_0^{+\infty} \left[e^{-\frac{\gamma}{L}(N_0+x)} - \frac{\gamma}{L}(N_0+x) E_1\left(\frac{\gamma}{L}(N_0+x)\right) \right] \frac{1}{L} E_1(x/L) dx \\
&= \frac{1}{L} \int_0^{+\infty} e^{-\frac{\gamma}{L}(N_0+x)} E_1(x/L) dx - \frac{\gamma N_0}{L^2} \int_0^{+\infty} E_1\left(\frac{\gamma}{L}(N_0+x)\right) E_1(x/L) dx - \frac{\gamma}{L^2} \int_0^{+\infty} x E_1\left(\frac{\gamma}{L}(N_0+x)\right) E_1(x/L) dx
\end{aligned} \tag{30}$$

We have:

$$\begin{aligned}
U &= \int_0^{+\infty} E_1\left(\frac{\gamma}{L}(N_0+x)\right) E_1(x/L) dx = L \int_0^{+\infty} E_1\left(\frac{\gamma}{L}(N_0+Lu)\right) E_1(u) du = L \int_0^{+\infty} E_1\left(\frac{\gamma}{L}N_0 + \gamma u\right) E_1(u) du \\
&= L \left(\left[[uE_1(u) + 1 - e^{-u}] E_1\left(\frac{\gamma}{L}N_0 + \gamma u\right) \right]_0^{+\infty} + \gamma \int_0^{+\infty} [uE_1(u) + 1 - e^{-u}] \left(\frac{e^{-\left(\frac{\gamma}{L}N_0 + \gamma u\right)}}{\frac{\gamma}{L}N_0 + \gamma u} \right) du \right) \\
&\approx L \left(\left[[uE_1(u) + 1 - e^{-u}] E_1(\gamma u) \right]_0^{+\infty} + e^{-\frac{\gamma}{L}N_0} \int_0^{+\infty} [uE_1(u) + 1 - e^{-u}] \frac{e^{-\gamma u}}{u} du \right) \\
&= L e^{-\frac{\gamma}{L}N_0} \left(\int_0^{+\infty} E_1(u) e^{-\gamma u} du + \int_0^{+\infty} \frac{1 - e^{-u}}{u} e^{-\gamma u} du \right) \tag{31}
\end{aligned}$$

$$\begin{aligned}
& \int_0^{+\infty} E_1(u) e^{-\gamma u} du \\
&= \left[[uE_1(u) + 1 - e^{-u}] e^{-\gamma u} \right]_0^{+\infty} + \gamma \int_0^{+\infty} [uE_1(u) + 1 - e^{-u}] e^{-\gamma u} du \\
&= \gamma \int_0^{+\infty} [uE_1(u) + 1 - e^{-u}] e^{-\gamma u} du \\
&= \gamma \int_0^{+\infty} \left[u \left[-\delta - \ln(u) + \sum_{k=1}^{+\infty} \frac{(-1)^{k+1} u^k}{k k!} \right] + 1 - e^{-u} \right] e^{-\gamma u} du \\
&= -\gamma \delta \int_0^{+\infty} u e^{-\gamma u} du - \gamma \int_0^{+\infty} u \ln(u) e^{-\gamma u} du + \gamma \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{k k!} \int_0^{+\infty} u^{k+1} e^{-\gamma u} du + \gamma \int_0^{+\infty} [1 - e^{-u}] e^{-\gamma u} du \\
&= -\gamma \delta \int_0^{+\infty} u e^{-\gamma u} du - \gamma \int_0^{+\infty} u \ln(u) e^{-\gamma u} du + \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{\gamma^{k+1} k k!} \int_0^{+\infty} t^{k+1} e^{-t} dt + \gamma \int_0^{+\infty} [1 - e^{-u}] e^{-\gamma u} du \\
&= -\gamma \delta \int_0^{+\infty} u e^{-\gamma u} du - \gamma \int_0^{+\infty} u \ln(u) e^{-\gamma u} du + \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{\gamma^{k+1} k k!} (k+1)! + \gamma \int_0^{+\infty} [1 - e^{-u}] e^{-\gamma u} du \\
&= -\gamma \delta \int_0^{+\infty} u e^{-\gamma u} du - \gamma \int_0^{+\infty} u \ln(u) e^{-\gamma u} du + \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{\gamma^{k+1}} \left(1 + \frac{1}{k} \right) + \gamma \int_0^{+\infty} [1 - e^{-u}] e^{-\gamma u} du \quad (32)
\end{aligned}$$

We have:

$$S_k = \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{\gamma^{k+1}} \left(1 + \frac{1}{k} \right) = \frac{1}{\gamma^2} \sum_{k=0}^{+\infty} \frac{(-1)^k}{\gamma^k} - \frac{1}{\gamma} \sum_{k=1}^{+\infty} \frac{(-1)^k}{k \gamma^k}$$

And we have the simple relationship: $\int \frac{(-1)^{k+1} dx}{x^{k+1}} = \frac{(-1)^k}{k x^k}$, this yields :

$$\sum_k \frac{(-1)^k}{k x^k} = \int \sum \frac{(-1)^{k+1} dx}{x^{k+1}} = \int \left(\sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{x^{k+1}} \right) dx = \int \frac{1}{x^2} \left(\sum_{k=0}^{+\infty} \frac{(-1)^k}{x^k} \right) dx = \int \frac{1}{x^2} \frac{1}{1 + \frac{1}{x}} dx$$

$$\sum_k \frac{(-1)^k}{k x^k} = \int \frac{dx}{x(x+1)} = \int \left(\frac{1}{x} - \frac{1}{x+1} \right) dx = -\ln\left(1 + \frac{1}{x}\right)$$

And thus:

$$\begin{aligned}
S_k &= \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{\gamma^{k+1}} \left(1 + \frac{1}{k} \right) = \frac{1}{\gamma^2} \sum_{k=0}^{+\infty} \frac{(-1)^k}{\gamma^k} - \frac{1}{\gamma} \sum_{k=1}^{+\infty} \frac{(-1)^k}{k \gamma^k} \\
S_k &= \frac{1}{\gamma^2} \frac{1}{1 + \frac{1}{\gamma}} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right) \quad (33) \\
S_k &= \frac{1}{\gamma(\gamma+1)} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right)
\end{aligned}$$

We have also:

$$\int_0^{+\infty} ue^{-\gamma u} du = \left[\frac{-u}{\gamma} e^{-\gamma u} \right]_0^{+\infty} - \int_0^{+\infty} \frac{-1}{\gamma} e^{-\gamma u} du = \frac{1}{\gamma} \int_0^{+\infty} e^{-\gamma u} du = \frac{1}{\gamma} \left[-\frac{e^{-\gamma u}}{\gamma} \right]_0^{+\infty} = \frac{1}{\gamma^2} \quad (34)$$

And, thus:

$$\begin{aligned} \int_0^{+\infty} u \ln(u) e^{-\gamma u} du &= \int_0^1 u \ln(u) e^{-\gamma u} du + \int_1^{+\infty} u \ln(u) e^{-\gamma u} du \\ \int_0^{+\infty} u \ln(u) e^{-\gamma u} du &= \int_{-\infty}^0 te^{2t} e^{-\gamma e^t} dt + \int_1^{+\infty} u \ln(u) e^{-\gamma u} du \\ \int_0^{+\infty} u \ln(u) e^{-\gamma u} du &= - \int_0^{+\infty} ue^{-2u} e^{-\gamma e^{-u}} du + \int_1^{+\infty} u \ln(u) e^{-\gamma u} du \end{aligned}$$

We have :

$$\begin{aligned} \int ue^{-\gamma u} du &= -\frac{u}{\gamma} e^{-\gamma u} + \frac{1}{\gamma} \int e^{-\gamma u} du = -\frac{u}{\gamma} e^{-\gamma u} - \frac{1}{\gamma^2} e^{-\gamma u} \\ \int_1^{+\infty} u \ln(u) e^{-\gamma u} du &= \left[\left(-\frac{u}{\gamma} e^{-\gamma u} - \frac{1}{\gamma^2} e^{-\gamma u} \right) \ln(u) \right]_1^{+\infty} + \int_1^{+\infty} \left(\frac{u}{\gamma} e^{-\gamma u} + \frac{1}{\gamma^2} e^{-\gamma u} \right) \frac{du}{u} \\ \int_1^{+\infty} u \ln(u) e^{-\gamma u} du &= \frac{1}{\gamma} \int_1^{+\infty} e^{-\gamma u} du + \frac{1}{\gamma^2} \int_1^{+\infty} \frac{e^{-\gamma u}}{u} du = \frac{1}{\gamma^2} e^{-\gamma} + \frac{1}{\gamma^2} \int_{\gamma}^{+\infty} \frac{e^{-t}}{t} dt = \frac{1}{\gamma^2} e^{-\gamma} + \frac{1}{\gamma^2} E_1(\gamma) \end{aligned} \quad (35)$$

$$\begin{aligned} \int_0^{+\infty} ue^{-2u} e^{-\gamma e^{-u}} du &= \sum_n \frac{(-1)^n \gamma^n}{n!} \int_0^{+\infty} ue^{-2u} e^{-nu} du = \sum_n \frac{(-1)^n \gamma^n}{n!} \int_0^{+\infty} ue^{-(n+2)u} du \\ \int_0^{+\infty} ue^{-2u} e^{-\gamma e^{-u}} du &= \sum_n \frac{(-1)^n \gamma^n}{n!} \left(\left[\frac{-ue^{-(n+2)u}}{n+2} \right]_0^{+\infty} + \frac{1}{n+2} \int_0^{+\infty} e^{-(n+2)u} du \right) \\ \int_0^{+\infty} ue^{-2u} e^{-\gamma e^{-u}} du &= \sum_n \frac{(-1)^n \gamma^n}{n!(n+2)^2} \end{aligned} \quad (36)$$

For the integral $\int_0^{+\infty} [1 - e^{-u}] e^{-\gamma u} du$, we have immediately:

$$\int_0^{+\infty} [1 - e^{-u}] e^{-\gamma u} du = \int_0^{+\infty} e^{-\gamma u} du - \int_0^{+\infty} e^{-(\gamma+1)u} du = \frac{1}{\gamma} - \frac{1}{\gamma+1} \quad (37)$$

Concerning the last integral, we have:

$$\begin{aligned} \int_0^{+\infty} \frac{1 - e^{-u}}{u} e^{-\gamma u} du &= \int_0^{+\infty} \sum_n (-1)^{n+1} \frac{u^{n-1}}{n!} e^{-\gamma u} du \\ &= \sum_n (-1)^{n+1} \int_0^{+\infty} \frac{u^{n-1}}{n!} e^{-\gamma u} du = \sum_n \frac{(-1)^{n+1}}{\gamma^n} \int_0^{+\infty} \frac{t^{n-1}}{n!} e^{-t} dt = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n\gamma^n} \end{aligned} \quad (38)$$

We obtain:

$$\begin{aligned}
& \int_0^{+\infty} E_1(u)e^{-\gamma u} du \\
&= -\gamma\delta \int_0^{+\infty} ue^{-\gamma u} du - \gamma \int_0^{+\infty} u \ln(u)e^{-\gamma u} du + \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{\gamma^{k+1}} \left(1 + \frac{1}{k}\right) + \gamma \int_0^{+\infty} [1 - e^{-u}]e^{-\gamma u} du \\
&= -\gamma\delta \frac{1}{\gamma^2} - \gamma \left[\frac{1}{\gamma^2} e^{-\gamma} + \frac{1}{\gamma^2} E_1(\gamma) - \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^n}{n!(n+2)^2} \right] + \frac{1}{\gamma(\gamma+1)} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right) + \gamma \left[\frac{1}{\gamma} - \frac{1}{\gamma+1} \right] \quad (39) \\
&= -\frac{\delta}{\gamma} - \frac{1}{\gamma} e^{-\gamma} - \frac{1}{\gamma} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^{n+1}}{n!(n+2)^2} + \frac{1}{\gamma(\gamma+1)} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right) + \frac{1}{\gamma+1}
\end{aligned}$$

Where δ is the Euler's constant: $\delta \approx 0,577$ thus:

$$\begin{aligned}
& \int_0^{+\infty} E_1\left(\frac{\gamma}{L}(N_0 + x)\right) E_1(x/L) dx \\
&= Le^{-\frac{\gamma}{L}N_0} \left(\int_0^{+\infty} E_1(u)e^{-\gamma u} du + \int_0^{+\infty} \frac{1-e^{-u}}{u} e^{-\gamma u} du \right) \\
&= Le^{-\frac{\gamma}{L}N_0} \left(-\frac{\delta}{\gamma} - \frac{1}{\gamma} e^{-\gamma} - \frac{1}{\gamma} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^{n+1}}{n!(n+2)^2} + \frac{1}{\gamma(\gamma+1)} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right) + \frac{1}{\gamma+1} + \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n\gamma^n} \right) \\
&= Le^{-\frac{\gamma}{L}N_0} \left(-\frac{\delta}{\gamma} - \frac{1}{\gamma} e^{-\gamma} - \frac{1}{\gamma} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^{n+1}}{n!(n+2)^2} + \frac{1}{\gamma(\gamma+1)} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right) + \frac{1}{\gamma+1} + \ln\left(1 + \frac{1}{\gamma}\right) \right) \\
&= Le^{-\frac{\gamma}{L}N_0} \left(-\frac{\delta}{\gamma} - \frac{1}{\gamma} e^{-\gamma} - \frac{1}{\gamma} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^{n+1}}{n!(n+2)^2} + \frac{1}{\gamma(\gamma+1)} + \left(\frac{1}{\gamma} + 1\right) \ln\left(1 + \frac{1}{\gamma}\right) + \frac{1}{\gamma+1} \right) \\
&= Le^{-\frac{\gamma}{L}N_0} \left(-\frac{\delta}{\gamma} - \frac{1}{\gamma} e^{-\gamma} - \frac{1}{\gamma} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^{n+1}}{n!(n+2)^2} + \frac{1}{\gamma} + \left(\frac{1}{\gamma} + 1\right) \ln\left(1 + \frac{1}{\gamma}\right) \right) \quad (40)
\end{aligned}$$

Concerning the integral form:

$$T = \int_0^{+\infty} x E_1\left(\frac{\gamma}{L}(N_0 + x)\right) E_1(x/L) dx = L^2 \int_0^{+\infty} u E_1\left(\frac{\gamma}{L}(N_0 + uL)\right) E_1(u) du$$

We have:

$$\begin{aligned}
\int_0^{+\infty} u E_1(u) du &= \frac{u^2}{2} E_1(u) + \int \frac{u^2}{2} \frac{e^{-u}}{u} du \\
&= \frac{u^2}{2} E_1(u) + \frac{1}{2} [-ue^{-u} - e^{-u}] + \frac{1}{2} = \frac{u^2}{2} E_1(u) - \frac{1}{2} ue^{-u} + \frac{1}{2} (1 - e^{-u})
\end{aligned}$$

Hence:

$$\begin{aligned}
T &= L^2 \left[\left(\frac{u^2}{2} E_1(u) - \frac{1}{2} u e^{-u} + \frac{1}{2} (1 - e^{-u}) \right) E_1\left(\frac{\gamma}{L} N_0 + \gamma u\right) \right]_0^{+\infty} + \dots \\
&\dots + L^2 \int_0^{+\infty} \left(\frac{u^2}{2} E_1(u) - \frac{1}{2} u e^{-u} + \frac{1}{2} (1 - e^{-u}) \right) \frac{e^{-\left(\frac{\gamma}{L} N_0 + \gamma u\right)}}{\frac{\gamma}{L} N_0 + \gamma u} \gamma du \\
T &\approx L^2 e^{-\frac{\gamma}{L} N_0} \int_0^{+\infty} \left(\frac{u^2}{2} E_1(u) - \frac{1}{2} u e^{-u} + \frac{1}{2} (1 - e^{-u}) \right) \frac{e^{-\gamma u}}{u} du \\
T &= L^2 e^{-\frac{\gamma}{L} N_0} \left[\frac{1}{2} \int_0^{+\infty} u E_1(u) e^{-\gamma u} du - \frac{1}{2} \int_0^{+\infty} e^{-(\gamma+1)u} du + \frac{1}{2} \int_0^{+\infty} \frac{(1 - e^{-u})}{u} e^{-\gamma u} du \right]
\end{aligned}$$

The integral form: $\int_0^{+\infty} u E_1(u) e^{-\gamma u} du$ has been already calculated:

$$\begin{aligned}
&\int_0^{+\infty} u E_1(u) e^{-\gamma u} du \\
&= \int_0^{+\infty} \left[u \left[-\delta - \ln(u) + \sum_{k=1}^{+\infty} \frac{(-1)^{k+1} u^k}{k k!} \right] \right] e^{-\gamma u} du \\
&= -\delta \int_0^{+\infty} u e^{-\gamma u} du - \int_0^{+\infty} u \ln(u) e^{-\gamma u} du + \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{k k!} \int_0^{+\infty} u^{k+1} e^{-\gamma u} du \\
&= -\delta \int_0^{+\infty} u e^{-\gamma u} du - \int_0^{+\infty} u \ln(u) e^{-\gamma u} du + \frac{1}{\gamma} \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{\gamma^{k+1}} \left(1 + \frac{1}{k} \right) \\
&= -\delta - \frac{1}{\gamma^2} e^{-\gamma} - \frac{1}{\gamma^2} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^n}{n!(n+2)^2} + \frac{1}{\gamma} \left(\frac{1}{\gamma(\gamma+1)} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right) \right) \\
&= -\delta - \frac{1}{\gamma^2} e^{-\gamma} - \frac{1}{\gamma^2} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^n}{n!(n+2)^2} + \frac{1}{\gamma^2(\gamma+1)} + \frac{1}{\gamma^2} \ln\left(1 + \frac{1}{\gamma}\right)
\end{aligned} \tag{41}$$

We have:

$$\begin{aligned}
&\int_0^{+\infty} \frac{1 - e^{-u}}{u} e^{-\gamma u} du = \int_0^{+\infty} \sum_n (-1)^{n+1} \frac{u^{n-1}}{n!} e^{-\gamma u} du \\
&= \sum_n (-1)^{n+1} \int_0^{+\infty} \frac{u^{n-1}}{n!} e^{-\gamma u} du = \sum_n \frac{(-1)^{n+1}}{\gamma^n} \int_0^{+\infty} \frac{t^{n-1}}{n!} e^{-t} dt = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n \gamma^n} = \ln\left(1 + \frac{1}{\gamma}\right)
\end{aligned} \tag{42}$$

This yields:

$$T = L^2 e^{-\frac{\gamma}{L} N_0} \left[\frac{1}{2} \int_0^{+\infty} u E_1(u) e^{-\gamma u} du - \frac{1}{2} \int_0^{+\infty} e^{-(\gamma+1)u} du + \frac{1}{2} \int_0^{+\infty} \frac{(1 - e^{-u})}{u} e^{-\gamma u} du \right]$$

$$T = \frac{L^2}{2} e^{-\frac{\gamma}{L} N_0} \left(-\delta - \frac{1}{\gamma^2} e^{-\gamma} - \frac{1}{\gamma^2} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^n}{n!(n+2)^2} + \frac{1}{\gamma^2(\gamma+1)} + \frac{1}{\gamma^2} \ln\left(1 + \frac{1}{\gamma}\right) \right) - \frac{L^2 e^{-\frac{\gamma}{L} N_0}}{2(\gamma+1)} + \dots$$

$$\dots + \frac{L^2 e^{-\frac{\gamma}{L} N_0}}{2} \ln\left(1 + \frac{1}{\gamma}\right) \quad (43)$$

The remaining integral form is:

$$V = \int_0^{+\infty} e^{-\frac{\gamma}{L}(N_0+x)} E_1(x/L) dx = L e^{-\frac{\gamma}{L} N_0} \int_0^{+\infty} E_1(u) e^{-\gamma u} du$$

We have already calculated:

$$\int_0^{+\infty} E_1(u) e^{-\gamma u} du$$

$$= -\gamma \delta \int_0^{+\infty} u e^{-\gamma u} du - \gamma \int_0^{+\infty} u \ln(u) e^{-\gamma u} du + \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{\gamma^{k+1}} \left(1 + \frac{1}{k}\right) + \gamma \int_0^{+\infty} [1 - e^{-u}] e^{-\gamma u} du$$

$$= -\gamma \delta \frac{1}{\gamma^2} - \gamma \left[\frac{1}{\gamma^2} e^{-\gamma} + \frac{1}{\gamma^2} E_1(\gamma) - \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^n}{n!(n+2)^2} \right] + \frac{1}{\gamma(\gamma+1)} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right) + \gamma \left[\frac{1}{\gamma} - \frac{1}{\gamma+1} \right]$$

$$= -\frac{\delta}{\gamma} - \frac{1}{\gamma} e^{-\gamma} - \frac{1}{\gamma} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^{n+1}}{n!(n+2)^2} + \frac{1}{\gamma(\gamma+1)} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right) + \frac{1}{\gamma+1}$$

Then:

$$\text{Proba}\left(\frac{P_i |h_i|^2}{N_0 + P_j |h_j|^2} \geq \gamma\right)$$

$$= \int_0^{+\infty} \left[e^{-\frac{\gamma}{L}(N_0+x)} - \frac{\gamma}{L} (N_0+x) E_1\left(\frac{\gamma}{L}(N_0+x)\right) \right] \frac{1}{L} E_1(x/L) dx$$

$$= \frac{1}{L} \int_0^{+\infty} e^{-\frac{\gamma}{L}(N_0+x)} E_1(x/L) dx - \frac{\gamma N_0}{L^2} \int_0^{+\infty} E_1\left(\frac{\gamma}{L}(N_0+x)\right) E_1(x/L) dx - \frac{\gamma}{L^2} \int_0^{+\infty} x E_1\left(\frac{\gamma}{L}(N_0+x)\right) E_1(x/L) dx$$

$$= e^{-\frac{\gamma}{L} N_0} \left[-\frac{\delta}{\gamma} - \frac{1}{\gamma} e^{-\gamma} - \frac{1}{\gamma} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^{n+1}}{n!(n+2)^2} + \frac{1}{\gamma(\gamma+1)} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right) + \frac{1}{\gamma+1} \right] + \dots$$

$$\dots - \frac{\gamma N_0}{L} e^{-\frac{\gamma}{L} N_0} \left(-\frac{\delta}{\gamma} - \frac{1}{\gamma} e^{-\gamma} - \frac{1}{\gamma} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^{n+1}}{n!(n+2)^2} + \frac{1}{\gamma} + \left(\frac{1}{\gamma} + 1\right) \ln\left(1 + \frac{1}{\gamma}\right) \right) + \dots$$

$$\dots - \frac{\gamma e^{-\frac{\gamma}{L} N_0}}{2} \left[-\frac{\delta}{\gamma^2} - \frac{1}{\gamma^2} e^{-\gamma} - \frac{1}{\gamma^2} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^n}{n!(n+2)^2} + \frac{1}{\gamma^2(\gamma+1)} + \left(\frac{1}{\gamma^2} + 1\right) \ln\left(1 + \frac{1}{\gamma}\right) - \frac{1}{(\gamma+1)} \right]$$

$$\begin{aligned}
&= e^{-\frac{\gamma}{L}N_0} \left[-\frac{\delta}{\gamma} - \frac{1}{\gamma} e^{-\gamma} - \frac{1}{\gamma} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^{n+1}}{n!(n+2)^2} + \frac{1}{\gamma} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right) + \dots \right. \\
&\dots - \frac{\gamma N_0}{L} \left(-\frac{\delta}{\gamma} - \frac{1}{\gamma} e^{-\gamma} - \frac{1}{\gamma} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^{n+1}}{n!(n+2)^2} + \frac{1}{\gamma} + \left(\frac{1}{\gamma} + 1\right) \ln\left(1 + \frac{1}{\gamma}\right) \right) + \dots \\
&\dots \left. - \frac{\gamma}{2} \left(-\frac{\delta}{\gamma^2} - \frac{1}{\gamma^2} e^{-\gamma} - \frac{1}{\gamma^2} E_1(\gamma) + \sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^n}{n!(n+2)^2} + \frac{1}{\gamma^2(\gamma+1)} + \left(\frac{1}{\gamma^2} + 1\right) \ln\left(1 + \frac{1}{\gamma}\right) - \frac{1}{(\gamma+1)} \right) \right]
\end{aligned}$$

To conclude, we obtain:

$$\begin{aligned}
&\text{Proba}\left(\frac{P_i |h_i|^2}{N_0 + P_j |h_j|^2} \geq \gamma\right) \\
&= e^{-\frac{\gamma}{L}N_0} \left(\frac{1}{2} - \frac{\gamma}{L} N_0 \right) \left[\sum_{n=0}^{+\infty} \frac{(-1)^n \gamma^{n+1}}{n!(n+2)^2} - \frac{E_1(\gamma)}{\gamma} - \frac{\delta}{\gamma} - \frac{1}{\gamma} e^{-\gamma} \right] + \dots \quad (44) \\
&\dots + e^{-\frac{\gamma}{L}N_0} \left[\frac{1}{\gamma} + \frac{1}{\gamma} \ln\left(1 + \frac{1}{\gamma}\right) - \frac{\gamma N_0}{L} \left(\frac{1}{\gamma} + \left(\frac{1}{\gamma} + 1\right) \ln\left(1 + \frac{1}{\gamma}\right) \right) - \frac{\gamma}{2} \left(\frac{1}{\gamma^2} + 1\right) \ln\left(1 + \frac{1}{\gamma}\right) - \frac{1}{2} \frac{1-\gamma}{\gamma} \right] \\
&= F(L, \gamma, N_0)
\end{aligned}$$

Then, the throughput efficiency can be calculated in this case as:

$$\eta(M, B) = M \left[\left(1 - \frac{1}{B}\right)^{M-1} + \left(1 - \frac{1}{B}\right)^{M-2} F(L, \gamma, N_0) \right] \quad (45)$$

And, using the Poisson law:

$$\mathbb{E}[\eta(M, B)] = B\lambda e^{-\lambda} + B\lambda e^{-\lambda} F(L, \gamma, N_0) = B\lambda e^{-\lambda} [1 + F(L, \gamma, N_0)] \quad (46)$$