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Improvement on Delay-Dependent Robust Controller Design for Uncertain Takagi-Sugeno Fuzzy Systems with Time-Varying Delays

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Abstract—This paper deals with the analysis and design of robust controllers for a class of uncertain continuous time of Takagi-Sugeno (T-S) fuzzy systems with time varying delays. The closed-loop T-S fuzzy model is obtained using a Parallel Distributed Compensation (PDC) control law with including both memoryless and delayed state feedback gains. Sufficient delay-dependent controller design conditions for uncertain Takagi-Sugeno (T-S) fuzzy systems with time varying delays are derived in terms of linear matrix inequality (LMI). From a convenient choice of a Lyapunov-Krasovskii Functional (LF) associated with free weighting matrices techniques and Finsler's lemma, relaxed the LMI conditions are proposed to reduce the conservatism. A numerical example is presented to demonstrate the effectiveness of the proposed approach and the conservatism improvement regarding to previous results.

I. INTRODUCTION

In the recent decades, Takagi-Sugeno (T-S) fuzzy models [1] have been extensively investigated due to their effectiveness in nonlinear control theory. They are described by fuzzy IF-THEN rules which represent local linear input-output relations of nonlinear systems. Moreover, when they are obtained through the sector nonlinearity approach [2], they are able to match exactly a nonlinear system in a compact sets of its state space and thus becomes a powerful tool to deal with complex systems, including time delay systems.

Time-delays are often observed in many areas of engineering systems such like networked control systems, chemical processes, pneumatic or hydraulic processes, nuclear reactors, telecommunications and so on. Thus, to control such dynamical systems, time-delays must be taken into account for controller synthesis in order to avoid instability and/or undesirable oscillations, i.e. the degradation of the considered system's performances.

According to the size or the nature of time-delays, the stability analysis of T-S fuzzy system with time-delay using Lyapunov-Krasovskii Functionals (LKF) can be classified into two major categories: delay-independent stability conditions [3], [4] and delay-dependent ones [5], [6], [7], [8]. Delay-independent criterions are able to check overall, uniform and asymptotic stability of time-delay systems for any arbitrary positive value of the delay. However, delay-dependent criterions guarantee stability only for all delay belonging into a

specified range, i.e. including information regarding to the delays such as their maximal values or the bounds of their derivatives. It turns out that delay-dependent criterions lead to more relaxed results than delay-independent ones, especially when some informations about the delay are known with small sizes.

As usual, it is generally impossible to fully describe the dynamical behavior of a physical system for three main reasons: The first one is related to the presence of parasitic parameters or processes that are not completely known. The second one is that some control systems must operate in different operating ranges. The third reason relives to use of relatively simple or approximated models to get closer to a practical system, because of the limitation of available mathematical tools. Hence, to cope with these problems, robust controllers have to be designed to guarantee the stability of the controlled system in the presence of uncertainties, i.e. to achieve robust closed-loop stability. There exist recent studies in the field of robust control of uncertain nonlinear systems that focus on uncertain T-S fuzzy systems with time-varying delay see e.g. [5], [9], [6], [10], [11] and references therein. In this works, the Lyapunov-Krasovskii functional (LKF) are widely used in order to determine the maximum allowable delay value with the aid of linear matrix inequalities (LMIs).

In order to get less conservative results, robust stability of fuzzy large-scale systems with time-varying delays by descriptor model transformation and Park's inequality was addressed in [12]. A free-weighting matrices method associated with the extension of the Jensen's inequality has been proposed in [13], [6]. In [11], an novel LMI-based robust H^∞ controller design criterion for uncertain T-S fuzzy systems with state and input time-delays have been presented. Recently, a delay partitioning approach has been proposed to further reduce the conservatism, see e.g. [9], [14] and references therein. We can find many other approaches in the literature based on various mathematical tools to reduce the conservatism, e.g. using the Jensen's inequality [15], using the Wirtinger's inequality approach [16] or using the Finsler's lemma [7].

In this paper, new relaxed LMI-based delay-dependent conditions for robust PDC controller design stabilizing uncertain T-S fuzzy models with state time-varying delays is proposed. The main contribution of this paper is summarized through

three points: 1) the choice of a convenient augmented LKF candidate, 2) the application of an extension of the Jensen's inequality, 3) the application of the Finsler's lemma. In this context, a robust Parallel Distributed Compensation (PDC) control law, which includes both memoryless and delayed state feedbacks, will be considered for generalization purposes. Indeed, despite the fact that such controller requires to assume that the time-varying delay is available online, it will be shown that designing the delayed state feedback gains allows to significantly improve the conservatism reduction for high variation rates of the time-varying delay. To validate the proposed results, the conservatism of the proposed LMI conditions is compared to several previous results through an academic example.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Let us consider an uncertain T-S fuzzy system with time-delays. The i^{th} rule of this T-S fuzzy model ($i = 1, \dots, r$) is given by:

Rule i : IF $z_1(t)$ is μ_{i1} and ... and $z_p(t)$ is μ_{ip} THEN

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i(t))x(t) + (A_i^d + \Delta A_i^d(t))x(t - \tau(t)) \\ \quad + (B_i + \Delta B_i(t))u(t) \\ x(t) = \phi(t), \forall t \in [-\bar{\tau}, 0] \end{cases} \quad (1)$$

where $z(t) = [z_1(t) \dots z_p(t)] \in \mathbb{R}^P$ is the vector of premises which is assumed to depend only on the state variables, i.e. the entries of the state vector $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ is the input vector, μ_{ij} are fuzzy sets, the scalar function $\tau(t) \in [0, \bar{\tau}]$ represents a time-varying delay with $\bar{\tau} < +\infty$ and $\dot{\tau}(t) \leq \eta < +\infty$, $\phi(t)$ is a vector-valued initial function for $t \in [-\bar{\tau}, 0]$, $A_i \in \mathbb{R}^{n \times n}$, $A_i^d \in \mathbb{R}^{n \times n}$ and $B_i \in \mathbb{R}^{n \times m}$ are known constant matrices, $\Delta A_i(t) \in \mathbb{R}^{n \times n}$, $\Delta A_i^d(t) \in \mathbb{R}^{n \times n}$ and $\Delta B_i(t) \in \mathbb{R}^{n \times m}$ are unknown matrices representing Lebesgue measurable uncertainties, which can be rewritten as :

$$\Delta A_i = H_i \delta(t) E_{ai} \quad (2)$$

$$\Delta A_i^d = H_i \delta(t) E_{ai}^d \quad (3)$$

$$\Delta B_i = H_i \delta(t) E_{bi} \quad (4)$$

where $H_i \in \mathbb{R}^{n \times q}$, $E_{ai} \in \mathbb{R}^{q \times n}$, $E_{ai}^d \in \mathbb{R}^{q \times n}$ and $E_{bi} \in \mathbb{R}^{q \times m}$ are known constant real matrices and $\delta(t) \in \mathbb{R}^{q \times q}$ is an unknown real time-varying matrix satisfying:

$$\delta^T(t) \delta(t) \leq I \quad (5)$$

By using the center-average defuzzification, product inference and singleton fuzzifier, the global dynamics of the T-S fuzzy system (1) can be inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(z(t)) ((A_i + \Delta A_i(t))x(t) \\ \quad + (A_i^d + \Delta A_i^d(t))x(t - \tau(t)) \\ \quad + (B_i + \Delta B_i(t))u(t)) \end{cases} \quad (6)$$

where $h_i(z(t))$ are normalized membership functions obtained as:

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^r w_i(z(t))}, w_i(z(t)) = \prod_{i=1}^r \mu_{ij}(z(t)) \quad (7)$$

where $\mu_{ij}(z_j(t)) \in [0, 1]$ is the grade of membership of $z_j(t)$ in μ_{ij} and $h_i(t) \geq 0$ hold the convex sum property $\sum_{i=1}^r h_i(z(t)) = 1$.

Assumption 1: For stabilization purpose and when not explicitly stated in the sequel, the time-varying delay $\tau(t)$ is assumed to be available online at any time t .

To stabilize the uncertain T-S fuzzy models (1) we propose the following PDC control law:

$$u(t) = \sum_{i=1}^r h_i(z(t)) (K_i X^{-1} x(t) + K_i^d X^{-1} x(t - \tau(t))) \quad (8)$$

where, for $i = 1, \dots, r$, $K_i \in \mathbb{R}^{m \times n}$, $K_i^d \in \mathbb{R}^{m \times n}$ and $X > 0$ are the controller gain matrices to be designed.

Note that the control law (8) requires assumption 1, which is considered as a general case to derive new design conditions. Then, it will be shown that straightforward simplifications may apply for particular cases such like constant delays.

In the sequel, the following notations are employed to simplify mathematical expressions.

Notations: Stars * in matrices denote bloc transpose quantities. One denotes $\mathcal{H}_e(M) = M + M^T$ and the set of integer $\mathcal{I}_r = \{1, \dots, r\}$. Let us denote $\bar{A}_i = A_i + \Delta A_i(t)$, $\bar{A}_i^d = A_i^d + \Delta A_i^d(t)$ and $\bar{B}_i = B_i + \Delta B_i(t)$. Moreover, for any set of matrices M_i of appropriate dimensions, one denotes $M_h = \sum_{i=1}^r h_i(z(t)) M_i$ and $M_{hh} = \sum_{i=1}^r \sum_{j=1}^r h_i(z(t)) h_j(z(t)) M_{ij}$.

Thanks to these notations, the considered T-S models with time-varying delays (6) can be rewritten as:

$$\dot{x}(t) = \bar{A}_h x(t) + \bar{A}_h^d x(t - \tau(t)) + \bar{B}_h u(t) \quad (9)$$

as well as the control law (8) as:

$$u(t) = u_m(t) + u_d(t) \quad (10)$$

with $u_m(t) = K_h X^{-1} x(t)$ and $u_d(t) = K_h^d X^{-1} x(t - \tau(t))$.

Thus, from (9) and (10), the closed-loop dynamics can be represented as:

$$\dot{x}(t) = (\bar{A}_h + \bar{B}_h K_h X^{-1}) x(t) + (\bar{A}_h^d + \bar{B}_h K_h^d X^{-1}) x(t - \tau(t)) \quad (11)$$

The purpose of this paper is to propose delay-dependent LMI-based conditions for the design of (10) such that the closed-loop system (11) is globally asymptotically stable (GAS). Before deriving the main results, some lemmas, given below, will be useful to derive the proposed LMI-based conditions. Lemma 1 is derived from the Jensen's integral inequality [15]. It will be used to provide much tighter bounding for cross terms and improve the conservatism as proposed in [17].

Lemma 1: [17] For any constant matrices $Q_{11} = Q_{11}^T$, $Q_{22} = Q_{22}^T$, and $Q_{12} \in \mathbb{R}^{n \times n}$ satisfying $\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \geq 0$, a positive scalar function $\tau(t) \leq \bar{\tau} < +\infty$, and a vector function $\dot{x}(t) : [-\bar{\tau}, 0] \rightarrow \mathbb{R}^n$, such that the following

integrations are well defined, then:

$$-\bar{\tau} \int_{t-\bar{\tau}}^t \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \leq \theta^T(t) \begin{bmatrix} -Q_{22} & Q_{22} & -Q_{12}^T \\ Q_{22} & -Q_{22} & Q_{12}^T \\ -Q_{12} & Q_{12} & -Q_{11} \end{bmatrix} \theta(t) \quad (12)$$

$$\text{with } \theta(t) = \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ \int_{t-\tau(t)}^t x(s) ds \end{bmatrix}.$$

Next, the Finsler's lemma [18], given below, will be used to relax the proposed LMI conditions by adding slack decision variables and decoupling the system's matrices from the Lyapunov-Krasovskiy ones.

Lemma 2: [18] Let $\xi \in \mathbb{R}^n$, $G \in \mathbb{R}^{m \times n}$ and $Q = Q^T \in \mathbb{R}^{n \times n}$ such that $\text{rank}(G) < n$. The following statements are equivalent.

$$\xi^T Q \xi < 0, \quad \forall \xi \in \{\xi \in \mathbb{R}^n : \xi \neq 0, G\xi = 0\} \quad (13)$$

$$\exists R \in \mathbb{R}^{n \times m} : Q + \mathcal{H}_e(RG) \prec 0 \quad (14)$$

Then, to cope with bounded uncertainties, the following usual lemma will be employed.

Lemma 3: [19] Let $Q = Q^T$, H , E and be real matrices of appropriate dimensions and uncertain matrix $\delta(t)$ satisfying (3). The inequality:

$$Q + H\delta(t)E + E^T\delta^T(t)H^T \leq 0 \quad (15)$$

is satisfied if there exists a scalar $\lambda > 0$ such that:

$$Q + \lambda HH^T + \lambda^{-1} E^T E \leq 0 \quad (16)$$

Finally, Lemma 4 will be used as relaxation scheme to reduce the conservatism due to the double sum fuzzy structure of the obtained parametrized LMIs [20]. Note that, among relaxation lemmas, it constitutes a good compromise between complexity and computational burden, see [21] for more details on relaxation schemes.

Lemma 4: [20] For $(i, j) \in \{1, \dots, r\}^2$, Let Γ_{ij} be matrices of appropriate dimensions. $\Gamma_{hh} \prec 0$ is satisfied if both the following conditions hold:

$$\begin{cases} \Gamma_{ii} \prec 0, \forall i \in \{1, \dots, r\} \\ \frac{2}{r-1} \Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} \prec 0, \forall (i, j) \in \mathcal{I}_r^2, i \neq j \end{cases} \quad (17)$$

III. MAIN RESULT

This section aims at developing a novel LMI-based delay dependent conditions for the design of PDC controllers (10) which globally asymptotically stabilizes the uncertain T-S fuzzy system with time-varying delays (9). The main result is proposed by the following theorem.

Theorem 1: Let $(i, j) \in \mathcal{I}_r^2$. For given scalars $\bar{\tau} > 0$ and $\eta \geq 0$ such that $\tau(t) \in [0, \bar{\tau}]$ with $|\dot{\tau}(t)| \leq \eta$, the uncertain T-S fuzzy model with time varying delays (11) is globally asymptotically stabilized by the PDC controller (10) if there exist the real matrices with appropriate dimensions $L = L^T >$

0 , K_j , K_j^d , X , $P_{11} = P_{11}^T$, $P_{22} = P_{22}^T$, P_{12} , $Q_{11} = Q_{11}^T$, $Q_{22} = Q_{22}^T$ and Q_{12} , and the scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$ and $\lambda_i > 0$ such that the following LMI-based conditions hold:

$$\Gamma_{ii} < 0, \quad \forall i \in \mathcal{I}_r \quad (18)$$

$$\frac{2}{r-1} \Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0, \quad \forall (i, j) \in \mathcal{I}_r^2 / i \neq j \quad (19)$$

$$\begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} > 0 \quad (20)$$

$$\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0 \quad (21)$$

with:

$$\Gamma_{ij} = \begin{bmatrix} \tilde{\Pi} + \mathcal{H}_e(\tilde{G}_{ij}) + \lambda_i \tilde{H}_i \tilde{H}_i^T & \tilde{E}_{ij}^T \\ * & -\lambda_{ij} I \end{bmatrix}$$

where:

$$\tilde{\Pi} = \begin{bmatrix} \tilde{\Pi}_{11} & \tilde{\Pi}_{12} & P_{22} - Q_{12}^T & P_{11} + \bar{\tau}^2 Q_{12} \\ * & \tilde{\Pi}_{22} & \tilde{\Pi}_{23} & 0 \\ * & * & -Q_{11} & P_{12}^T \\ * & * & * & \bar{\tau}^2 Q_{22} \end{bmatrix}$$

with:

$$\tilde{\Pi}_{11} = L + P_{12} + P_{12}^T + \bar{\tau}^2 Q_{11} - Q_{22},$$

$$\tilde{\Pi}_{12} = -(1-\eta)P_{12} + Q_{22},$$

$$\tilde{\Pi}_{22} = -(1-\eta)L - Q_{22},$$

$$\tilde{\Pi}_{23} = -(1-\eta)P_{22} + Q_{12}^T,$$

$$\tilde{G}_{ij} = \begin{bmatrix} A_i X + B_i K_j & A_i^d X + B_i K_j^d & 0 & -X \\ \varepsilon_1 (A_i X + B_i K_j) & \varepsilon_1 (A_i^d X + B_i K_j^d) & 0 & -\varepsilon_1 X \\ \varepsilon_2 (A_i X + B_i K_j) & \varepsilon_2 (A_i^d X + B_i K_j^d) & 0 & -\varepsilon_2 X \\ \varepsilon_3 (A_i X + B_i K_j) & \varepsilon_3 (A_i^d X + B_i K_j^d) & 0 & -\varepsilon_3 X \end{bmatrix},$$

$$\tilde{H}_i = [H_i^T \quad \varepsilon_1 H_i^T \quad \varepsilon_2 H_i^T \quad \varepsilon_3 H_i^T]^T,$$

and:

$$\tilde{E}_{ij} = [E_{ai} X + E_{bi} K_j + E_{ai}^d X + E_{bi} K_j^d \quad 0 \quad 0]$$

Proof: Let us consider the following LKF candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (22)$$

with:

$$V_1(t) = \theta_1^T(t) M \theta_1(t), \quad (23)$$

$$V_2(t) = \int_{t-\tau(t)}^t x^T(s) S x(s) ds, \quad (24)$$

$$V_3(t) = \bar{\tau} \int_{-\bar{\tau}}^0 \int_{t+s}^t \theta_2^T(w) N \theta_2(w) dw ds, \quad (25)$$

where

$$\theta_1(t) = \begin{bmatrix} x(t) \\ \int_{t-\tau(t)}^t x(s) ds \end{bmatrix}, \theta_2(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$$

$$M = \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix} > 0, N = \begin{bmatrix} N_{11} & N_{12} \\ * & N_{22} \end{bmatrix} > 0$$

The closed-loop system (11) is GAS if:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) < 0 \quad (26)$$

Let us first focus on the time derivative of (23), one has:

$$\dot{V}_1(t) = 2 \left[\begin{array}{c} x(t) \\ \int_{t-\tau(t)}^t x(s) ds \end{array} \right]^T M \left[\begin{array}{c} \dot{x}(t) \\ \frac{d}{dt} \int_{t-\tau(t)}^t x(s) ds \end{array} \right] \quad (27)$$

Since $\frac{d}{dt} \left(\int_{t-\tau(t)}^t x(s) ds \right) = x(t) - (1 - \dot{\tau}(t))x(t - \tau(t))$, and assuming that $\dot{\tau}(t) \leq \eta$, from (27) we have:

$$\dot{V}_1(t) \leq \zeta^T(t) \Pi_1 \zeta(t) \quad (28)$$

with:

$$\zeta(t) = \left[\begin{array}{c} x(t) \\ x(t - \tau(t)) \\ \int_{t-\tau(t)}^t x(s) ds \\ \dot{x}(t) \end{array} \right] \quad (29)$$

and:

$$\Pi_1 = \begin{bmatrix} \mathcal{H}_e(M_{12}) & -(1-\eta)M_{12} & M_{22} & M_{11} \\ * & 0 & -(1-\eta)M_{22} & 0 \\ * & * & 0 & M_{12}^T \\ * & * & * & 0 \end{bmatrix}$$

Now, let us focus on the time derivative of (24), we have:

$$\dot{V}_2(t) = x^T(t) S x(t) - (1 - \dot{\tau}(t)) x^T(t - \tau(t)) S x(t - \tau(t)) \quad (30)$$

That is to say:

$$\dot{V}_2(t) = \zeta^T(t) \Pi_2 \zeta(t) \quad (31)$$

where:

$$\Pi_2 = \begin{bmatrix} S & 0 & 0 & 0 \\ 0 & -(1 - \dot{\tau})S & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then, let us focus on the time derivative of (25), one has:

$$\dot{V}_3(t) = \bar{\tau}^2 \theta_2^T(t) N \theta_2(t) - \bar{\tau} \int_{t-\bar{\tau}}^t \theta_2^T(s) N \theta_2(s) ds \quad (32)$$

Then applying lemma 1 on the second right hand term of (32), we obtain:

$$\dot{V}_3(t) \leq \bar{\tau}^2 \theta_2^T(t) N \theta_2(t) + \theta(t)^T \tilde{N} \theta(t) \quad (33)$$

with:

$$\tilde{N} = \begin{bmatrix} -N_{22} & N_{22} & -N_{12}^T \\ N_{22} & -N_{22} & N_{12}^T \\ -N_{12} & N_{12} & -N_{11} \end{bmatrix}$$

and $\theta(t)$ defined in (12).

The inequality (32) can be rearranged as follows:

$$\dot{V}_3(t) \leq \zeta^T(t) \Pi_3 \zeta(t) \quad (34)$$

with:

$$\Pi_3 = \begin{bmatrix} \bar{\tau}^2 N_{11} - N_{22} & N_{22} & -N_{12}^T & \bar{\tau}^2 N_{12} \\ * & -N_{22} & N_{12}^T & 0 \\ * & * & -N_{11} & 0 \\ * & * & * & \bar{\tau}^2 N_{22} \end{bmatrix}$$

Thus, from (28), (31), (34), the condition (26) is satisfied if:

$$\zeta^T(t) \Pi \zeta(t) < 0 \quad (35)$$

where:

$$\Pi = \Pi_1 + \Pi_2 + \Pi_3$$

Now, let us rewrite the closed-loop T-S system with time-varying delays (11) as:

$$(\mathcal{G}_{hh} + \Delta \mathcal{G}_{hh}) \zeta(t) = 0 \quad (36)$$

with:

$$\mathcal{G}_{hh}^T = \begin{bmatrix} (A_h + B_h K_h X^{-1})^T \\ (A_h^d + B_h K_h^d X^{-1})^T \\ 0 \\ -I \end{bmatrix}, \quad \Delta \mathcal{G}_{hh}^T = \begin{bmatrix} (\Delta A_h + \Delta B_h K_h X^{-1})^T \\ (\Delta A_h^d + \Delta B_h K_h^d X^{-1})^T \\ 0 \\ 0 \end{bmatrix}$$

where $\zeta(t)$ is defined in (29).

From (35) and (36), we can apply the lemma 2 and the uncertain closed-loop T-S system with time-varying delay (11) is stable if there exists $\mathcal{R} \in \mathbb{R}^{4n \times n}$ such that:

$$\Pi + \mathcal{H}_e(\mathcal{R}(\mathcal{G}_{hh} + \Delta \mathcal{G}_{hh})) < 0 \quad (37)$$

Let $\mathcal{R}^T = [X^{-T} \ \varepsilon_1 X^{-T} \ \varepsilon_2 X^{-T} \ \varepsilon_3 X^{-T}]$ with $X \in \mathbb{R}^{n \times n}$ invertible and arbitrary scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3$. Let also $D_X = \text{diag}[X \ X \ X \ X]^T$, after matrix expansion and multiplying (37) left by D_X and right by D_X^T , and with the change of variables $P_{ab} = X^T M_{ab} X$, $Q_{ab} = X^T N_{ab} X$ ($\forall (a, b) \in \{1, 2\}^2, a \leq b$), $L = X^T S X$, the inequality (37) yields:

$$\tilde{\Pi} + \mathcal{H}_e(\tilde{\mathcal{G}}_{hh} + \Delta \tilde{\mathcal{G}}_{hh}) < 0 \quad (38)$$

with $\tilde{\Pi}$ defined in theorem 1 and:

$$\tilde{\mathcal{G}}_{hh} = \begin{bmatrix} \tilde{A} & \tilde{A}^d & 0 & -X \\ \varepsilon_1 \tilde{A} & \varepsilon_1 \tilde{A}^d & 0 & -\varepsilon_1 X \\ \varepsilon_2 \tilde{A} & \varepsilon_2 \tilde{A}^d & 0 & -\varepsilon_2 X \\ \varepsilon_3 \tilde{A} & \varepsilon_3 \tilde{A}^d & 0 & -\varepsilon_3 X \end{bmatrix}, \quad \Delta \tilde{\mathcal{G}}_{hh} = \begin{bmatrix} \Delta \tilde{A} & \Delta \tilde{A}^d & 0 & -X \\ \varepsilon_1 \Delta \tilde{A} & \varepsilon_1 \Delta \tilde{A}^d & 0 & -\varepsilon_1 X \\ \varepsilon_2 \Delta \tilde{A} & \varepsilon_2 \Delta \tilde{A}^d & 0 & -\varepsilon_2 X \\ \varepsilon_3 \Delta \tilde{A} & \varepsilon_3 \Delta \tilde{A}^d & 0 & -\varepsilon_3 X \end{bmatrix}$$

where $\tilde{A} = A_h X + B_h K_h$, $\tilde{A}^d = A_h^d X + B_h K_h^d$, $\Delta \tilde{A} = \Delta A_h X + \Delta B_h K_h$ and $\Delta \tilde{A}^d = \Delta A_h^d X + \Delta B_h K_h^d$.

Expanding $\Delta \tilde{\mathcal{G}}_{hh}$ with (2), (3) and (4), the inequality (38) can be rewritten as:

$$\tilde{\Pi} + \mathcal{H}_e(\tilde{\mathcal{G}}_{hh}) + \mathcal{H}_e(\tilde{H}_h \delta(t) \tilde{E}_{hh}) < 0. \quad (39)$$

with

$$\tilde{H}_h = [H_h^T \ \varepsilon_1 H_h^T \ \varepsilon_2 H_h^T \ \varepsilon_3 H_h^T]^T,$$

$$\tilde{E}_{hh} = [E_{ah} X + E_{bh} K_h + E_{ah}^d X + E_{bh} K_h^d \ 0 \ 0]$$

Then, applying lemma 3 and from (5), the inequality (39) is satisfied if:

$$\tilde{\Pi} + \mathcal{H}_e(\tilde{\mathcal{G}}_{hh}) + \lambda_h \tilde{H}_h \tilde{H}_h^T + \lambda_h^{-1} \tilde{E}_{hh}^T \tilde{E}_{hh} < 0 \quad (40)$$

where λ_h is a scalar function. Now, applying the Schur complement, yields:

$$\begin{bmatrix} \tilde{\Theta} + \mathcal{H}_e(\tilde{\mathcal{G}}_{hh}) + \lambda_h \tilde{H}_h \tilde{H}_h^T & \tilde{E}_{hh}^T \\ * & -\lambda_{hh} I \end{bmatrix} < 0 \quad (41)$$

Then, applying lemma 2, one obtains the conditions proposed in theorem 1. \blacksquare

Remark 1 Let us point-out that the conditions expressed in theorem 1 are LMIs if the scalars ε_1 , ε_2 and ε_3 are prefixed. Note that, according to the proof of theorem 1 these scalars can be arbitrarily chosen. Nevertheless, they are useful to provide more degree of freedom to the four X^{-1} block elements of the slack decision matrix \mathcal{R} , which comes from the use of the Finsler's lemma. Hence in practice, to solve such kind of LMI conditions, these scalars are obtained from linear programming and searched in a logarithmically spaced family, e.g. $(\varepsilon_1, \varepsilon_2, \varepsilon_3) \in \{10^{-6}, 10^{-5}, \dots, 10^6\}^3$. As stated in [22], this way of doing is generally outperforming the results obtained without these scalars.

Remark 2 One acknowledges that assumption 1, which is required to implement the control law (8), may be challenging in practical applications. Nevertheless a straightforward simplification of theorem 1 can be considered for constant delays by setting $\eta = 0$. In this case, if the constant time delay is known, the implementation of the controller (8) is no longer challenging. Moreover, if the time delay is available online but its variation rate is unknown, a straightforward simplification of theorem 1 hold with $L = 0$, $P_{12} = 0$ and $P_{22} = 0$.

IV. ILLUSTRATIVE EXAMPLE

In this section, we will provide a numerical example to illustrate the effectiveness and the conservatism reduction of the above proposed theorem regarding to previous results. Therefore, let us consider the following academic uncertain T-S systems with time-varying delay given by:

$$\dot{x}(t) = \sum_{i=1}^2 h_i(z(t)) (\bar{A}_i x(t) + \bar{A}_i^d x(t - \tau(t)) + \bar{B}_i u(t)) \quad (42)$$

with:

$$A_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, A_1^d = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix},$$

$$A_2^d = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$H_1 = H_2 = \begin{bmatrix} -0.03 & 0 \\ 0 & 0.03 \end{bmatrix}$$

$$E_{a1} = E_{a2} = \begin{bmatrix} -0.15 & 0.2 \\ 0 & 0.04 \end{bmatrix}$$

$$E_{a1}^d = E_{a2}^d = \begin{bmatrix} -0.05 & -0.35 \\ 0.08 & -0.45 \end{bmatrix}$$

and with the membership functions given by:

$$h_1(x_1(t)) = 1 / \left(1 + e^{-2(x_1(t) + \pi)} \right),$$

$$h_2(x_1(t)) = 1 - h_1(x_1(t))$$

Note that this academic uncertain system has been considered in many previous studies for conservatism comparison (see the references in Table 1).

Let us first illustrate the conservatism improvement provided by theorem 1 regarding to some previous studies, according to the feasibility fields of their respective LMI conditions. Figure 1 shows a comparison of the feasibility fields obtained from theorem 1, theorem 3 in [5] and corollary 2 in [6], for $\eta \in [0, 3]$ and $\bar{\tau} \in [0, 2.5]$. As one can notice, theorem 1 provides the widest feasibility field.

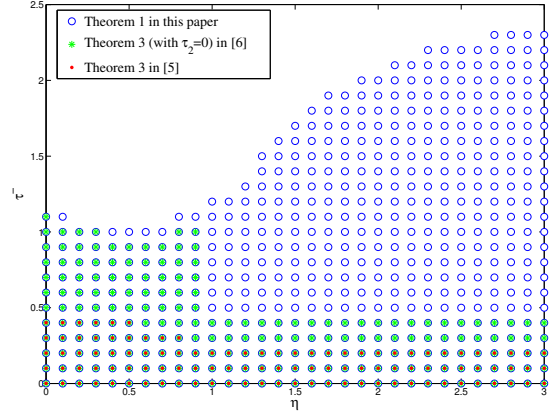


Fig. 1. Feasibility fields obtained from theorem 1, theorem 3.3 in [5] and theorem 3 in [6].

In Table 1, the maximal allowable upper bound of $\tau(t)$ (denoted $maub(\bar{\tau})$) are collected from several previous studies in comparison to the ones obtained from theorem 1. The symbol "-" indicates that the result is non available from the considered study. Moreover, "m" denotes the number of delay partitions considered in [9]. As one can notice, among all the provided results, the ones obtained from theorem 3 provide the biggest allowable values of $\bar{\tau}$.

TABLE I. MAXIMUM ALLOWABLE UPPER BOUND OF $\tau(t)$

Considered results	$\eta = 0$	$\eta = 0.6$	$\eta = 3$	η unknown
Theorem 4 in [10]	-	-	-	0.3991
Theorem 3 in [5]	0.461	0.3941	0.2283	0.2321
Theorem 2 in [23]	-	0.6072	-	-
Theorem 2 in [24]	-	0.7896	-	-
Theorem 2 in [25]	-	0.7780	-	-
Theorem 3 in [6]	1.1644	1.0534	0.4780	0.4144
Theorem 4 in [9]	-	-	1.135	-
Theorem 1 in [13]	1.1648	1.0594	2.1813	0.9632
Theorem 1 in this paper	1.1648	1.0731	2.3521	0.9654

The optimal solution of theorem 1 (for $\bar{\tau} = 2.3521$ and $\eta = 3$) provides the following PDC gains for the control law (8):

$$K_1 = [21.989 \quad -64.212], K_2 = [20.834 \quad -62.531],$$

$$K_1^d = [0.349 \quad -2.801], K_2^d = [-1.125 \quad -1.461].$$

A simulation of the designed closed-loop uncertain system has been realized with the initial conditions $x(0) = [2 \quad 1]^T$,

$\forall t \in [-\bar{\tau}, 0], \phi(t) = x(0)$ and with a time-varying delay $\tau(t) = \frac{\bar{\tau}}{2} (1 + \sin(\frac{2\eta t}{\bar{\tau}}))$, which satisfies $\bar{\tau} = 2.3521$ and $\eta = 3$. Moreover for this simulation, a random uncertain signal $\delta(t)$ has been considered. Figure 2 shows the obtained state trajectories, the control signal, the uncertain signal, as well as the time-varying delay. This clearly shows that the designed fuzzy controller is able to asymptotically stabilize the uncertain closed-loop fuzzy system (42) despite the presence of the uncertainties and the time-varying delay.

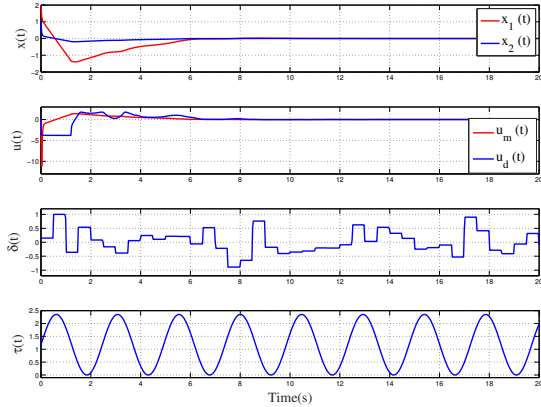


Fig. 2. Closed-loop simulation: state response $x_i(t)$, control signal $u(t)$ and uncertain signal $\delta(t)$

V. CONCLUSION

In this paper, new delay-dependent LMI-based conditions for the design of robust PDC controllers dedicated to stabilize uncertain T-S fuzzy systems with time-varying delays have been presented. These have been shown less conservative than previous related results in the literature through an numerical example. The conservative improvement of the proposed result come from a convenient choice of a LKF, the application of the Finsler's Lemma and an improved Jensen's inequality. Further works will be conducted to alleviate the requirement on the time-varying delay $\tau(t)$ to be available online at any time t , for instance by introducing dedicated observers for its online estimation.

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