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► **To cite this version:**

Zhuocheng Li, Tuong-Bach Nguyen, Quentin Lampin, Isabelle Sivignon, Steeve Zozor. Ensuring k -coverage in low-power wide area networks for Internet of Things. 2016. hal-01982357v1

HAL Id: hal-01982357

<https://hal.science/hal-01982357v1>

Preprint submitted on 14 Aug 2016 (v1), last revised 15 Jan 2019 (v3)

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Ensuring k -coverage in low-power wide area networks for Internet of Things

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Abstract—Low-Power Wide Area Networks (LPWANs) promise to enable nation-wide and low-power connectivity to Internet of Things (IoT) devices. While transmission ranges meet expectations, concerns on the packet delivery rates are voiced. Receiver-side diversity is often thought as a solution to improve on delivery rates, as well as an enabler to the geolocation of devices. In this context, we study the infrastructure density required to achieve k -coverage of the IoT devices, which precludes k -reception. To that purpose, we make use of k -order Voronoi diagram to compute an estimate of the gateway density in reference deployment setups: rural, urban and dense urban networks. We thus provide insights on the feasibility of a geolocation-enabled deployment in those scenarios.

I. INTRODUCTION

Trading higher bit-rates for better link-budgets, LPWANs provide long range wireless connectivity to IoT devices. Such networks provide a promising alternative to traditional cellular or multi-hop networks. LPWANs are indeed envisioned to provide nation-wide connectivity over industrial, scientific and medical (ISM) frequencies to battery-powered IoT devices that require to transmit little amount of data over long periods of time, *e.g.*, water & gas meters. As stated in European Telecommunications Standards Institute (ETSI) requirements for smart city applications [1], typical use cases exhibit stringent Quality of Service (QoS) requirements, *e.g.*, high packet delivery ratio, which pose a challenge to LPWANs such as [2, 3] that mostly rely on random, uncoordinated access schemes. Those requirements, as well as harsh propagation environments, *e.g.*, meters deploying in water pits, motivate the use of redundancy mechanisms at the emitter side, *i.e.*, retransmissions, and/or the receiver side, *i.e.*, multiple-reception or receiver diversity. Unlike retransmission mechanisms that apply a higher pressure on the medium usage, receiver diversity comes at virtually no cost for IoT devices and medium usage. However, receiver diversity requires a higher infrastructure density and thus higher deployment costs.

In this context, we investigate the dimensioning of the gateway density to ensure k -coverage, *i.e.*, k gateways in communication range of each IoT device. With the number of IoT devices being huge, we consider k -covering the network region of IoT devices. To this purpose, we introduce some notions based on k -order Voronoi diagram to devise the inter-gateway distance and gateway density in three regular deployment patterns: triangular, square and hexagonal. We

express the inter-gateway distance and gateway density as a function of the target coverage degree k and communication range r .

Our results provide an estimate of the infrastructure density required in real-world use cases. To this purpose, we apply our results to three reference IoT scenarios: a dense urban setup, an urban setup and a rural setup. Our estimate is rough in the sense that we only consider the factor of k -coverage. As we will see, the k -coverage is not the primary dimensioning factor in some use cases. Capacity of gateways should also be taken into account. We further provide insights on the feasibility of LPWANs in those scenarios, by considering both dimensioning factors, *i.e.*, capacity and k -coverage.

The paper is organized as follows. In section II, we provide a critical view of methodologies and results of related works in the domain of wireless sensor networks (WSNs). In section III, we introduce our problem setting and essential definitions before proposing our methodology in section IV. Section V is finally dedicated to the numerical application of our methodology on three network dimensioning use cases.

II. RELATED WORKS

k -order Voronoi diagram and k -coverage in WSNs are both well-studied subjects in both the geometry and the network literatures. Without pretending to be exhaustive on the matter, we present works whose methodology is similar to that of this paper. WSNs have been first thought to be a viable solution to monitor physical events, be it for military or civil surveillance applications. From this context arose the concept of sensing coverage, *i.e.*, ensuring the detection of events in a monitored area. The requirement of coverage degree k comes from factors such as detection precision. In [4–9], a dense network of randomly scattered sensors is assumed and the authors propose algorithms, either distributed or centralized, to compute minimal sets of active nodes to ensure a complete k -coverage of the area of interest while maximizing the network lifetime. While the coverage objective is similar to covering an area of sensor nodes with gateways, such algorithms are ill-suited to compute the minimal spatial density of an infrastructure tailored to ensure k -coverage.

Pattern-based deployments have the advantage of guaranteeing a regular inter-site distance. It's especially adapted for planned infrastructure deployment. In [10], by combining the

notion of grid coverage degree, and that of k -perimeter coverage, a sufficient yet not necessary condition for k -coverage in regular deployment patterns is given. In this article, we propose a different methodology based on k -order Voronoi diagram that gives the necessary and sufficient condition for k -coverage, thus resulting in minimal spatial densities of regular deployments. Our results confirm those obtained in [11, 12] in terms of optimal 1 and 2-coverage deployment patterns, and can give some insights for the more general case $k \geq 3$.

k -order Voronoi diagram has been used to study distributed and randomized k -coverage issues in WSNs [13, 14]. However, pattern-based deployments is more adapted for infrastructure deployment and requires probably smaller density, we apply the notion to regular patterns and find the optimal k -coverage pattern as well as an overview of the deployment cost for achieving k -coverage.

III. PROBLEM FORMULATION

As in [10], we choose 3 regular patterns coming from the three possible edge-to-edge tilings of the 2D plane by mutually congruent regular square, triangle and hexagon polygons [15] to model the gateway deployment. The vertices of the polyons represent the position of gateways, as shown in figure 1. As the deployment thus obtained is spatially periodical, a single period is shown. We first introduce several definitions necessary to our problem formulation.

We denote the Euclidean distance between two points $p, q \in \mathbb{R}^2$ by $d(p, q)$. The region to provide k -coverage is denoted by $\Omega \subseteq \mathbb{R}^2$, that can be bounded or not. Let G be the deployment pattern of gateways, $G = \{c_i\}_{i \in I}$, where $I \subseteq \mathbb{N}$. G is not necessarily limited to Ω , nor finite (assuming that the grid is regular, when $\Omega = \mathbb{R}^2$ the grid is necessarily infinite). Let $|\cdot|$ denotes the cardinality, and r the communication range which is supposed to be identical for all devices. We denote by $G(p, r)$ the subset of G enclosed by a disc of radius r , centered at $p \in \Omega$, i.e., $G(p, r) = \{c_i \in G \mid d(p, c_i) \leq r\}$. Finally, the k -distance of p to set G is defined as $d_k(p, G) = \min\{r \geq 0 \mid |G(p, r)| \geq k\}$.

k -coverage means every device in Ω is able to communicate with k gateways in G with range r as stated by the following definition and proposition.

Definition 1 (k -coverage). Ω is said to be k -covered by the deployment G , if for any $p \in \Omega$ representing a device, $|G(p, r)| \geq k$.

Proposition 1. The minimal communication range to k -cover Ω is $r_k = \max_{p \in \Omega} d_k(p, G)$.

Proof. Note that since G is defined as a regular pattern, this maximum is well defined as long as $k \leq |G|$. If r_k is the minimal communication range achieving k -coverage, then we have $r_k \geq \max_{p \in \Omega} d_k(p, G)$: indeed, if $r_k < d_k(p, G)$, there exists at least a point p such that $|G(p, r_k)| < k$ and p is not k -covered. For $r = \max_{p \in \Omega} d_k(p, G)$, we have for any $p \in \Omega$, $r \geq d_k(p, G)$, according to the definition of $d_k(p, G)$, we have $|G(p, r)| \geq k$, thus Ω is k -covered. \square

In order to find r_k , let's introduce the following definitions.

Definition 2 (k -order Voronoi cell). Let us denote by $\mathcal{P}_k(G) = \{\mathcal{C} \subseteq G \mid |\mathcal{C}| = k\}$ the collection of the subsets of G having the cardinality k , with $k \geq 1$. For any $\mathcal{C}_0 \in \mathcal{P}_k(G)$, the k -order Voronoi cell of \mathcal{C}_0 is defined as $C_k(\mathcal{C}_0) = \{p \in \Omega \mid \forall \mathcal{C} \in \mathcal{P}_k(G), d_k(p, \mathcal{C}_0) \leq d_k(p, \mathcal{C})\}$.

Definition 3 (k -order Voronoi cell of a vertex). The k -order Voronoi cell of a vertex $c_i \in G$ is defined as $V_k(c_i) = \bigcup_{\mathcal{C} \ni c_i} C_k(\mathcal{C})$.

$C_k(\mathcal{C}_0)$ is the set of points that have \mathcal{C}_0 as the k nearest gateways among G . $V_k(c_i)$ is the set of points that have c_i as one of its k nearest gateways among G . A Voronoi polygon of a c_i is a special k -Voronoi cell given by $k = 1$, i.e., $C_1(c_i) = V_1(c_i)$. Note that there is no overlap between adjacent Voronoi cells C_k , but the V_k overlap in general. An example of $C_1(c_i) = V_1(c_i)$, $C_3(\mathcal{C})$ and $V_3(c_i)$ are shown in figure 1. With all the necessary definitions given, in section IV we express the minimal k -coverage ensuring density $D(k, r)$ in regular patterns by fixing a communication range r .

IV. ENSURING k -COVERAGE

The spatial density of a deployment is defined as the number of gateways per unit of surface to cover, i.e., $D = \frac{|G|}{S_\Omega}$, where S_Ω stands for the surface of Ω , when both quantities are finite and $D = \frac{1}{S_{vp}}$, where S_{vp} is the surface of the Voronoi polygon $V_1(c_i)$ for the regular grid and $\Omega = \mathbb{R}^2$. In this section, we first consider the infinite case. A realistic finite scenario with Ω being a rectangle region is briefly discussed at the end of the section.

A. Necessary and sufficient condition of k -coverage

Proposition 2. We denote by a the inter-gateway distance, i.e., the distance between nearest gateways. By fixing an a , the necessary and sufficient condition for \mathbb{R}^2 to be k -covered is that every point c_i in G covers $V_k(c_i)$. r_k equals to the distance between an arbitrary vertex c_i and the most distant points from c_i in $V_k(c_i)$. We have $r_k(a) = \max_{p \in \mathbb{R}^2} d_k(p, G) =$

$$\max_{p \in V_k(c_i)} d(p, c_i).$$

Proof. Since $C_k(\mathcal{C}_0)$ perfectly tessellate the 2D plane, i.e., $\bigcup_{\mathcal{C}_0 \in \mathcal{P}_k(G)} C_k(\mathcal{C}_0) = \mathbb{R}^2$, we have $r_k(a) =$

$$\max_{c_i \in G} \max_{p \in C_k(\mathcal{C}_0)} d_k(p, G). \text{ According to the definition of } d_k(p, G) \text{ and } C_k(\mathcal{C}_0), \text{ we have for any } p \in C_k(\mathcal{C}_0), d_k(p, G) = \min_{\mathcal{C} \in \mathcal{P}_k(G)} d_k(p, \mathcal{C}) = d_k(p, \mathcal{C}_0) = \max_{c_i \in \mathcal{C}_0} d(p, c_i). \text{ Thus } r_k(a) = \max_{\mathcal{C}_0 \in \mathcal{P}_k(G)} \max_{p \in C_k(\mathcal{C}_0)} \max_{c_i \in \mathcal{C}_0} d(p, c_i). \text{ Let's change the order of max to have } r_k(a) = \max_{c_i \in G} \max_{\mathcal{C}_0 \in \mathcal{P}_k(G), \mathcal{C}_0 \ni c_i} \max_{p \in C_k(\mathcal{C}_0)} d(p, c_i), \text{ which can be rewritten as } r_k(a) = \max_{c_i \in G} \max_{p \in V_k(c_i)} d(p, c_i) \text{ according to the definition of } V_k(c_i). \text{ Recall that } G \text{ is regular pattern, naturally } V_k(c_i) \text{ takes}$$

the same form, we can thus remove the max on c_i , which gives us $r_k(a) = \max_{p \in V_k(c_i)} d(p, c_i)$. \square

Trivially, the coverage condition is scale-invariant and thus $r_k(a)$ is proportional to a , i.e., $r_k(a) = \beta(k)a$ where $\beta(k)$ depends only on k and on the pattern.

Proposition 3. *Given a communication range r , the minimal densities $D(k, r)$ ensuring k -coverage in regular patterns is given by $D(k, r) = \frac{1}{S_{V_k(a_k(r))}}$, where $a_k(r) = \frac{r}{\beta(k)}$ is the maximal inter-gateway distance that guarantees k -coverage.*

Proof. With a given r , if $a > a_k(r)$, $r < \max_{p \in V_k(c_i)} d(p, c_i)$ as $V_k(c_i)$ gets larger, k -coverage is not ensured any more. If $a = a_k(r) = \frac{r}{\beta(k)}$, we have $r = \beta(k)a = r_k(a)$, according to proposition 2, we guarantee k -coverage. \square

In figure 1, an example of $V_3(c_i)$, p_3^{crit} and $a_3(r)$ for three regular patterns is also illustrated.

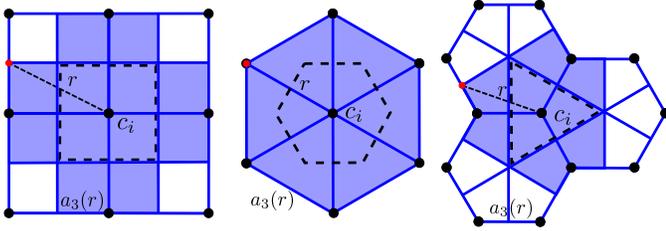


Fig. 1. Illustration of 3 regular patterns based deployments. Black points are the positions of gateways. r denotes a given communication range. Polygons with dashed black edges are $C_1(c_i) = V_1(c_i)$. Polygons with blue edges are $C_3(\mathcal{C})$. Regions painted in light blue are $V_3(c_i)$, i.e., $\bigcup_{\mathcal{C} \ni c_i} C_3(\mathcal{C})$. Red points are the points where $r = r_k$. $a_3(r)$ are maximal inter-gateway distance for 3-coverage for the r .

Obviously, $D(k, r)$ is proportional to $\frac{1}{a_k(r)^2}$, and thus to $\frac{1}{r^2}$, i.e., $D(k, r) = \frac{\alpha(k)}{r^2}$. $\alpha(k)$ is the k -coverage coefficient which depends only on k and on the pattern. $D(k, r)$ can be expressed as: $D_{\text{sqr}}(k, r) = \frac{1}{a_k(r)^2}$, so that $\alpha_{\text{sqr}}(k) = \beta_{\text{sqr}}(k)^2$ for the square grid; $D_{\text{tri}}(k, r) = \frac{2\sqrt{3}}{3a_k(r)^2}$, so that $\alpha_{\text{tri}}(k) = \frac{2\sqrt{3}\beta_{\text{tri}}(k)^2}{3}$ for the triangular grid; $D_{\text{hex}}(k, r) = \frac{4\sqrt{3}}{9a_k(r)^2}$, so that $\alpha_{\text{hex}}(k) = \frac{4\sqrt{3}\beta_{\text{hex}}(k)^2}{9}$ for the hexagonal grid. The density is thus directly linked to $\beta(k)$, i.e., to the critical point, which can be determined exactly by some algorithm of the literature on k -Voronoi diagrams (see for instance [16]).

B. Numerical results

We implemented algorithm [16] for the three regular patterns given table I. The obtained $\alpha(k)$ are listed in table I. We observe that $\alpha_{\text{tri}}(1) \approx 0.385$ is the smallest for 3 regular patterns, which confirms the theory of [11], i.e., the triangular pattern is asymptotically optimal in terms of the number of discs necessary to provide 1-coverage on a plane. We define the minimal k -coverage achieving coefficient in congruent patterns as α_{min} . Obviously, we have $\alpha_{\text{min}}(1) = \alpha_{\text{tri}}(1)$. In [12], the authors find that $\alpha_{\text{min}}(2) = \frac{4}{3\sqrt{3}} \approx 0.770$,

which coincides with $2\alpha_{\text{tri}}(1)$, meaning a superposition of two 1-coverage achieving triangular pattern is optimal among congruent patterns. Note that hexagonal pattern can be seen as a superposition of two triangular pattern, that's why we have $\alpha_{\text{hex}}(2) = 2\alpha_{\text{tri}}(1) = \alpha_{\text{min}}(2)$. The advantage of hexagonal pattern over any 2 randomly superposed triangular patterns is the inter-gateway distance can not be arbitrarily small. Since $\alpha_{\text{min}}(1)$ and $\alpha_{\text{min}}(2)$ are found among regular patterns, we conjecture that for $k \geq 3$, $\alpha_{\text{min}}(k) = \min\{\alpha_{\text{tri}}(k), \alpha_{\text{sqr}}(k), \alpha_{\text{hex}}(k)\}$.

In literature, many heuristics for optimal general k -coverage have been proposed by superposing the regular triangular pattern [17, 18]. Note that this may be true for $k = 3$, as we have $\alpha_{\text{tri}}(3) = 3\alpha_{\text{tri}}(1)$ and α_{tri} is the smallest among the 3 regular patterns. For $k = 4$, although we have $\alpha_{\text{tri}}(4) = 4\alpha_{\text{tri}}(1)$, it's $\alpha_{\text{sqr}}(4)$ that's the smallest of the three. For $k = 5$, $\alpha_{\text{tri}}(5)$ is the smallest of the three but $\alpha_{\text{tri}}(5) > 5\alpha_{\text{tri}}(1)$, meaning superposition heuristics have better performance. However, for $k = 6$ we have $\alpha_{\text{tri}}(6) < 6\alpha_{\text{tri}}(1)$, meaning superposition heuristics requires greater density. Although it's not our primary objective to deduce optimal k -coverage pattern, our work can give some guidelines for this effort.

TABLE I
 $\alpha(k)$ IN FUNCTION OF k FOR 3 REGULAR PATTERNS

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
α_{sqr}	$\frac{1}{2}$	1	$\frac{5}{4}$	$\frac{25}{18}$	$\frac{5}{2}$	$\frac{5}{2}$
α_{tri}	$\frac{2\sqrt{3}}{9}$	$\frac{2\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$	$\frac{8\sqrt{3}}{9}$	$\frac{7\sqrt{3}}{6}$	$\frac{98\sqrt{3}}{81}$
α_{hex}	$\frac{4\sqrt{3}}{9}$	$\frac{4\sqrt{3}}{9}$	$\frac{7\sqrt{3}}{9}$	$\frac{196\sqrt{3}}{225}$	$\frac{4\sqrt{3}}{3}$	$\frac{4\sqrt{3}}{3}$

Figure 2 depicts the behavior of α versus k for the three regular patterns. We can see that the $\alpha(k)$ increases almost linearly with k . Note that the curve is the minimum over all 3 regular patterns considered. For a given k and r , it's not always the same pattern that gives the minimum $D(k, r)$.

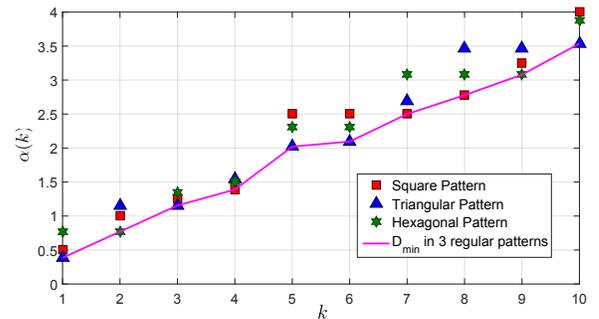


Fig. 2. $\alpha(k)$ in function of k in 3 regular patterns

C. Edge effect consideration

In this section, we consider a realistic finite scenario. We first provide the following proposition.

Proposition 4. *Let $\Omega \subset \mathbb{R}^2$ a bounded region and r a given communication range. If gateways are deployed following any*

regular pattern with inter-gateway distance $a_k(r)$ inside the dilation of Ω by r , denoted by $\Omega^{\oplus r} = \{y \in \mathbb{R}^2 \mid \exists x \in \Omega, d(x, y) \leq r\}$, then Ω is k -covered.

Proof. If we deploy gateways following any regular pattern with inter-gateway distance $a_k(r)$ to the whole \mathbb{R}^2 , according to the proposition 3, we have k -coverage over \mathbb{R}^2 . As $\Omega \in \mathbb{R}^2$, Ω is k -covered. For any $c_i \in \mathbb{R}^2 / \Omega^{\oplus r}$, according to the definition of $\Omega^{\oplus r}$, we have $\forall p \in \Omega, d(p, c_i) > r$. Recall that the coverage degree of p is $|G(p, r)|$, with $G(p, r) = \{c_i \in G \mid d(p, c_i) \leq r\}$. Removing all the $c_i \in \mathbb{R}^2 / \Omega^{\oplus r}$ doesn't change $|G(p, r)|$. \square

Let's take an example of providing 3-coverage with triangular pattern to a rectangle region Ω of dimensions $5\text{km} \times 2.5\text{km}$. r is chosen to be a realistic communication range for LPWANs in dense urban scenario, *i.e.*, 0.79 km , see table II. $a_3(r) = r = 0.79\text{km}$ for triangular pattern. There are several ways to interpret this proposition. When Ω is seen as the interest region filled with sensor nodes to provide 3-coverage, this proposition provides a way to fully 3-cover Ω by deploying gateways in the dilation of it, *i.e.*, $\Omega^{\oplus r}$. Gateway density in this scenario can be calculated as $D_{\text{full}} = \frac{|G \cap S_{\Omega^{\oplus r}}|}{S_{\Omega}} = \frac{48}{12.5\text{km}^2} = 3.84/\text{km}^2$, which is much greater than the density calculated in infinite case, *i.e.*, $D_{\text{tri}}(3, r) = \frac{1}{S_{\text{VP}}(a_3(r))} = \frac{2\sqrt{3}}{3r^2} \approx \frac{1.155}{r^2} \approx 1.85/\text{km}^2$. However, if we consider $\Omega^{\oplus r}$ as the interest area, certainly the border area of it is not 3-covered, but we guarantee a fully 3-covered center area Ω . Gateway density in this scenario is calculated as $D_{\text{real}} = \frac{|G \cap S_{\Omega^{\oplus r}}|}{S_{\Omega^{\oplus r}}} \approx \frac{48}{26.31\text{km}^2} \approx 1.82/\text{km}^2$. In the fully 3-covered area Ω , we have $D_{\text{center}} = \frac{|G \cap S_{\Omega}|}{S_{\Omega}} \approx \frac{24}{12.5\text{km}^2} \approx 1.92/\text{km}^2$. In reality, the border area is usually less populated, it's not necessary and realistic to perfectly k -cover it, especially when the cost is too high. Moreover, it's difficult to consider edge effect which depends on region shape etc. It's of little interest when providing general laws. In section V, we neglect the edge effect and stick to the approximation $D(k, r) = \frac{1}{S_{\text{VP}}(a_k(r))}$ and evaluate the efficiency of deployments of a real LPWANs technology.

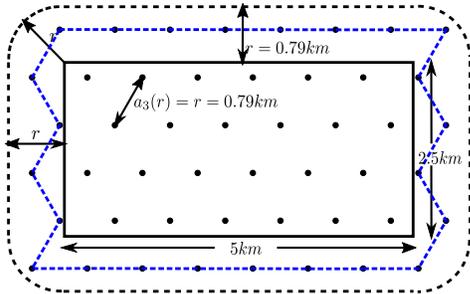


Fig. 3. Illustration of edge effect. Region enclosed by black solid line is Ω , by black dashed line $\Omega^{\oplus r}$. The area between black and blue dashed line is the border area.

V. APPLICATION

In this section, by applying our results in IV, we aim at visualizing the dimensioning of a real LPWANs deployment

according to the k -coverage criterion in some real-world scenarios. In order to quantify the efficiency of a deployment we introduce the average number of sensor nodes per gateway when achieving k -coverage, denoted by $N_{\text{avg}} = d_s / D(k, r)$, in which d_s indicates the sensor node density in an area. $D(k, r)$ depends on communication range r . We thus consider 3 scenarios, *i.e.*, dense urban, urban and rural, in which the sensor node density d_s and typical communication range r are intrinsically different, due to different population density and radio environment.

We consider a real LPWANs technology LoRaWANTM, which operates on 868MHz frequency band in Europe and has a typical link budget of 154dB [2]. For the path loss model, we choose Hata model for $150\text{MHz} \leq f \leq 1000\text{MHz}$ as recommended by ETSI for GSM 900MHz [19]. We add shadowing margin and penetration loss as we consider sensor nodes deployed indoor. These 2 parameters are not included in the Hata model, typical values are found in [20]. The parameters and calculated typical r are listed in table II.

TABLE II
PATH LOSS PARAMETERS FOR LORAWANTM

Parameters	dense urban	urban	rural
shadowing margin (dB)	13	13	13
penetration loss (dB)	18	15	10
typical r (km)	0.79	0.92	2.43

d_s can be calculated as the product of the house density and number of sensor nodes per house. For house density, we gathered some statistics from INSEE [21] for different scenarios. For dense urban scenario, we take the city of Paris in 2013 as an example. For urban and rural scenarios, we averaged statistics from several cities and villages. The estimation of number of sensor nodes per house are also listed in table III.

TABLE III
STATISTICS AND ESTIMATION IN DIFFERENT SCENARIOS

Statistics	dense urban	urban	rural
house density (/km ²)	10946	2845	102
sensor nodes /house	10	8	5
d_s (nodes/km ²)	109460	22760	510

$N_{\text{avg}} = \frac{d_s}{D(k, r)} = d_s S_{\text{VP}}(a_k(r))$, which is the number of sensor nodes in the Voronoi polygon of a gateway c_i . These sensor nodes are nearer to c_i than to any other gateways. However, the number of sensor nodes that can actually communicate with c_i should be calculated as $N_{\text{real}} = d_s \pi r^2$. N_{avg} and N_{real} are both represented in figure 4. N_{avg} reflects the number of sensor nodes "allocated" to c_i . $D(k, r)$ increases with k , and N_{avg} decreases with k . A denser deployment can in fact reduce the number of sensor nodes "allocated" to every gateway, thus reducing its traffic pressure.

A gateway can only support packets from a limited number of sensor nodes, *i.e.*, the load-capacity of gateway denoted by C_{load} . It depends on the number of frequency channels,

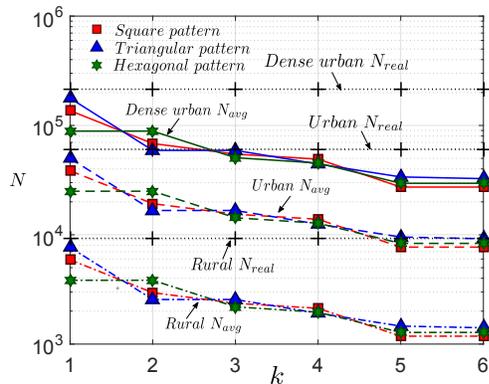


Fig. 4. Comparison of N_{avg} and N_{real} in 3 scenarios

the duration of packets, the interference phenomenon between packets etc. Note that C_{load} of a gateway also increases with k in that packets benefit from an increase in reception diversity. One can imagine an increasing curve representing C_{load} in figure 4, which probably crosses the N_{real} and N_{avg} curves. As sensor nodes in LPWANs send their packets using uncoordinated access schemes, the level of interferences increases with N_{real} and N_{avg} . When N_{real} and N_{avg} are too high compared to C_{load} , we are in the interference-limited regime. Even with k -reception, there could be heavy packet loss. Therefore the position of C_{load} curve is essential to decide when the dimensioning factor is the k -coverage or C_{load} . In certain scenarios, even though we deploy gateways to achieve a certain k -coverage, N_{real} and N_{avg} may be too high compared to C_{load} . One solution is to allocate more frequency resources to gateways, which is not always available. Another solution is to reduce the communication range r by a tilt of the gateway antenna or an artificial reduction of the reception sensibility, in order to reduce N_{real} and N_{avg} and thus the interferences, but more gateways needs to be deployed in this case. The primary dimensioning factor is thus C_{load} . In other cases, by achieving k -coverage, N_{real} and N_{avg} don't exceed C_{load} of gateways, no more gateways need to be deployed. The primary dimensioning factor is thus k -coverage.

In [22], the phenomenon of interferences between packets is studied using a stochastic geometry model for typical LPWANs access schemes, in order to deduce C_{load} without considering k -reception. An extra reference C_{load} curve could be drawn by combining the models in [22] and in this article in order to give a more refined study of the dimensioning factors of LPWANs.

VI. CONCLUSION AND FUTURE WORKS

In this article, we studied the problem of minimal gateway density ensuring k -coverage in regular deployment patterns in order to give some insights of the dimensioning of the LPWANs. We note that the gateway density increases almost linearly with coverage degree. Our method allows to evaluate or compare two deployments, which one is optimal in terms of gateway density. We confirm the results obtained in [11, 12],

that the minimal deployment density among congruent patterns for $k = 1$ and $k = 2$ is achieved respectively for a regular triangular pattern and a superposition of 2 regular triangular patterns. We conjecture that minimal density for general k among congruent patterns is always achieved among the 3 regular patterns that we considered.

From 3 real-world scenarios, we show that k -coverage alone is probably unable to dimension the gateway density in urban scenarios, load-capacity of gateway being another essential factor. In future works, we are going to combine the load-capacity model and k -coverage model in order to give a more refined dimensioning of LPWANs.

REFERENCES

- [1] "ETSI." http://www.etsi.org/deliver/etsi_tr/103000/_103099/linebreak/103055/01.01.01_60/tr_103055v010101p.pdf. Accessed: 2016-08-03.
- [2] "LoRaWANTM Specification." <https://www.lora-alliance.org/portals/0/documents/whitepapers/LoRaWAN/linebreak101.pdf>. Accessed: 2016-08-03.
- [3] "Sigfox." <http://www.sigfox.com/fr>. Accessed: 2016-08-03.
- [4] S. Kumar, T. H. Lai, and J. Balogh, "On k -coverage in a mostly sleeping sensor network," in *Proc. of the 10th annual int. conf. on Mobile computing and networking*, pp. 144–158, 2004.
- [5] G. Yang and D. Qiao, "Critical conditions for connected- k -coverage in sensor networks," *IEEE Com. Letters*, vol. 12, no. 9, pp. 651–653, 2008.
- [6] G. Xing, X. Wang, Y. Zhang, C. Lu, R. Pless, and C. Gill, "Integrated coverage and connectivity configuration for energy conservation in sensor networks," *ACM Trans. on Sensor Networks*, vol. 1, no. 1, pp. 36–72, 2005.
- [7] Z. Zhou, S. Das, and H. Gupta, "Connected k -coverage problem in sensor networks," in *Proc. of the 13th IEEE Int. Conf. on Computer Communications and Networks*, pp. 373–378, 2004.
- [8] M. Hefeeda and M. Bagheri, "Randomized k -coverage algorithms for dense sensor networks," in *26th IEEE Int. Conf. on Computer Communications*, pp. 2376–2380, 2007.
- [9] H. M. Ammari and S. K. Das, "Centralized and clustered k -coverage protocols for wireless sensor networks," *IEEE Trans. on Computers*, vol. 61, no. 1, pp. 118–133, 2012.
- [10] Y.-h. Kim, C.-M. Kim, D.-S. Yang, Y.-j. Oh, and Y.-H. Han, "Regular sensor deployment patterns for p -coverage and q -connectivity in wireless sensor networks," in *Int. Conf. on Information Networking*, pp. 290–295, 2012.
- [11] R. Kershner, "The number of circles covering a set," *Am. J. of math.*, vol. 61, no. 3, pp. 665–671, 1939.
- [12] J. Ge, *Planar Thinnest Deployment Pattern of Congruent Discs which Achieves 2-coverage*. PhD thesis, School of Mathematical Sciences, Soochow University, Suzhou 215006, P. R. China, 2010.
- [13] C. Zhu, C. Zheng, L. Shu, and G. Han, "A survey on coverage and connectivity issues in wireless sensor net-

- works,” *Journal of Network and Computer Applications*, vol. 35, no. 2, pp. 619–632, 2012.
- [14] F. Li, J. Luo, S.-Q. Xin, W.-P. Wang, and Y. He, “LAACAD: Load balancing k-area coverage through autonomous deployment in wireless sensor networks,” in *IEEE 32nd Int. Conf. on Distributed Computing Systems*, pp. 566–575, 2012.
- [15] B. Grunbaum and G. C. Shephard, “Tilings by regular polygons,” *Math. Mag.*, vol. 50, no. 5, pp. 227–247, 1977.
- [16] The CGAL Project, *CGAL User and Reference Manual*. CGAL Editorial Board, 4.8.1 ed., 2016.
- [17] J.-E. Kim, J. Han, and C.-G. Lee, “Optimal 3-coverage with minimum separation requirements for ubiquitous computing environments,” *Mobile Networks and Applications*, vol. 14, no. 5, pp. 556–570, 2009.
- [18] Z. Yuan, W. Li, J. Zhu, and W. Zhao, “A cost-efficiency method on beacon nodes placement for wireless localization,” in *IEEE Int. Conf. on Computing, Networking and Communications*, pp. 546–550, 2015.
- [19] “ETSI.” http://www.etsi.org/deliver/etsi/_tr/101300/_101399/linebreak/101362/06.00.01/_60/tr/_101362v060001p.pdf. Accessed: 2016-08-03.
- [20] A. R. Mishra, *Advanced cellular network planning and optimisation*. John Wiley & Sons, 2007.
- [21] “INSEE.” http://www.insee.fr/en/themes/theme.asp?theme\linebreak=1\&sous_theme=2. Accessed: 2016-08-03.
- [22] S. Zozor, Z. Li, Q. Lampin, and J.-M. Brossier, “Time-frequency aloha-like random access: A scalability study of low-power wide-area networks for iot using stochastic geometry.” <http://arxiv.org/abs/1606.04791>.