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Romain Mesnil, Cyril Douthe, Olivier Baverel, Christiane Richter, Bruno Leger. Structural exploration of a fabrication-aware design space with Marionettes Meshes. IASS Annual Symposium 2016: Spatial Structures in the 21st Century, Sep 2016, Tokyo, Japan. hal-01978811

HAL Id: hal-01978811

<https://hal.science/hal-01978811>

Submitted on 11 Jan 2019

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Structural exploration of a fabrication-aware design space with Marionettes Meshes

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Abstract

This paper presents a new methodology for generating doubly-curved shapes covered with planar facets. The proposed method, called *marionette method* relies mainly on descriptive geometry, and it is shown that specifying appropriate projections, e.g. plane view and elevations, defines uniquely a free-form shape. This methodology is highly intuitive and can be used in real-time, mainly because only linear systems of equations are solved. The design-space offered by the marionette method is extremely large, and easily parameterized. Some applications show the potential of this technique for the parameterization of a fabrication-aware design space for structural optimization.

Keywords: Fabrication-aware design, structure, optimization, architectural geometry, PQ-mesh

1. Introduction

The design of thin shells often merges fabrication and structural preoccupations. Structural artists have been able to build structurally efficient and cost-effective shells by using shapes that are appropriate both for structural behavior and fabrication (Billington [3]). Felix Candela for example used ruled surfaces as primary vocabulary of his structural language. The use of straight elements simplified the construction of formworks, showing the concern of Candela for fabrication. Frei Otto and his team developed the compass method to cover free-form shapes with constant member length and rationalize fabrication (Hennicke [11]). Likewise Jörg Schlaich and Hans Schober used surfaces of translation or scale-trans surfaces to build elegant grid shells covered with planar facets (Glymph *et al.* [10], Schlaich and Schober [21]).

Nowadays, the planarity constraint for facets is seen as one of the defining factors on the economy of free-form envelopes, especially for glazed structures. Several methods for optimization of façade layouts towards facet planarity have been proposed recently (Liu *et al.* [13], Mesnil *et al.* [14]). While some methods are computationally efficient and allow geometrical optimization in real-time (Deng *et al.* [8]), they generally give little insight to the designer (Bagneris *et al.* [2]). Geometrical rationalization methods are also generally envisioned with little concern to the structural behavior of the modelled shapes. On the other hand, shape optimization of shell structures is an active topic of research (Rahm *et al.* [20], Ohmori *et al.* [18]) and the problem of design-space parameterization for structural optimization is crucial. However, the community of structural engineers does not benefit from the most recent advances in the field of computational geometry.

The aim of this paper is to introduce a new method for fabrication-aware design that offers another perspective on the use of numerical tools. Rather than proposing an approach of *design by analysis* where the computer provides results without building an understanding of the underlying geometry (Baker [3]), the proposed method constructs complex shapes from simple rules derived from descriptive geometry.

The main contribution of this paper is the investigation on the use of the marionette technique for the parameterization of shape optimization problems. The second section presents the methodology for shape generation, introduced by the authors in (Mesnil *et al.* [15]) and structural optimization. The third section shows some case studies that illustrate the potential of marionette meshes for structural optimization and a comparison with NURBS. The fourth section introduces a study on creased structures, which are tedious to model either with NURBS or pure node-based approaches.

2. Methodology

2.1. Descriptive geometry and the marionette quad

Descriptive geometry is a tool that represents surfaces with planar projections. It was developed during the eighteenth century and the mathematician Gaspard Monge gathered all the existing theories at the end of the eighteenth century (Monge [16]). Among many problems treated by descriptive geometry, an exercise of particular interest for the modelling of free-form surfaces is the description of a planar quadrilateral from a planar projection and two elevations.

Consider Figure 1: it turns out that a planar quad ($A'B'C'D'$) can be described with a planar view (ABCD) and two compatible elevations. In the drawing, the point C' is uniquely defined once the points ($A'B'C'$) and its projection C is chosen. The point is found by doing the intersection of a plane and a vertical line, which has a unique solution. Figuratively, the solution of the problem requires to construct vertical lines, which recall the strings of a marionette. We propose to turn the solution of this problem of shape description into a shape-generation tool. Note that only a linear equation has to be solved for the lifting of a marionette quad.

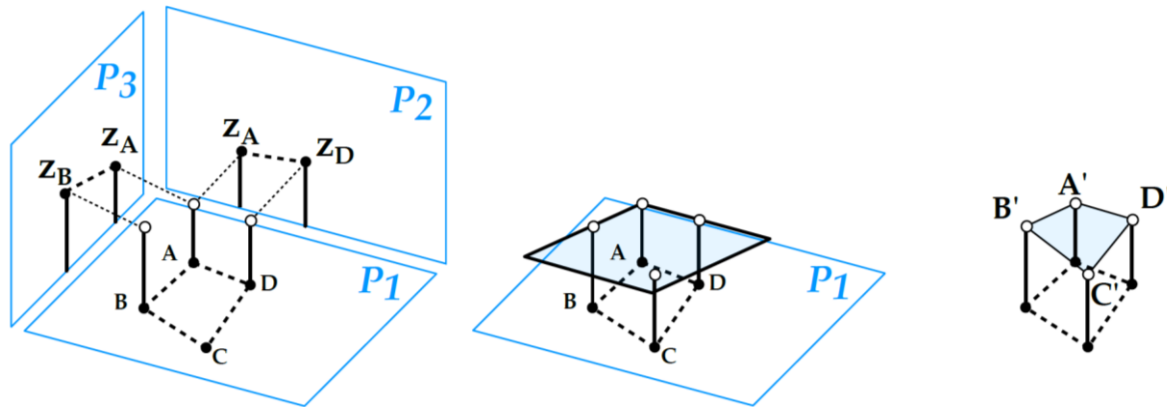


Figure 1: A planar 'marionette' quad and a representation from descriptive geometry (Mesnil *et al.* [15])

2.2. Marionette mesh

Consider now a quadrangular mesh without singularity. Like in the marionette quad, we prescribe a plane view and two elevations of curves crossing the mesh. The lifting technique can be used for the quad at the intersection of the two curves (since three altitudes are prescribed), and the planar quad defines a unique altitude for the fourth point. The information can then be propagated along each strip, as shown in Figure 2.

Prescribing a planar view and two elevations is thus sufficient for the definition of a quadrilateral meshes with planar facets. Other projections, e.g. on cylinders or tori can be used, which does not restrict this framework to height field (Mesnil *et al.* [15]). Likewise, different patterns or mesh topologies can be lifted, for example the Kagome or dual-Kagome patterns (Mesnil *et al.* [15]). We focus in the following on quadrilateral meshes constructed from a projection in the (XY) plane and vertical elevations.

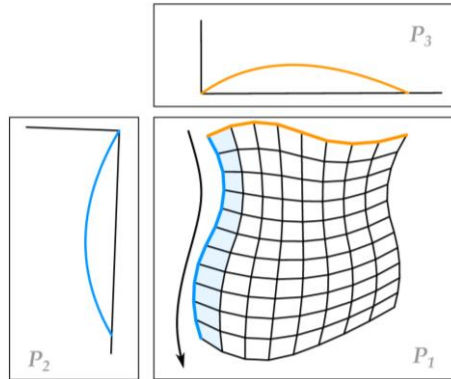


Figure 2: Marionette mesh with a prescribed planar view and two elevations (Mesnil et al. [15])

The marionette technique can be used as a meshing tool, where the planar view is parameterized by the (XY) coordinates of each point. It can also be used in the manner of NURBS, if the planar view and the guide curves are controlled with NURBS and Bézier curves. These two approaches can be linked with the current approaches for shape parameterization in structural optimization problem. The two main strategies are node-based approaches, where every vertex position is taken as a variable (Firl *et al.* [9], Stavropoulou *et al.* [22]), and CAD based approaches, where the shapes are modelled with an underlying construction rule (often NURBS patches) (Bletzinger *et al.* [5, 6]). Node-based approaches often require regularization techniques, and the high number of variables makes it harder to make sensitivity analysis. For that reason, the following of the paper focuses on CAD-based approach with marionette meshes.

2.2.1. Node-based approach

Let us estimate the number of degrees of freedom offered by the marionette method. If we consider the method as a node-based approach, then for a grid with NM faces, we have $2(N+1)(M+1)$ degrees of freedom for the plane view and $(N+M+1)$ degrees of freedom for the elevation curves. The size of the design space is then:

$$S_{\text{Marionette}} = 2(N+1)(M+1) + N + M + 1 \quad (1)$$

This is less than the full model with no planarity constraints for the facets:

$$S_{\text{unconstrained}} = 3(N+1)(M+1) \quad (2)$$

The size of the design space compared to classical node-based approaches is reduced by 30%. This leaves a very large design space for typical gridshells, which have hundreds of facets.

2.2.2. CAD-based approach

If the method is considered as a CAD tool, then the plane view is controlled by a NURBS patch with $2n$ degrees of freedom, where n is the number of control points. Each guide curve can then be controlled by control points. We write N_{CP} the number of control points of the NURBS patch and N_{Curve1} , N_{Curve2} the number of control points for each curve. The number of parameters describing the design space follows:

$$S_{\text{Marionette}} = N_{\text{curve1}} + N_{\text{curve2}} + 2N_{CP} \quad (3)$$

Generally, this number represents 2/3 of the numbers of parameters offered by NURBS. The planarity constraints reduces the size of the design space, but still gives sufficient degrees of freedom. The differences between NURBS and Marionette approaches will be discussed in Section 3.

2.3. Constraints and linear programming

The marionette method deals only with linear equations. Mathematically, the set of solutions for the planarity constraint is a vector space: the admissible altitudes Z are indeed solution of a linear system: for a given planar view there is a matrix A , and a set of prescribed altitudes on the boundaries so that:

$$AZ_p = Z \quad (4)$$

The solutions belong thus to the image of the matrix A , which is a vector space, whose dimension is given by equation (1). It is then easy to introduce linear constraints. If the number of independent linear constraints $n_{constraints}$ is inferior to the dimension of the space of solutions, then the dimension of the new design space is given by:

$$d = S_{Marionette} - n_{constraints} \quad (5)$$

Note that equation (4) corresponds to the constraint of facet planarity. It is extremely simple to optimize quadratic functional under linear constraints. Examples of linear constraints include: node position, maximum altitude, encapsulated volume. Examples of quadratic functional include distance measurements for surface fitting or smoothness functions (Mesnil *et al.* [15]). All these remarks indicate that the proposed framework can be treated very efficiently with usual numerical tools of linear programming.

2.4. Structural optimization

The following of this paper considers application of the marionette method as a CAD-based tool for the parameterization of structural optimization problem. In order to show the pertinence of this technique, several optimization methods are compared. The nonlinear optimization library NLopt is used in this paper: this library uses derivative-free algorithms (Johnson [12]). We use the algorithms COBYLA (Constrained Optimization BY Linear Approximations), BOBYQA (BOund Optimization by Quadratic Approximation) algorithms.

In the followings, we focus on shell design, as it is well-documented in shape optimization problems. Two objective functions usually studied in the literature on structural optimization were used. The first one is elastic energy with a first order calculation: optimal structures tend to minimize this energy. The second one is the linear buckling load: optimal structures tend to maximize this value.

2.5. Computational set-up

The marionette technique is implemented within the software Grasshopper, a plug-in of the CAD software Rhinoceros. The generation of meshes with hundreds of facets requires typically 1 millisecond: the user can manipulate fabrication-aware surfaces in real-time. The governing factor in the structural exploration of the design space proposed in this paper remains thus the computation related to the mechanical behavior: the planarity constraints is handled in a transparent manner by our framework. We use the Grasshopper plug-in *Goat* for the implementation of the NLopt library.

The computations are done using the finite element method and the software Karamba (Presinger *et al.* [19]), which implements a shell element developed by (Argyris *et al.* [1]). A convergence study on mesh refinement was performed by the authors: the convergence at 95% for the elastic energy is reached for a mesh with 3,600 elements. This mesh density is used in our study, except for creased structures, where refinements were necessary.

3. Application for the shape optimization of shell structures

3.1. Definition of the model

The case study focuses on a shell supported on three corners. The geometry of the shell is illustrated by Figure 3. The geometry has six planes of symmetry and can be decomposed into six domains without singularity, where the marionette technique can be applied. Both NURBS and marionette meshes are generated. The NURBS models are built from patches of degree 2 with 9 control points. The planar view of the marionette meshes is built with the same kind of patch, whose control points are shown in Figure 3. The orange area shows the admissible area for the supports of the structure (where $z=0$). The shapes are trimmed with a horizontal plane.

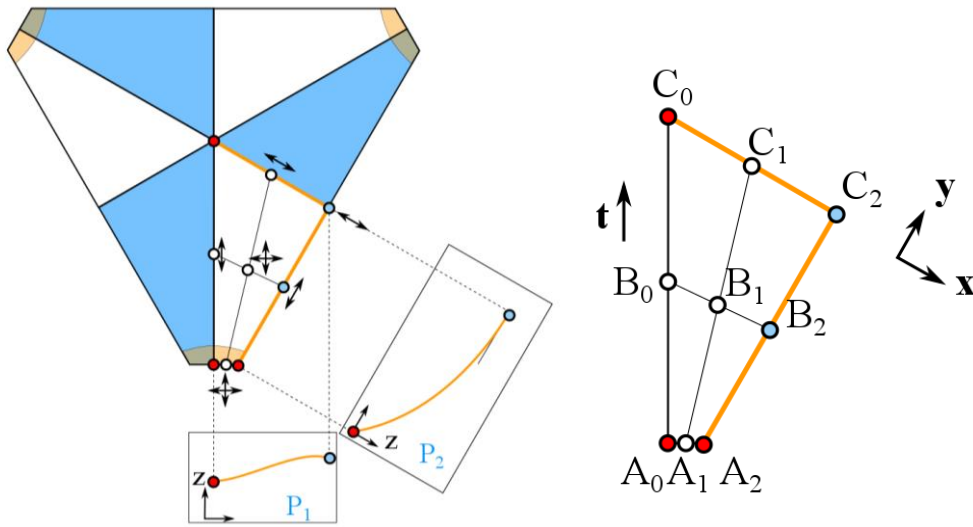


Figure 3: Geometry of the shell supported on three corners

The symmetry implies some constraints to the position of the control points. We recall the constraints in Table 1:

Table 1: available degrees of freedom

	A ₀	A ₁	A ₂	B ₀	B ₁	B ₂	C ₀	C ₁	C ₂
Marionette	Fixed	x,y	Fixed	t	x,y	x	Fixed	x	$C_{2,x} = B_{2,x}$
NURBS	Fixed	x,y	Fixed	t,z	x,y,z	x,z	z	x,z	$C_{2,x} = B_{2,x}, C_{2,z} = B_{2,z}$

The marionette and NURBS models have both 7 degrees of freedom in the horizontal plane. The elevations of the marionette mesh are controlled with polynomial curves of degree 3 drawn in Figure 4. In plane P_2 , we impose horizontal tangency at the crown, which leaves two degrees of freedom: the height of the crown h and the slope at A_2 . In plane P_1 , we do not impose restrictions on tangency, which leaves three degrees of freedom: the height of C_0 and the slopes at C_0 and C_2 .

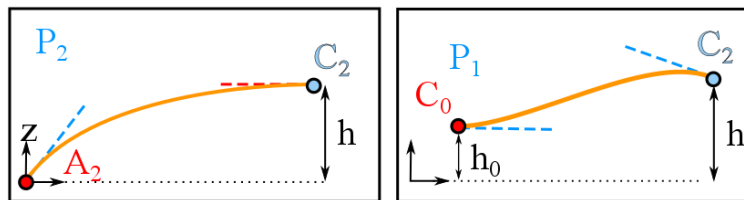


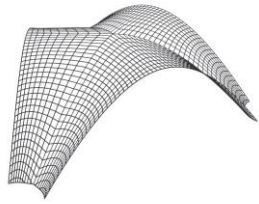
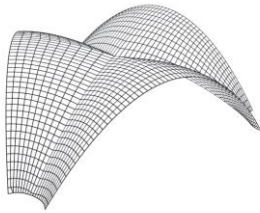
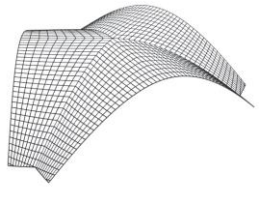
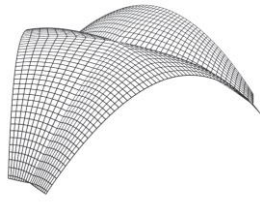
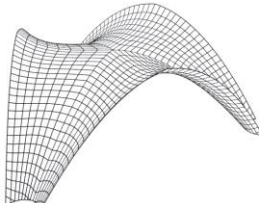
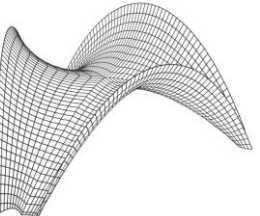
Figure 4: Parameterisation of the guide curves for the marionette mesh

In total, the marionette mesh and NURBS model have 11 and 14 parameters. The size of the design space is thus similar. All the optimisations start from the same point: the curve in the plane P_1 is a straight line and the rise-over-span ratio is of 30%. The shell has a span of 140 meters and a thickness set arbitrarily to 10cm and a Young modulus of 30GPa. It is subject to a vertical projected load of 1kN/m², non-symmetrical load cases were also considered with similar results but are not shown for the sake of conciseness. The supports are fully restricted in translation, but allow rotations.

3.2. Structural optimization

Several optimization algorithms were tested for the full problem. Two objectives were considered: linear buckling load and elastic energy. The objective functions have different optima, and are non-convex. As a result, several local optima are found by the different optimization algorithms. Table 2 illustrates several results obtained with different algorithms, found in less than 3 minutes. All the found shapes are subject to compression dominant loads (no tensile axial force N_1 or N_2), and the bending energy represents less than 1 percent of the total elastic energy. Like expected, the optimization tends to produce different funicular shapes and the algorithms find different local optima for buckling or elastic energy.

Table 2: Some results of the optimization, design with bolded figures belong to the Pareto front

	Marionette Mesh	NURBS
COBYLA, Energy	 <p>$E_{el}=17.36\text{kNm}$, $p_{cr}=1.61\text{kPa}$</p>	 <p>$E_{el}=31.29\text{kNm}$, $p_{cr}=1.14\text{kPa}$</p>
BOBYQA, Energy	 <p>$E_{el}=22.54\text{kNm}$, $p_{cr}=0.49\text{kPa}$</p>	 <p>$E_{el}=33.32\text{kNm}$, $p_{cr}=1.20\text{kPa}$</p>
COBYLA, Buckling	 <p>$E_{el}=19.88\text{kNm}$, $p_{cr}=6.79\text{kPa}$</p>	 <p>$E_{el}=42.63\text{kNm}$, $p_{cr}=7.65\text{kPa}$</p>

Some shapes recall the CNIT (energy minima with BOBYQA algorithm), others recall the aquarium of Valencia by Candela (optimized buckling with COBYLA algorithm). The optimized marionette mesh for elastic energy with the COBYLA algorithm has vanishing edge curvature like the highway service

area in Deitingen by Heinz Isler. These remarks show the mastery of these great engineers who came up with optimal designs without any computing power available, but they also show the richness of the proposed design space. Note that three designs are dominated both in terms of elastic energy and buckling by the others. Two of the Pareto optima are Marionette meshes.

3.3. Local exploration of the design space

This study shows that the performance of optimization algorithms is similar for the marionette design space and the NURBS design space. It illustrates that sufficient design freedom is left to the architects and engineers, and that other exploration techniques of the design space can be used.

As an example, we start from the structure minimizing elastic energy found by the COBYLA algorithm, which is shown in Figure 5. The top of the structure remains quite flat, and the edges have a vanishing curvature. Since the mesh is aligned with the free-edge, it is easy to introduce a curvature to the edge by manipulating just one parameter (the slope at the point C_2 in the plane P_1 in Figure 3). In this way, we mimic some of Heinz Isler's shells (Billington [3]). The model with curved edges has an elastic energy superior by 30%, but its linear buckling strength is multiplied by 6 compared to the model with vanishing edge curvature.

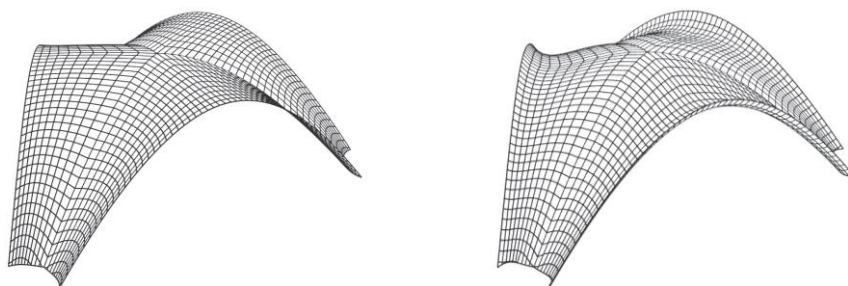


Figure 5: A local optimum of elastic energy with vanishing edge curvature (left), and a variation with curved edges (right)

3.4 Dome

Domes are objects of interest for structural engineers and can be described with marionette method. As an illustration, we show in Figure 6 two results of optimization on the elevation alone where the maximal height of the structure was constrained with the Marionette framework. In this way, the designer can study shallow or deep shells and focus on some designs easily. Like in the previous case-study, the pictured structures are under compression dominant loads.

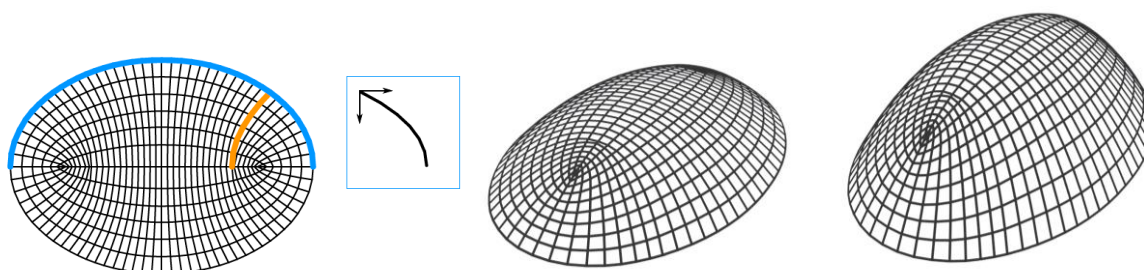


Figure 6: Results of optimisation of a dome structure, shallow (middle) and deep (right) structures.

4. Comments and application to creased structures

4.1. Comments

The marionette mesh shows good performance in the optimization problem. An interesting feature of this representation is that it separates the description of the vertical and horizontal components. Recall that

moving the z components of a shell does not change the horizontal equilibrium if the structure is only subject to vertical loads. This property was used in (Block & Ochsendorf [7]) for form-finding of funicular shells. The geometrical description of marionette meshes is derived from similar principles: the description of the elevations can be enriched independently from the planar projection, for example to add corrugation.

We notice also that the definition of the parameters can be made meaningful with marionette meshes. The example of Section 3.3 shows that one parameter can control the curvature of the edge, whereas three parameters are required with NURBS modelling. This fact can explain why the optimization algorithms do not find the same optima for the two modelling framework. Like in many optimization problems, the know-how of the user is a key to the good convergence of the calculations: the problem presented here can easily be parameterized, but finding appropriate mesh topology for more complex configurations still remains a key to the efficient use of the proposed technique.

4.2. Corrugation and non-smooth surfaces

An interesting application of the separation of vertical and horizontal components is the introduction of creases. This technique was used by Nicolas Esquillan for the Marignane hangar and the CNIT, which remains the largest span for a concrete shell to this day (Motro & Maurin [17]). It is possible to add a crease perturbation defined by equation (6) to the equation of a guide curve.

$$f(k, u, N) = k \cdot |\sin(\pi Nu)| \tag{6}$$

The number N corresponds to the number of waves, and the number k is the wave amplitude. The crease is therefore easily parameterized with the marionette method, NURBS would require insertion of multiple knots.

4.3. Case study: CNIT

Consider now the previous model of the shell supported on three supports. In the manner of Esquillan, we add creases to the curve in the plane P₁. We set the number of waves to 6 for each subdomain and study the influence of the crease amplitude. Table 3 shows the main results: the values on the x-axis represent the ratio k/h in both graphs. The introduction of creases can drastically improve the structural behavior, and especially the buckling capacity. The two objective have different optima ($k=0.05$ for the displacement, and $k=0.08$ for the buckling).

Table 3: Displacement and buckling load with respect to the crease amplitude

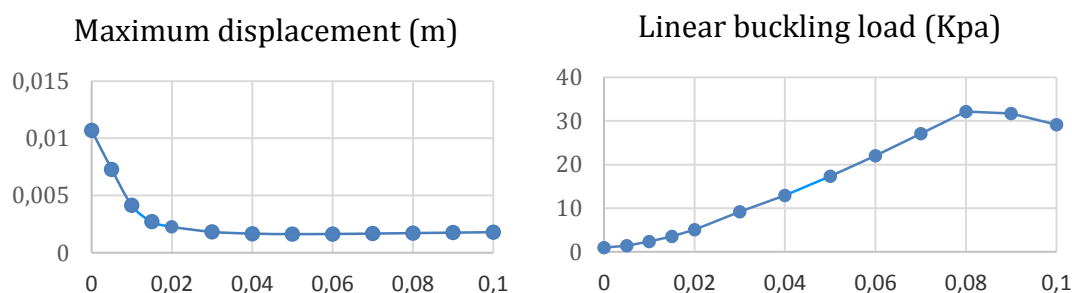


Figure 7 shows a smooth structure and a corresponding creased shell. The buckling capacity of the creased shell is 20 times the one of the smooth shape. The creased shell clearly recalls the work on Esquillan, and we noticed that none of the smooth shapes found by optimization reaches its critical buckling load. The creasing strategy is thus very efficient in the presented case and reveals Esquillan's great intuition.

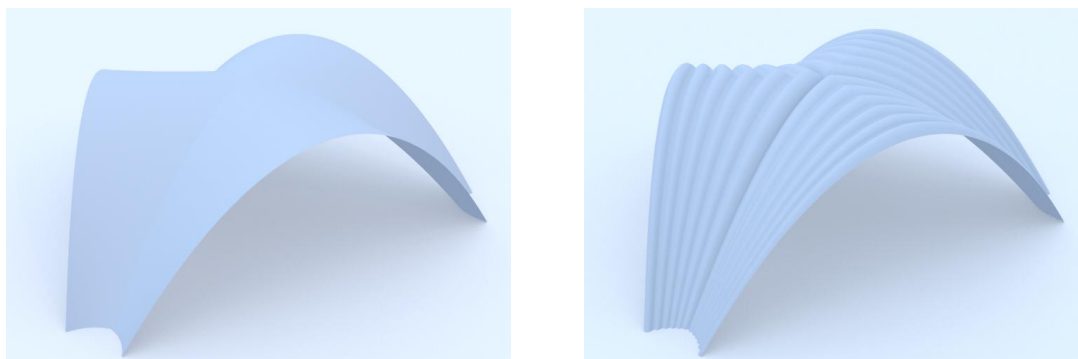


Figure 7: Smooth and creased structures

The exploration of creased structures is much more demanding in computational power, as the mesh has to be refined in order to capture the variations of curvature. The convergence is reached with approximately 20 times more elements with creased shells, which makes the exploration of these solutions more difficult and demonstrates the pertinence of simple parameterization like the one proposed here.

5. Comments and conclusion

This paper proposed an innovative method for fabrication-aware shape parameterization for structural optimization. A case-study showed that this technique can be compared to NURBS in terms of richness of the design space. In particular, it is possible to separate the shape description of the horizontal and vertical components, which is meaningful for architectural shells.

The optima found are funicular shapes in the two examples presented in this paper, which hints that the marionette technique could be combined with Thrust Network Analysis to find compression dominant structures covered with planar facets. Marionette technique and TNA share similar methods and could potentially be unified. The lifting technique shown in this paper can also be adapted to other patterns, like the Kagome pattern, which could yield new typologies for gridshells covered with planar facets.

We believe that this framework can be applied to the structural optimization of shells and gridshells while taking into account fabrication constraints. More sophisticated tools for the exploration of the design space, like multi-objective optimization could be used to harness the full potential of Marionette Meshes.

Acknowledgements

This work was made during Mr. Mesnil doctorate within the framework of an industrial agreement for training through research (CIFRE number 2013/1266) jointly financed by the company Bouygues Construction SA, and the National Association for Research and Technology (ANRT) of France.

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