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To cite this version:

Pierre Normandajc Lecanu, Joël Bréard, Dominique Mouazé. Theoretical calculation of the power of wind turbine or tidal turbine. 2019. hal-01982516v7

HAL Id: hal-01982516

https://hal.science/hal-01982516v7

Preprint submitted on 17 Oct 2019 (v7), last revised 21 Jun 2023 (v13)

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Theoretical calculation of the power of wind turbine or tidal turbine

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October 17, 2019

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1 Abstract

The subject of this article is to define the power of a wind turbine or marine current turbine. As it was admitted that the maximum power coefficient for a turbine type wind turbine or tidal, is that defined by Betz. Many people consider that Betz limit is even a stated law.
The research has long been for efficient turbines to optimize the angle of incidence, the shape of the profiles, the shape of the ends of the blade tips, the wakes caused by the turbine, etc. The number of references to these subjects are numerous see PARASCHIVOIU, LI et al. ((2018)), BURTON et al., JAMIESON, PAILLARD et al. ((2013)).
The article presented below is not an optimal search of the power coefficient in order to approach or to exceed the coefficient defined by Betz, but to reformulate the definition of the power coefficient The power factor can be much higher than that defined by Betz, which is presented in this article

2 preliminary

The German mathematician A. Betz has shown that the power of this turbine is(cf. annex 1): see BETZ ((1920))

$$P_k = F\ V\ = [\frac{C_k}{a}\ \frac{1}{2}\ \rho\ S\ V_{fluid}^2]\ V = C_k\ \frac{1}{2}\ \rho\ S\ V_{fluid}^3$$

$$with \quad V\ =\ a\ V_{fluid}$$

P_k the kinetic turbine power,
C_k the kinetic power coefficient,
ρ the fluid density,
V_{fluid} the fluid velocity.
V the fluid velocity at the position of the turbine.

According to the work of Betz, a kinetic energy approach shows that the maximum power coefficient C_T can not exceed a maximum of $\frac{16}{27}$

$$C_{T\ maxi} = C_{p\ Betz} = \frac{16}{27} \qquad C_k \leq C_{T\ maxi} \qquad P_{T\ maxi} = C_{p\ Betz}\ \frac{1}{2}\ \rho\ S\ V_{fluid}^3$$

The theory of the Betz limit is correct, it is based on the calculation of the kinetic energy. The objective to increase the efficiency of the turbine is to transform the potential energy into kinetic.

3 Power calculation

3.1 Stress in turbine blade :

The blades are subjected to stresses due to a thrust force of fluid

$$F_x = C_x \ \frac{1}{2} \ \rho \ S \ V_{fluid}^2$$

The power of this thrust force is equal to

$$P_p = V \ F_x \quad with \quad V \ = \ a \ V_{fluid} \quad and \quad \frac{1}{2} \ \leq \ a \ \leq \ 1$$

(cf. annexe 2)

$$P_p \ = \ a \ C_x \ \frac{1}{2} \ \rho \ S \ V_{fluid}^3$$

$$by\ asking \quad C_p \ = \ a \ C_x \qquad P_p \ = \ C_p \ \frac{1}{2} \ \rho \ S \ V_{fluid}^3$$

with V the fluid velocity at the position of the turbine.

C_p the potential power coefficient,

The stresses are the source of a potential energy because of the position of each internal particles that interact with each other see JAIN and C.
Using piezo-electric materials see BRISSAUD, it is possible to transform these stresses into electric energy.
The objective is rather to transform this potential energy into kinetic mechanical energy

3.2 Total power coefficient :

The potential or kinetic power derived from the potential and kinetic energy respectively.

The kinetic power of the fluid is

$$P_{fluid} = \frac{1}{2} \rho S_{fluid} V_{fluid}^3$$

The power of the turbine is

$$P = \ = \ C_T \ \frac{1}{2} \rho S V_{fluid}^3 \qquad S_{fluid} \ = \ a \ S \qquad P = \ \frac{C_T}{a} \ P_{fluid}$$

C_T Turbine coefficient

variations in kinetic and potential energy vary simultaneously and in opposite directions.(cf. annexe 6)

Thus the power coefficients are equal $C_k \ = \ C_p$

The calculation of the C_k *or* C_p maximum is following

$$\frac{dC_k}{da} = -\frac{dC_p}{da} \qquad a\ (2\ -\ 3\ a)\ =\ 0 \qquad as \quad a\ \geq\ 0.5 \quad a\ =\ \frac{2}{3}$$

The power of the turbine for $a \ = \ \frac{2}{3}$ is

$$P = \ \frac{3}{2}\ C_T\ P_{fluid}$$

The maximum power of the turbine is

$$C_T \ = \ \frac{2}{3}(\approx 0.67) \quad > \quad C_{p\ Betz}(\approx 0.59)$$

The Betz coefficient is in accordance with this inequation.

3.3 Conversion :

In the case of horizontal wind turbines (HAWT, fast wind turbine type), the stresses in the blades for a defined wind speed, are constant.

$$\frac{d\sigma}{d\beta} = 0$$

σ Stress in turbine blade
β rotation angle of the blades.
In fact, some variations of the stresses are existing due to gravitational forces and the differencial velocity within the boundary layer depending on the elevation.

In the case of vertical axis turbines (VAWT, Darrieus type) the blade and arm stresses are depending on the rotation angle of the blades (for a given wind speed).

$$\frac{d\sigma}{d\beta} \neq 0 \qquad \frac{1}{2\ \pi}\int_0^{2\ \pi} \sigma d\beta \ \ = \ \ \epsilon \ \ (\epsilon\ small)$$

During a half-turn, the arms are submitted to compression stresses whereas extending stresses are dominant during the next half-turn.

In the case of a HAWT, it is not possible to convert with a dynamic mechanical system, these stresses into additional energy as the stresses are constant for a given wind speed. Alternative stresses, encountered in a vertical axis wind turbine VAWT can allow the extraction of additionnal energy.

3.4 Coefficient of power of a turbine with a conversion system :

Energy conservation : the total energy remains constant see JOULE
Total energy is equal to the sum of kinetic energy and potential energy.

$$E_T = E_k + E_p$$

The energy is conserved during the transformations in particular to the transformations the resistance of the materials (computation of the stresses) and the kinematics (study of the movements).
The variation of the total energy during the time is zero.

$$\frac{dE_{total}}{dt} = 0 \qquad \frac{dE_{kinetic}}{dt} = -\frac{dE_{potential}}{dt}$$

The variations of energy kinetic and potential vary simultaneously and in opposite sense.

$$E_k = m\,\frac{v^2}{2} \qquad P_k = \frac{dE_{kinetic}}{dt} = \frac{dm}{dt}\frac{v^2}{2} + \frac{1}{2}m\frac{v^2}{dt}$$

$$E_p = m\,\frac{p}{\rho} \qquad P_p = \frac{dE_{potential}}{dt} = \frac{dm}{dt}\frac{p}{\rho} + \frac{m}{\rho}\frac{dp}{dt}$$

$$\frac{dv}{dt} = 0 \qquad \frac{dp}{dt} = 0 \qquad dm = \rho s v dt \qquad P_k = \frac{1}{2}\rho s v^3 \qquad P_p = svp$$

$$\frac{dE_{kinetic}}{dt} = -\frac{dE_{potential}}{dt} \qquad P_k = -P_p \qquad p = -\frac{1}{2}\,\rho\,v^2$$

Pressure variations vary with variations in speed

the pressure difference creates stresses on the turbine. A reaction energy (which results in constraints) is opposed to the potential energy (due to the pressure difference).

The stresses are maximum when the forces due to the kinetic energy are maximum.
The coefficient C_p can not be greater than or equal to C_k.

$$V_w = V_f(2a-1) \qquad V = aV_f \qquad (p_w - p_f) = \frac{1}{2}(V_f^2 - V_w^2) = \frac{1}{2}4a(1-a)V_f^2$$

$$P_p = pSV = apSV_f$$

$$C_k \;=\; 4\ a^2\ (1-a) \qquad C_p \;=\; a\ 4\ a\ (1-a) = \ 4\ a^2\ (1-a)$$

At the power level, taking into account the differential of kinetic energy and the differential of the potential energy, the power coefficient is

$$C_T \;=\; C_k \;+\; C_p$$

3.5 Balance sheet of the powers

In the case of horizontal wind turbines (HAWT, fast wind turbine type), the power coefficient $C_{T\ HAWT}$ is

$$C_{T\ HAWT} \;=\; C_k \;=\; 4\ a^2\ (1-a)$$

In the case of vertical axis turbines (VAWT, Darrieus type) the power coefficient $C_{T\ VAWT\ Darrieus}$ is

$$C_{T\ VAWT\ Darrieus} \;=\; C_k \;=\; 4\ a^2\ (1-a)$$

In the case of vertical axis turbines (VAWT with conversion) with convert these stresses into additional energy, the power coefficient $C_{T\ VAWT\ with\ conversion}$ is

$$C_{T\ VAWT\ with\ conversion} \;=\; C_k \;+\; C_k \;=\; 8\ a^2\ (1-a)$$

At the level of Powers

$$P = \ C_T\ \frac{1}{2}\rho S V_{fluid}^3$$

$$P_{T\ HAWT} = \ 4\ a^2\ (1-a)\ \frac{1}{2}\rho S V_{fluid}^3$$

$$P_{T\ VAWT\ Darrieus} = \ 4\ a^2\ (1-a)\ \frac{1}{2}\rho S V_{fluid}^3$$

$$P_{T\ VAWT\ with\ conversion} = \ 8\ a^2\ (1-a)\ \frac{1}{2}\rho S V_{fluid}^3$$

3.6 HAWT-VAWT comparison

Following the work of E.Hau , the power coefficient of different turbines is compared see HAU ((2000))

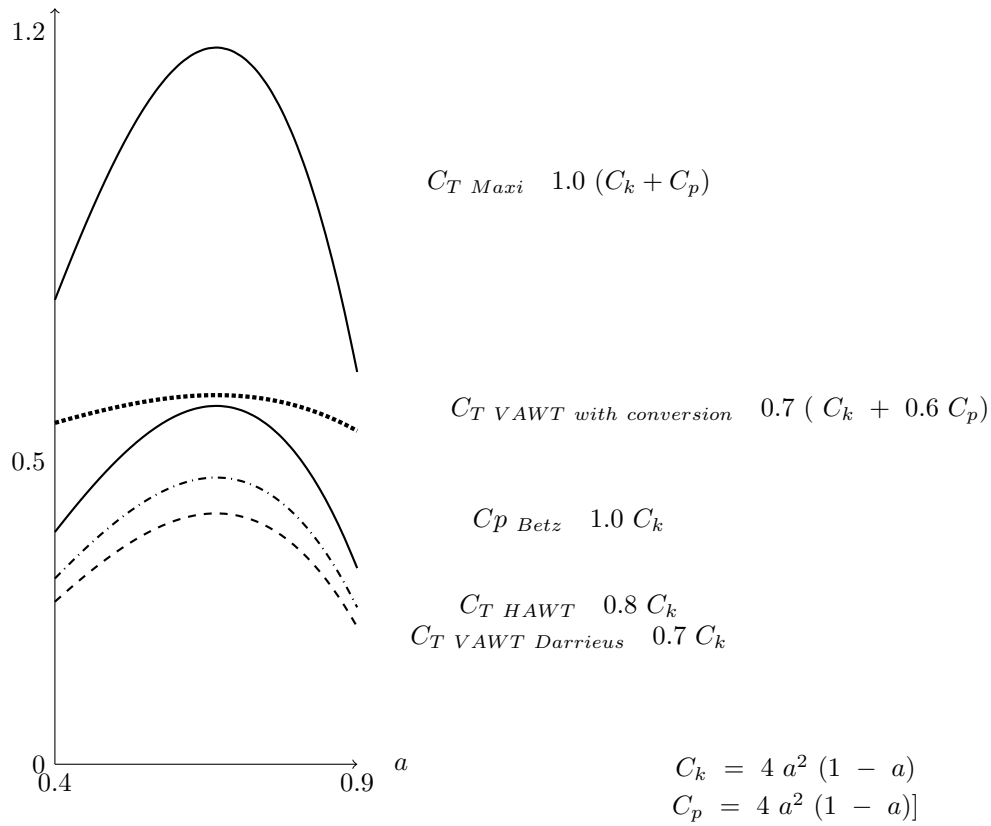
(A performance of 0.6 is applied for the supplementary energy recovery system.)

$$With\ a = \frac{2}{3} \qquad C_k \approx 60\% \qquad C_p \approx 60\%$$

case	Coef.	HAWT	VAWT(Darrieus)	VAWT(with conversion)
$perfect$	C_k	60%	60%	60%
$perfect$	C_p *with conversion*	0%	0%	60%
$perfect$	C_T	60%	60%	120%
in practice	C_k	$0.8 \times 60\% \approx 48\%$	$0.7 \times 60\% \approx 42\%$	$0.7 \times 60\% \approx 42\%$
in practice	C_p *with conversion*	0%	0%	$0.6 \times 0.7 \times 60\% \approx 25\%$
in practice	C_T	48%	42%	67%
.....				
gain / HAWT		+ 0%	−12 %	+ 39%
gain / $C_k(Betz)$		− 20%	−30 %	+ 11%

$$With\ a \approx 0.8 \qquad C_k \approx 50\% \qquad C_p \approx 50\%$$

case	Coef.	HAWT	VAWT(Darrieus)	VAWT(with conversion)
in practice	C_k	$0.8 \times 50\% \approx 40\%$	$0.7 \times 50\% \approx 35\%$	$0.7 \times 50\% \approx 35\%$
in practice	C_p *with conversion*	0%	0%	$0.6 \times 0.7 \times 50\% \approx 21\%$
in practice	C_T	40%	35%	56%
.....				
gain / HAWT		+ 0%	−12 %	+ 27%
gain / C_k		− 20%	−30 %	+ 12%



4 Project - Active Lift Turbine :

The project "Active lift turbine" is an transformation' example of potential energy into kinetic energy.

$$C_k = \frac{9\pi}{2^7}b^3 - \frac{2^3}{3}b^2 + \frac{\pi}{2}b$$

$$C_p = \frac{e}{R}\lambda(\frac{9\pi}{2^7}b^3 - \frac{2^2}{3}b^2 + \frac{\pi}{2}b)$$

$$with \quad b = \sigma\lambda$$

σ stiffness coefficient
λ Tip speed ratio
e eccentric distance
R Turbine radius
See preprint : Simplified theory of an active lift turbine with controlled displacement see LECANU et al. ((2016)) LECANU et al. ((2018)).

5 Synthesis :

With a mechanical conversion sytem, the powers for the different turbines are

$$P_{T\ HAWT} = \ 4\ a^2\ (1-a)\ \frac{1}{2}\rho S V_{fluid}^3$$

$$P_{T\ VAWT\ Darrieus} = \ 4\ a^2\ (1-a)\ \frac{1}{2}\rho S V_{fluid}^3$$

$$P_{T\ VAWT\ with\ conversion} = \ 8\ a^2\ (1-a)\ \frac{1}{2}\rho S V_{fluid}^3$$

Compared to a HAWT turbine, the gain of a VAWT Turbine with an energy recovery system is in practice from 20% to 50%.

For a vertical axis turbine with a conversion system, the power factor is higher than that defined by betz.
In the comparative table, a yield of 0.6 was chosen for the mechanical conversion system of the potential energy into mechanical energy. By choosing a powerful technology, this yield can be greatly increased, which will increase the performance of the turbine.

The definition of the maximum power coefficient is that established by Betz remains valid for the horizontal axis turbine HAWT and no longer makes sense for vertical axis turbines VAWT.

The results given are examined a conversion of the potential energy into kinetic energy through a mechanical system which is not applicable for horizontal axis turbines HAWT.

The calculation of the powers are the sum of the powers which one wants to consider.

The maximum power coefficient for a wind turbine or tidal turbine is

$$C_{T\ maxi} = \ \frac{32}{27}\ (\approx\ 118.5\%)$$

$C_{T\ maxi}$ is a limit value with $C_p\ \leq\ C_k$.

The coefficient is greater than 1 because Betz defined its coefficient from only the kinetic power of the fluid.

Using piezo-electric materials as an example, it is possible to transform potential energy into electrical energy.
Then the power coefficient is $C_T\ =\ C_k\ +\ C_p$ whatever the type of turbine.

6 Conclusion :

Betz had the maximum power coefficient $C_{p\ Betz}(= \frac{16}{27})$ of a wind turbine or tidal turbine from the calculation of kinetic energy.

Taking into account the kinetic energy and the potential energy, the coefficient of maximum power becomes $C_{T\ maxi}(= \frac{32}{27})$:

Transforming potential energy into kinetic energy greatly increases turbine performance.

$$P_{T\ maxi} = C_{T\ maxi}\ \frac{1}{2}\ \rho\ S\ V_{fluid}^{3} \quad with \quad C_{T\ maxi} = \frac{32}{27}$$

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Maximum wind power (from kinetic energy)

$$E_k = \frac{1}{2}\, m\, V^2$$

$$\frac{dE_k}{dt} = \frac{1}{2}\,\frac{dm}{dt}\,V^2 + \frac{1}{2}\,m\,\frac{dV^2}{dt} \qquad \frac{dV}{dt} = 0$$

$$\frac{dE_k}{dt} = \frac{1}{2}\,\dot{m}\,V^2 = \frac{1}{2}\rho\, S\, V^3$$

V Wind speed at the turbine level

Force applied by the wind on the rotor

$$F = m\frac{dV}{dt} = \dot{m}\Delta V \;=\; \rho SV\,(V_{fluid} \;-\; V_{wake})$$

V_{wake} **streamwise velocity in the far wake**

$$P = FV \;=\; \rho SV^2\,(V_{fluid} \;-\; V_{wake})$$

$$P = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2}mV_{fluid}^2 - \frac{1}{2}mV_{wake}^2}{\Delta t}$$

$$P = \frac{\Delta E}{\Delta t} \;=\; \frac{1}{2}\dot{m}(V_{fluid}^2 \;-\; V_{wake}^2) = \frac{1}{2}\rho SV(V_{fluid}^2 \;-\; V_{wake}^2)$$

From theses equalities

$$V = \bar{V} = \frac{V_{fluid} + V_{wake}}{2}$$

$$F = \;\rho SV\,(V_{fluid} \;-\; V_{wake}) \;=\; \frac{1}{2}\rho S(V_{fluid}^2 \;-\; V_{wake}^2)$$

$$P = FV \;=\; \rho SV^2\,(V_{fluid} \;-\; V_{wake})$$

defining $a = \frac{V}{V_{fluid}}$

$$V_{wake} \;=\; V_{fluid}\;(2\;a \;-\; 1) \quad as \quad V_{wake} \geq 0 \quad a \geq \frac{1}{2}$$

$$P = \;\; 4\;a^2\;(1-a)\;\frac{1}{2}\;\rho SV_{fluid}^3$$

defining power coefficient $C_k = \frac{P}{\frac{1}{2}\rho SV_{fluid}^3} \;=\; 4\;a^2\;(1-a)$

Search of maximum power coefficient

$$\frac{dC_k}{da} = 0 \quad a\;(2 \;-\; 3\;a) \;=\; 0 \qquad a = 0 \qquad or \qquad a = \frac{2}{3}$$

$$a = \frac{2}{3} \qquad C_k = \frac{16}{27} = 0.593$$

The maximum power coefficient C_{kmaxi} is defined by Betz

$$C_{kmaxi} \;=\; C_{pBetz} = \frac{16}{27} \approx 60\%$$

The maximum power of the fluid is

$$P_{fluid} = \frac{1}{2}\rho S_{fluid} V_{fluid}^3$$

$S_{fluid} \;=\; \frac{S\ V}{V_{fluid}} \;=\; a\ S$

The power of the turbine is

$$P = \frac{C_k}{a}\ P_{fluid} \;=\; C_k\ \frac{1}{2}\rho \frac{S_{fluid}}{a} V_{fluid}^3 \;=\; C_k\ \frac{1}{2}\rho S V_{fluid}^3$$

The maximum power of the turbine is

$$P_{max} = \frac{C_{p\ Betz}}{\frac{2}{3}}\ P_{fluid} \;=\; \frac{8}{9}\ P_{fluid} \;=\; C_{p\ Betz}\ \frac{1}{2}\rho S V_{fluid}^3 \;=\; \frac{16}{27}\ (\frac{1}{2}\rho S V_{fluid}^3)$$

$$P_{max} \;=\; C_{p\ Betz}\ \frac{1}{2}\rho S V_{fluid}^3 \tag{1}$$

Additional recovery power (from potential energy)

The fluid creates stresses in the blade. They are due to thrust force. The energy of this force is

$$E_p \;=\; m\;\frac{f_s}{\rho}$$

f_s **thrust force**
For HAWT horizontal wind turbines (fast wind turbine type), the thrust force F_s are constant.

$$\frac{dE_p}{dt} \;=\; 0$$

For a VAWT, the thrust force depends on the time or the rotation angle $F_s(t)$ **or** $F_s(\beta)$ $\quad \beta = \omega\; t$
$\omega \;=\; \frac{d\dot{\beta}}{dt}$ **angular frequency**

$$\frac{dE_p}{dt} \;\neq\; 0 \quad and \quad \frac{1}{2\;\pi}\int_0^{2\;\pi} E_p(\beta)d\beta \;=\; \epsilon \;\; (\epsilon\; small)$$

As

$$F_s \;=\; f_s S \;=\; C_x\frac{1}{2}\;\rho\;S\;V_{fluid}^2$$

$$E_p \;=\; m\;C_x\frac{1}{2}\;V_{fluid}^2$$

V **fluid speed at the level turbine**
The power is

$$P_p \;=\; \frac{dE_p}{dt} \quad dm = \rho S V dt$$

$$P_p \;=\; \frac{dE_p}{dt} \;=\; \frac{dm}{dt}\;C_x\;\frac{1}{2}\;S\;V_{fluid}^2 \;+\; m\;C_x\frac{1}{2}\;S\;\frac{dV_{fluid}^2}{dt} \qquad \frac{dV_{fluid}}{dt} \;=\; 0$$

$$P_p \;=\; a\;C_x\;\frac{1}{2}\;\rho\;S\;V_{fluid}^3 \quad with\;a = \frac{V}{V_{fluid}} \tag{2}$$

$$\frac{1}{2\;\pi}\int_0^{2\;\pi} E_p(\beta)d\beta \;=\; \epsilon \;\; (\epsilon\; small) \quad E_{p-max} \;\approx\; -E_{p-min}$$

the power depends on a potential energy difference

$$P_p \;=\; \frac{\Delta E_p}{\Delta t} \quad T \;=\; \frac{2\;\pi}{\omega} \quad P_p \;\leq\; \frac{E_{p-max} - E_{p-min}}{T} \quad P_p \;\leq\; \frac{E_{p-max}}{\pi}\;\omega$$

for a half-turn

$$E_p(\beta)\ R\ d\beta\ =\ dE_p\ \pi\ R$$

in particular

$$E_{p-max}\ =\ \frac{dE_p}{d\beta}\pi\ =\ \frac{dE_p}{dt}\ \frac{\pi}{\omega}$$

As

$$\frac{dE_p}{dt}\ =\ a\ C_x\ \frac{1}{2}\ \rho\ S\ V_{fluid}^3 \quad and \quad P_p\ \leq\ \frac{E_{p-max}}{\pi}\ \omega$$

$$P_p\ \leq\ a\ C_x\ \frac{1}{2}\ \rho\ S\ V_{fluid}^3$$

Variation of energy in opposite sense

Along a streamline, the Bernoulli's equation is see BERNOULLI ((1738))

$$\frac{p}{\rho} + \frac{v^2}{2} = constant \quad with \quad z = 0$$

By multiplying by m

$$m\,\frac{p}{\rho} + m\,\frac{v^2}{2} = constant$$

The differential of this equation is

$$d(\frac{1}{2}\,m\,v^2) = -\,d(m\,\frac{p}{\rho}) \tag{3}$$

the variations of energy vary simultaneously and in opposite sense.

$$As \quad \frac{dV}{dt} = 0 \quad and \quad \frac{dp}{dt} = 0 \quad p = -\frac{1}{2}\,\rho\,v^2$$