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► **To cite this version:**

Yasmina Bouzarour-Amokrane, Ayeley Tchangani, François Pérès. Defining and measuring risk and opportunity in BOCR framework for decision analysis. 9e International Conference of Modeling and Simulation - MOSIM'12, Jun 2012, Bordeaux, France. pp.0. <hal-01977490>

**HAL Id: hal-01977490**

**<https://hal.science/hal-01977490v1>**

Submitted on 10 Jan 2019

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**To cite this version:**

Bouzarour-Amokrane, Yasmina and Tchangani, Ayeley and Peres, François *Defining and measuring risk and opportunity in BOCR framework for decision analysis*. (2012) In: 9e International Conference of Modeling and Simulation - MOSIM'12 , 6-8 Jun 2012, Bordeaux, France.

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# DEFINING AND MEASURING RISK AND OPPORTUNITY IN BOCR FRAMEWORK FOR DECISION ANALYSIS

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**ABSTRACT :** Decision aid through analysis of benefit, opportunity, cost and risk (BOCR) offer the structural framework to get all necessary information to an effective decision making. It was investigated by researchers of different fields (economics, engineering, management, ...) and many evaluation and recommendation methods have been published in the literature to help decision makers to make their choice. However, most of the existing methods do not focus on quantification and evaluation of uncertain parameters of decision problem representing risk and opportunity. To address these issues, we propose in this paper a new BOCR analysis framework including definition and a measure of risk and opportunity. The basic idea highlights the bipolarity nature of the attributes that characterize alternatives with regard to objectives. Indeed, we consider that alternatives are evaluated against several objectives by using many features of them known as attributes. Taking into account uncertainty of some components or relationships, this paper proposes explicit modeling of risk and opportunity in the BOCR framework. The evaluation and recommendation because of the bipolar nature are made using the Satisficing game theory.

**KEYWORDS:** risk, opportunity, BOCR analysis, indicators, bipolarity of attributes, fuzzy measures.

## 1 INTRODUCTION

The characteristics of decision-making environment which is considered in this paper are (see Figure 1) as the following:

*Actors:* group of persons or organizations involved in different phases of decision process (identification and evaluation of objectives, alternatives, indicators and attributes).

*Objective:* something that decision makers want to achieve, realize, optimize, etc. We assume that a set  $O$  of  $k$  objectives has been identified and given by equation (1).

$$O = \{o_1, o_2, \dots, o_k\} \quad (1)$$

*Alternative:* Action, issue or possibility offered to decision makers to achieve their objectives. We consider that a discrete set of  $n$  alternatives is identified and given by equation (2).

$$U = \{u_1, u_2, \dots, u_n\} \quad (2)$$

*Attribute:* feature of an alternative  $u$  used to evaluate it with regard to objectives. Let us consider a couple

objective-alternative  $(o, u)$ , the set of attributes of  $u$  with regard to  $o$  is then given by equation (3).

$$A^o(u) = \{a^o_1, a^o_2, \dots, a^o_m\} \quad (3)$$

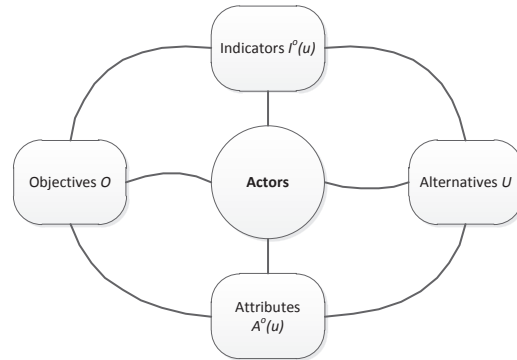


Figure 1: The considered decision problem framework.

*Indicator:* an index used to evaluate the degree of achievement of an objective, the set of indicators that allows the evaluation of an objective  $o$  is given by equation 4.

$$I^o = \{i^o_1, i^o_2, \dots, i^o_x\} \quad (4)$$

Figure 1 resumes the framework of decision making problem we are considering in this paper.

This framework constitutes a multi-objectives, multi-attributes and multi-actors decision making problems. Different approaches have been proposed in the literature to deal of versions (either multi-objectives or multiattributes) of these problems: Outranking Methods (Huron & Zopounidis, 1997) (Electre I, II, III, PROMETHEE, etc.); Multi-attribute Utility Theory (MAUT/MAVT) (Keeney & Raiffa, 1993), (Luce & P. Suppes, 1964), Bayesian Analysis (for structuring and evaluating decision making problem), voting (Peniwati, 2007), Analytical Hierarchy/Network Process AHP/ANP methods (Saaty, 1980), (Saaty, 2001a), etc. These methods mainly consider a common set of attributes for all the alternatives considering that the elicitation of attributes is independent of alternative, and ignore uncertain aspects that some components can present. However, attributes may depend on alternatives in terms of achieving objectives or not. Therefore, in this paper we consider the possibility that alternatives are characterized by different attributes.

The stages of decision making problem under consideration are described as follows:

1. Actors select objectives and indicators to measure achievement of these objectives.
2. Then potential alternatives are identified.
3. For each pair (objective, alternative), actors or experts determine a set of attributes that permit the evaluation of the alternative with regard to the objective.
4. Attributes values are assessed by expertise or direct measure.
5. Alternatives are evaluated with regard to the objectives for recommendation.

The work undertaken in this paper mainly concerns the last two points with a special focus on the evaluation procedure.

The remainder of this paper is organized as follows, the second section presents the BOCR analysis framework, section three provides a definition and a method of evaluating uncertain parameters of the approach, namely, risk (R) and opportunity (O). In fourth section we present the aggregation methods that can be used for final evaluation which is the object of section five. Section six provides an example of application. Conclusion is given in the last section.

## 2 BOCR ANALYSIS

The decision to undertake a project or to choose an alternative, usually requires investigating the ratio of positive and negative aspects that project or alternative presents'. The BOCR analysis (Saaty, 2001b), (Saaty & Özdemir, 2005) enables a riche analysis; it is based on the bipolarity nature of attributes with regard to objectives in terms of support and reject (Tchangani, 2010), (Tchangani *et al.*, 2011), but many of the aspects

that define the factors and their relationships are usually difficult to specify and quantify (Wijnmalen, 2007). To address this, we propose in the following a new framework of evaluation procedure, we define bipolarity by proposing supporting and rejecting notions. We propose then a method to identify attributes taking into account their bipolar nature in the BOCR analysis context.

*Definition 1:* An objective  $o$  is said to be supported (respect. rejected) by an attribute  $a$  if and only if its variation is positively (respect. negatively) correlated with the variation of that attribute. Otherwise this attribute is said to be neutral with regard to that objective (Tchangani *et al.*, 2011).

Thus, for a couple  $(o, u)$ , the set of attributes  $A^o(u)$  given by equation (3) is divided into a subset of attributes that support objective  $o$ ,  $A_s^o(u)$  and a subset  $A_r^o(u)$  of attributes that reject it. Elicitation of sets  $A_s^o(u) / A_r^o(u)$  can be done in response to questions like « what characteristics of alternative  $u$  'allows' / 'prevents' the realization of an objective? ».

Then, using the uncertainty parameter, we divide the set of supporting attributes of objective  $o$ ,  $A_s^o(u)$  between benefice (certain attributes) and opportunity (uncertain attributes). Similarly, the set  $A_r^o(u)$  is divided between cost (certain attributes) and risk (uncertain attributes). Figure 2 summarizes the BOCR hierarchy.

Benefit and cost attributes can be considered as immediate characteristics of alternative whereas opportunity and risk represent what could be expected from this alternative.

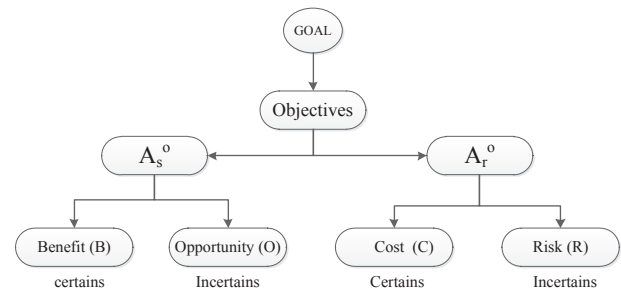


Figure 2: Bipolar hierarchy of attributes.

The next step in the decision process is to assess the strength of these relationships for each alternative with regard to objectives by using attributes. To this end, several assessment methods can be combined with BOCR analysis, among the most common, AHP/ANP approaches (see (Erdogmus *et al.*, 2005), (Feglar *et al.*, 2006), (Saaty, 2001b), (Saaty & Shang, 2007)) and fuzzy AHP approaches (see (Lee, 2009a), (Lee, 2009b)). After that, assessed results must be aggregated in a unique value for recommendation procedure, here again, several aggregation methods of b, o, c, r factors are proposed and discussed in literature (see for exemple (Saaty & Özdemir, 2005), (Saaty, 2001b), (Lee, 2009a) (Saaty, 2003)). In the next section, we present a new evaluation

approach that takes into account quantification of uncertain parameters.

## 2.1 Procedures in BOCR analysis

Solving a decision problem by BOCR analysis implies to evaluate alternatives with regard to objectives, based on their positive attributes  $(b, o)$  and negative attributes  $(c, r)$ . The evaluation process allows judging the ability of an alternative  $u$  to achieve an objective  $o$  based on its attributes  $A^o(u)$ .

In the approach that we propose, performance of alternatives with regard to an objective  $o$  is quantified using a set of indicators  $I^o$ . These indicators will help measuring the achievement of an objective  $o$  by an alternative  $u$  based on evaluation of its attributes  $a^o(u)$ . Given the distribution of attributes shown in Figure 2; performance of alternative  $u$  for objective  $o$  is given by equation (5).

$$I^o(u) = \varphi^o(a^o(u)) = \begin{cases} \varphi_b^o(a_b^o(u)) \\ \varphi_o^o(a_o^o(u)) \\ \varphi_c^o(a_c^o(u)) \\ \varphi_r^o(a_r^o(u)) \end{cases} \quad (5)$$

where

$\varphi_x^o$ : is the aggregated measure of the corresponding component related to the factor  $x$  ( $x = b, o, c, r$ ).

$a^o(u)$ : vector of attributes of alternative  $u$  divided into four components.

$$a^o(u) = [a_b^o(u) \ a_o^o(u) \ a_c^o(u) \ a_r^o(u)] \quad (6)$$

$a_x^o(u)$ : is the sub-vector of attributes of alternative  $u$  related to the factor  $x$ .

For ‘certain’ factors, given certain attributes  $a^o(u)$ , evaluation function  $\varphi_x^o(a_x^o(u))$  ( $x = b, c$ ) can be obtained by direct evaluation or by pairwise comparison methods like AHP (Tchangani *et al.*, 2011). For uncertain factors, the definition and measure of risk and opportunity must be established to obtain the corresponding evaluation functions  $\varphi_x^o(a_x^o(u))$  ( $x = r, o$ ). The following section provides a definition and measures of risk and opportunity.

## 3 EVALUATION PROCEDURE FOR RISK AND OPPORTUNITY

Any alternative may present some risks to avoid and opportunities to seize. To choose the ‘good’ alternative, a strict evaluation of these factors is necessary. To proceed to the estimation of risk and opportunity related to the corresponding alternatives, we propose in the following, definition and methodology for measuring these factors in BOCR analysis framework.

### 3.1 Definition and measure of risk and opportunity

Unlike a speculative risk that considers opportunity as a positive effect and risk as negative effect of the same factor or event (Institute P.M., 2000), (Hillson, 2002), risk and opportunity in our approach are strictly negative and positive respectively and related to independent events or factors (attributes).

To keep the common terminology used in literature, risk and opportunity attribute are named respectively ‘risk factor’ and ‘opportunity factor’. For an alternative  $u$ , the risk related to an objective  $o$  with regard to the risk factor depends on the occurrence of this factor and negative impact it may have on the objective. To identify a risk measure, it’s required to answer three questions (Haimes *et al.*, 2002):

- i) What can go wrong? (Identification of risk factor).
- ii) What is the likelihood of that happening? (Probability of occurrence).
- iii) What are the consequences on the objective  $o$  if it goes wrong?

The opportunity can be identified and measured through similar questions:

- i) What can go well?
- ii) What is the likelihood that it goes well?
- iii) What are the consequences on the objective  $o$  if it goes well?

*Definition 2:* Let  $R^o(u)$  and  $O^o(u)$  be respectively, the measure of risk and opportunity related to an objective  $o$  for an alternative  $u$ , these measures are given by:

$$R^o(u) = \varphi_r^o(a_r^o(u)) = S_r^o(a_r^o(u)) Pr(a_r^o(u)) \quad (7)$$

$$O^o(u) = \varphi_o^o(a_o^o(u)) = Imp_r^o(a_o^o(u)) Pr(a_o^o(u)) \quad (8)$$

where

$S_r^o$ : is the severity of risk factors on the achievement of objective  $o$ .

$Imp_r^o$ : is the importance of opportunity factors on the achievement of objective  $o$ .

Parameters  $S_r^o(\cdot)$  and  $Imp_r^o(\cdot)$  are often measured numerically and can take many forms. To obtain them, we must define what is meant by ‘achieving an objective’. For us, achieving an objective is measured through the achievement or not of its indicators. Thus, the severity of a risk factor for an objective  $o$  can be measured by the degree of non-achievement of these objective indicators.

As decision makers are better able to comment on trends rather than fixed values; one possibility to define this achievement is to use a fuzzy discretization. For instance, an indicator  $i$  may be evaluated over a set  $L_i$  of labels  $L_i = \{l_1, l_2, \dots, l_n\}$ , (such as {low, medium, high, very high, etc.}) used to assess the degree of achievement of an objective  $i$ . Among these labels, some

labels will be named 'green labels' that is they correspond to the realization of the indicator and other named 'red labels' because they must be avoided. Consequently, severity may be then given by equation:

$$S_r^o(a_r^o(u)) = Pr(i \text{ takes a red label} | a_r^o(u)) \quad (9)$$

Similarly, importance related to opportunity is given by equation (10):

$$\begin{aligned} Imp_o^o(a_o^o(u)) \\ = Pr(i \text{ takes a green label} | a_o^o(u)) \end{aligned} \quad (10)$$

We do think that decision makers and/or expert interpret better information requested of them, with these formulations. Data nature, quality and quantity will determine the methods for estimation of conditional probabilities. The degree of expert knowledge also plays a role in the choice of valuation method if the estimation is made by expertise. Data can be of various kinds: certain, uncertain, quantitative or qualitative. If data are quantitative and certain in large quantities, statistical methods can be deployed to calculate conditional probabilities. In the case where data are missing and / or are uncertain and qualitative, human expertise is involved and subjective judgments are made by expert decision makers. The (AHP/ANP) approaches may be proposed to allow the assessment of conditional probabilities by pairwise comparison or prospective methods such as Smic-Prob Expert (Godet, Monti, Meunier, & Roubelat, 2000), (Martino, 1992) which aims to determine simple and conditional probabilities of assumptions and / or events from expert opinion. However, the estimation can be characterized by a lack of precision, uncertainty and subjectivity. To prevent this, the fuzzy logic (Bouchon-Meunier, 1990), (Klir *et al.*, 1988) proposes to manage hesitation, ambiguity and overcome limitations of classical Boolean logic. It proposes to formalize the use of vague terms and makes them able to be manipulated. Finally, if data are ignored (totally or partially) the theory of evidence (Shafer, 1976) can treat uncertainty and handle events that are not necessarily exclusive. This allows the explicit representation of the uncertainty taking into account what is unknown.

To simplify evaluation and recommendation, data aggregation is needed. Initially, the severity / importance calculated with regard to risk / opportunity factors are aggregated for each objective (for example, we note  $S_{r,i}^o(\cdot)$  aggregation result of  $S_{r,i}^o(\cdot)$  where  $i$  is the index of risk factor  $a_{r,i}^o$  and we write  $S_r^o(\cdot) = \text{agregation of } S_{r,i}^o(\cdot)$ ). In second step, the global risk / opportunity of each alternative are aggregated over all the objectives.  $(R(u)/O(u) = \text{aggregation of } (R^o(u)/O^o(u))$ . The next section is devoted to methods of aggregation that can be used.

## 4 AGREGATION METHODS

We consider generically in this section a set  $X = \{x_1, x_2, \dots, x_n\}$  of positive numerical values to be aggregated to obtain a single value.

The aggregation concept is a common feature of all the multi-criteria decision-making problem evaluation procedures (Multi-Attributes Utility Theory (MAUT) approach or Outranking Methods). In the MAUT procedure, one-dimensional utility functions are aggregated into a single overall utility by combining all the attributes while in the outranking methods as ELECTRE, we aggregate the preference relations in pairs of alternatives (see (Grabisch, 1996) and (Schärlig, 1985)).

The most used aggregation method until recently is the well known Weighted Arithmetic Mean (JMarichal, 2000), (Grabisch & Labreuche, 2005), (Grabisch *et al.*, 2006).

However, these methods have the disadvantage of ignoring interaction between elements to aggregate, such as synergy, redundancy or independence. To correct this, fuzzy integrals have been set up to consider these interactions. Among these integrals, the Choquet integral (Marichal., 1999), (Marichal, 2002), (Grabisch, 1996) is going to be introduced in the next section.

### 4.1 The Choquet Integral

The Choquet integral is considered like an adequate substitute for the weighted arithmetic mean; it proposes to define weight not only for each element but also for each subset of elements (Marichal, 2000). It has been proposed by many authors to aggregate interacting elements in the case of cardinal unipolar scales, in multi-criteria decision making ((Marichal, 2002), (Grabisch, 1996)). To obtain it, we must define a fuzzy measure on  $X$  (Sugeno m, 1974).

*Definition 3:* The fuzzy measure (also called capacity) defined on the set  $X$  of elements to aggregate, represents the overall score of final elements to be evaluated. It is a function  $v: 2^X \rightarrow [0,1]$  satisfying the following conditions:

- i)  $v(\emptyset) = 0, v(X) = 1,$
- ii)  $S \subseteq T \Rightarrow v(S) \leq v(T), \forall S, T \subseteq X$

where,  $2^X$  is the set of parts of  $X$ .

For all  $S \subseteq X, v(S)$  can be interpreted as the weight of the importance of elements combination of the set  $S$  (weights relative to  $S$ ).

*Definition 4:* Let  $v$  be a fuzzy measure on  $X$ , the Choquet integral of numerical function  $x$  with regard to  $v$  is defined by:

$$C_v(x) := \sum_{i=1}^n (v(A_{(i)}) |x_{(i)} - x_{(i-1)}|) \quad (11)$$

Where  $(.)$  indicates permutation of  $X$  such that  $0 \leq x_{(1)} \leq \dots \leq x_{(n)}$  and  $A(i) = \{(i), \dots, (n)\}$ .

For more details on the axiomatic characterization of Choquet integrals, we refer the reader to references (Marichal J.-L., 1998), (Marichal J.-L., 1999).

One disadvantage of the Choquet integral use to define a fuzzy measure that requires the necessity to specify  $2^n - 2$  coefficients for a set of  $n$  elements, which can be difficult to obtain for large values of  $n$ . To overcome this difficulty, methods based on specific capacities requiring fewer coefficients have been proposed. Among these methods, decomposable capacities and  $k$ -additive capacities. Decomposable capacities are based on the idea that elements can be partitioned into  $g$  groups of elements in-distinguishable, while  $k$ -additive capacities are capacities with indexes of interaction zero beyond  $k$  elements. This allows to deduce that a 1-additive capacity amounts to an additive capacity; in this case, the Choquet integral is reduced to a weighted average; while 2-additive capacity considers interaction between two elements only. The latter approach has been experimentally favored in the sense that the passage of a 2-additive capacity on  $k$ -additive capacity provides little precision, while transition to a 1-additive capacity causes a significant loss of precision (Grabisch & Miranda, 2007), (M. Grabisch, 2006). By using some indices (namely interaction indices) some approaches such as that used by the software that will be employed in the application example, try to overcome this combinatorial difficulties.

#### 4.1.1 Interaction index

It helps to explain the phenomena of interaction between elements  $i$  and  $j$  related to  $v$ , it is defined by (J. L. Marichal, 2000) as:

$$I(v, ij) := \sum_{T \subseteq X \setminus ij} \frac{(n-t-2)! t!}{(n-1)!} [(\Delta_{ij} v)(T)] \quad (12)$$

Where

$$(\Delta_{ij} v)(T) = v(T \cup ij) - v(T \cup i) - v(T \cup j) + v(T)$$

If  $I(v, ij) < 0$ , then we say that  $i$  and  $j$  are positively correlated or competitive. In contrast, if  $I(v, ij) > 0$ ,  $i$  and  $j$  are negatively correlated, therefore complementary. Finally if  $I(v, ij) = 0$  elements  $i$  and  $j$  are independent.

There are other indexes to help results interpretation of the Choquet integral, as the importance index and the index of influence (Marichal, 2000). Tolerance of decision makers and the critical elements that impact on the overall score can also be identified and quantified (see (Marichal, 2000) and (Dubois & Koning, 1991)). Several special cases of the Choquet integral are

discussed in literature, the interested reader can refer to (Marichal, 2000) for a review article, and (Grabisch & Labreuche, 2005) for a more detailed study of the Choquet integral in a special way and fuzzy integrals in general. A generalization of Choquet integral has also been recently proposed by Greco and al (Greco *et al.*, 2011).

## 5 FINAL EVALUATION

The final evaluation process is to aggregate  $b, o, c, r$  factors related to each alternative, to represent evaluation of alternatives by a single degree. Saaty (Saaty, 2003) proposed five ways to combine  $b, o, c, r$  scores of each alternative; probabilistic additive, subtractive, multiplicative priority powers, multiplicative. In our approach, concept of bipolarity is reconsidered in the final evaluation by measuring supportability and rejectability of each alternative, as is the case of satisficing games theory that we propose as a final aggregation method (See (Tchangani, 2006), (Tchangani, 2009a), (Tchangani, 2009b)).

### 5.1 Aggregation by satisficing game theory and final recommendation

The use of satisficing game theory in the BOCR analysis framework consists in the aggregation of benefit and opportunity in selectability measure and cost and risk in rejectability measure. Aggregation is realized through the following equations (13) and (14).

$$\psi_s(u) = \delta B(u) + (1 - \delta)O(u) \quad (13)$$

$$\psi_r(u) = (1 - \delta)C(u) + \delta R(u) \quad (14)$$

where:

$B(u), O(u), C(u), R(u)$  are respectively, the results of aggregation of benefit, opportunity, cost, and risk components.

$\delta$ : is the risk aversion index. It permits to consider the risk aversion of a decision maker. Index  $\delta$  is between 0 and 1; more it is close to 1, greater is risk aversion of decision maker who, pessimistic, will tend to give more importance to risk than cost in rejectability measure (equation (14)) and penalize opportunity in favor of benefit in selectability measure (equation (13)).

Conversely, when the risk aversion index tends to 0, the decision maker is optimist. He will focus on opportunity to benefit in the selectability measure, and will overlook risk against cost in the rejectability measure.

Finally, selectability and rejectability functions are given respectively by the following equations (15) and (16).

$$\mu_s(u) = \frac{\psi_s(u)}{\sum_{v \in U} \psi_s(v)} \quad (15)$$

$$\mu_r(u) = \frac{\psi_r(u)}{\sum_{v \in U} \psi_r(v)} \quad (16)$$

From these selectability and rejectability functions or measures, one can define some interesting sets that can be used in final recommendation procedure, see (Tchangani *et al.*, 2011):

The satisficing set  $\Sigma q \subseteq U$  (at a boldness or caution index  $q$ ) is the set of alternatives defined by the following equation (17).

$$\Sigma q = \{u \in U : \mu_S(u) \geq q\mu_R(u)\} \quad (17)$$

The caution index  $q$  can be used to adjust the aspiration level: increase  $q$  if too many alternatives are declared satisficing or on the contrary decrease  $q$  if  $\Sigma q$  is empty for instance. But a satisficing alternative may be dominated that is there may exist other alternative for which the selectability measure is higher and the rejectability measure lower than its. Some let us define the equilibrium set  $E$  as alternatives for which there are no strictly better alternatives, it is given by equation (18).

$$E = \{u \in U : D(u) = \emptyset\} \quad (18)$$

where  $D(u)$  is the set of alternatives that are strictly better than  $u$ , and the satisficing equilibrium set  $E_q^s$  is given by equation (19).

$$E_q^s = E \cap \Sigma q \quad (19)$$

The dominance relationships between alternative is then given as:  $E_q^s$  contains the best alternatives; it is not easy to compare elements from  $E - \Sigma q$  (equilibrium but not satisficing) and  $\Sigma q - E$  (satisficing but not equilibrium) and  $U - E \cup \Sigma q$  contains completely irrelevant alternatives.

## 6 APPLICATION

In this section, we apply our approach to a real size problem addressed by Lee *et al.* (A. Lee *et al.*, 2009c). Lee *et al.* used AHP procedure associated with BOCR analysis to examine the feasibility of a selection of a wind farms project in an anonymous province in China.

### 6.1 Data

Five potential sites (alternatives) noted A to E were examined in the feasibility analysis for the installation of Wind farms. Sites were evaluated on their performance, business drivers and socio-economic needs (objectives). Attributes and sub-attributes sets had been selected by experts based on literature reviews and practical experiences. The expert committee was composed of 7-13 members who had relevant professional knowledge about the objectives to be evaluated and one third of the total number at least was outside experts or scholars. (A.

Lee *et al.*, 2009c) This construction facilitates the problem adaptation to our approach by considering ‘attributes’ as ‘indicators’ and ‘sub-attributes’ as ‘attributes’, the problem data are mentioned in table 1. Note that all data is extracted from Lee *et al.* who consider common attributes for every couple (objective, alternative).

Factors	Indicators	Attributes
<b>Benefits</b>	(a) Wind availability	(a1) Geographical distribution of wind speed frequency (a2) Mean wind power density (a3) Annual mean wind speed
	(b) Site advantage	(b1) Influence of selected height of installation (b2) Effect of wind gusting (b3) Micro-siting of WEGs
	(c) WEG functions	(c1) Real and technical availability (c2) Affordable, reliable, and maintenance free (c3) Power factor, capacity factor
<b>Opport.</b>	(d) Financial schemes	(d1) Switchable tariff (d2) Discount of tax rate and duty rate (d3) Other investment and production incentives
	(e) Policy support	(e1) Wind power concession program (e2) Clean development mechanisms program (e3) Other policy supports
	(f) Advanced technologies	(f1) Computerized supervisory (f2) Variable speed wind power generation (f3) Swept area of a turbine rotor (f4) Static reactive power compensator, etc.
<b>Costs</b>	(g) Wind turbine	(g1) Design and development (g2) Manufacturing (g3) Installation, maintenance
	(h) Connection	(h1) Electric connection (h2) Grid connection
	(i) Foundation	(i1) Main construction (i2) Peripheral construction
<b>Risks</b>	(j) Concept conflict	Entrepreneurs, policy makers, residents
	(k) Technical risks	Technical complexity and difficulties

Table 1: Indicators and attributes for wind farm project.

Experts have evaluated indicators and attributes with respect to the same upper level factors and indicators respectively. Pairwise comparisons results were combined using the geometric average method. Results are summarized in table 2.

Factors	Indicators	Weights	Attributes	Priority
<b>B</b>	(a)	0,6317	(a1)	0,191
			(a2)	0,497
			(a3)	0,312

<b>O</b>	(b)	0,1324	(b1)	0,489
			(b2)	0,195
			(b3)	0,316
	(c)	0,2359	(c1)	0,221
			(c2)	0,286
			(c3)	0,493
	(d)	0,3077	(d1)	0,2107
			(d2)	0,4476
			(d3)	0,3417
			(d4)	0,4247
	(e)	0,4579	(e1)	0,2082
			(e2)	0,3671
(e3)			0,1872	
(e4)			0,2781	
(f)	0,2344	(f1)	0,1045	
		(f2)	0,4302	
		(f3)	0,5595	
		(f4)	0,3195	
<b>C</b>	(g)	0,5595	a	0,5595
	(h)	0,3195	a	0,3195
	(i)	0,1209	a	0,1209
<b>R</b>	(j)	0,5639	b	0,5639
	(k)	0,1208	b	0,1208
	(l)	0,3153	b	0,3153

<sup>a</sup> The costs of attributes under each cost indicator are summed up in the evaluation.

<sup>b</sup> For indicators under the risks factor, there is no lower-level attributes, therefore indicators are considered as such.

Table 2: Relative Importance of indicators and attributes.

The performance results of different alternatives under various attributes are collected and summarized in table3.

Alternative	Factor	Attributes									
<b>A</b>	B	63	349	4,9	85	63	76	63	98	51	79
	O	82	87	90	67	75	73	83	84	75	
	C	140	170	140	35	40	25	20			
	R	78	75	78							
<b>B</b>	B	77	451	5,7	78	74	89	76	97	57	76
	O	82	81	80	84	81	88	77	76	79	
	C	150	190	150	60	55	35	30			
	R	74	70	73							
<b>C</b>	B	42	337	4,4	61	83	75	71	98	50	81
	O	78	76	73	73	68	70	73	88	73	
	C	150	180	155	30	25	35	25			
	R	80	75	83							
<b>D</b>	B	73	502	5,3	77	75	88	74	97	59	79
	O	83	84	83	81	78	85	78	75	73	
	C	155	180	155	65	50	40	30			
	R	76	71	75							
<b>E</b>	B	85	426	4,5	86	76	83	78	98	53	82
	O	85	86	88	85	80	81	81	77	74	
	C	160	200	160	80	85	50	50			
	R	72	70	68							

Table 3: Evaluation of alternatives for each attribute.

These evaluation values are normalized by dividing, for each alternative, its evaluation value given an attribute, by the sum of values evaluation of all alternatives for the same attribute. The goal is to have the same magnitude values between 0 and 1 for all attributes.

## 6.2 Results

The overall performances of alternatives are obtained by aggregating evaluations by Choquet integral.

Aggregation is done in two stages, the first is to aggregate all attributes of each indicator to represent them on a single value. The second step consists in aggregating the indicators of each factor to obtain the performance evaluation of alternatives on the b, o, c, r factors. To overcome difficulties related to the definition of fuzzy measures by specifying  $2^n - 2$  coefficients, we used a software for the calculation of fuzzy integral (see link (<http://www.isc.senshu-u.ac.jp/~thc0456/Efuzzyweb/fm11.html>)). It calculates the Choquet integral using a class of fuzzy measures named  $\lambda$ -fuzzy measures. The principle of  $\lambda$ -fuzzy measures is to generate fuzzy measures based on an index of interaction  $\lambda$  or  $\xi$ , and individual weights of elements. Indexes of interaction  $\lambda$  or  $\xi$  are equivalent and represent interaction associated with combinations of elements, they are defined as follows:

$\lambda \in [-1, +\infty[$	$-1 < \lambda < 0$	$\lambda = 0$	$1 < \lambda < +\infty$
$\xi \in [0, 1]$	$0.5 < \xi < 1$	$\xi = 0.5$	$0 < \xi < 0.5$
Relation	Concurrence	Indifference	Complementarity

Table 4: Interaction indexes used by software.

Once the fuzzy measures have been generated, the Choquet integral can be calculated and a sensitivity analysis realized.

As already mentioned, the first stage of aggregation is to represent each indicator of each factor by a single value. This means that all attributes of a given indicator will be aggregated by the Choquet integral.

Taking the example of the benefit factor, the attributes a1, a2, a3 will be aggregated at first to get a single value of the indicator 'a'. The stages are:

- Definition of the number of attributes and alternatives to deal with.
- Enter the weight of the attributes (Table 2).
- Selecting degree of interaction  $\xi$  or  $\lambda$  and quantification ( $\xi=0.3$ ).
- Identifying fuzzy measures.
- Display of fuzzy measures obtained.
- Entry alternative assessments for attributes a1, a2, a3 (normalized data obtained from Table 3).
- Display the values of Choquet, which gives a unique value to the indicator for each alternative.

The same operation is repeated for indicators b and c. Once these results are obtained, the second stage is to aggregate indicators a, b, c to obtain a single benefit factor value for each alternative  $B(u)$ .

The same operation is repeated for factors o, c, and r.

Table 5 presents Choquet values obtained for each alternative on each factor. We put  $\xi=0.3$ , this assumes

that the elements are fairly complementary (with  $\xi \in [0, 1]$ ).

		Alternatives				
Evaluation		A	B	C	D	E
Factor	B	0,178	0,212	0,158	0,212	0,198
	O	0,187	0,199	0,182	0,200	0,205
	C	0,153	0,200	0,145	0,201	0,234
	R	0,206	0,194	0,211	0,199	0,185

Table 5: Aggregation results of alternatives evaluation by the Choquet integral.

The selectability and rejectability measures of each alternative have been evaluated. For illustration, we assume that the risk aversion expressed by decision makers is average ( $\delta = 0.5$ ). This index value gives the same importance to b, o, c, r factors and allows comparison with Lee's results that use fixed weights for attributes and give the same importance for b, o, c, r factors. The results obtained with these settings are summarized in Table 6.

		Alternatives				
Bipolarity		A	B	C	D	E
$\mu_s$		0,1890	0,2129	0,1758	0,2135	0,2089
$\mu_r$		0,1860	0,2044	0,1848	0,2075	0,2173

Table 6: Selectability and rejectability measures of alternatives.

The graphical representation of these results in the plane  $(\mu_r, \mu_s)$  provides better visibility of results, thus, better interpretation. Assuming that  $\delta = 0.5, q = 1$ , figure 3 shows that satisficing set of alternatives is composed of Alternatives A, B and D ( $\Sigma_1 = \{A, B, D\}$ ), but alternative D is dominated by alternative B. Therefore, the satisficing equilibrium set is composed of Alternative A and B ( $E_1^s = \{A, B\}$ ). These results coincide with first conclusions of sensitivity analysis. By varying value of  $q$ , we can resize set of satisficing alternatives as required by the decision maker. For example, caution index  $q = 0.95$  permits to enlarge the set of satisficing alternatives, which increased from two to four (see Figure 4).

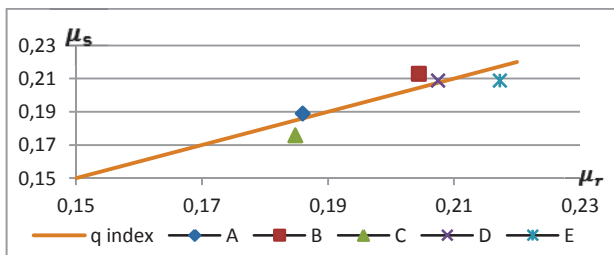


Figure 3: Graphic representation of alternatives in the plane  $(\mu_r, \mu_s)$  ( $\delta = 0.5, q = 1$ ).

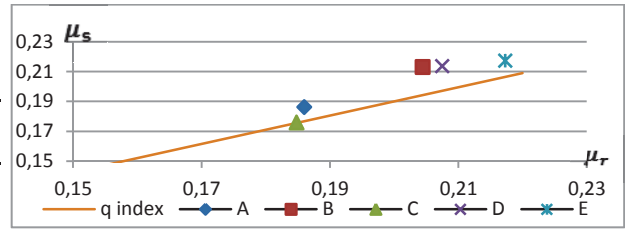


Figure 4: Graphic representation of alternatives in the plane  $(\mu_r, \mu_s)$  ( $\delta = 0.5, q = 0.95$ ).

Also, by varying risk aversion index we can observe some changes. Assuming that decision makers present a high risk aversion (for instance  $\delta = 0.9$ ) meaning that they prefer to consider certain factor (benefit) for positive factors and uncertain factor (risk) for negative factors, graphical representation of alternatives in the plane  $(\mu_r, \mu_s)$  (see figure 5) shows that the satisficing alternatives are alternatives B, D and E that are characterized by a good benefit and low risk. Note that alternative E was not part of the satisficing set when  $\delta = 0.5$ . This is due to the fact that decision maker when ( $\delta = 0.5$ ) gives the same importance of any b, o, c, r factor, which means that the high cost of alternative E was considered unlike the case  $\delta = 0.9$  where it is neglected. However, alternative C keeps its lowest, with its low benefit and its high degree of risk.

By putting  $\delta = 0.1$ , we promote opportunity to benefit and cost to risk which denotes a high risky decision making attitude, figure 6 shows that alternatives C and A are in the satisficing alternatives set, that is explained by the fact that alternatives A and C are characterized by an average cost and a good opportunity.

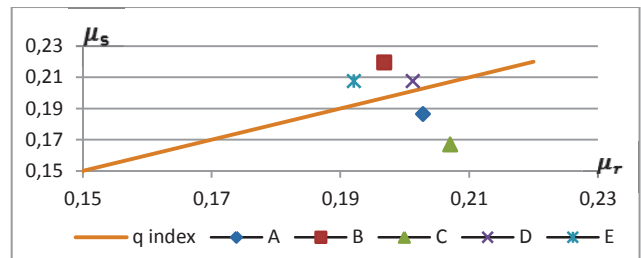


Figure 5: Graphic representation of alternatives in the plane  $(\mu_r, \mu_s)$  ( $\delta = 0.9, q = 1$ ).

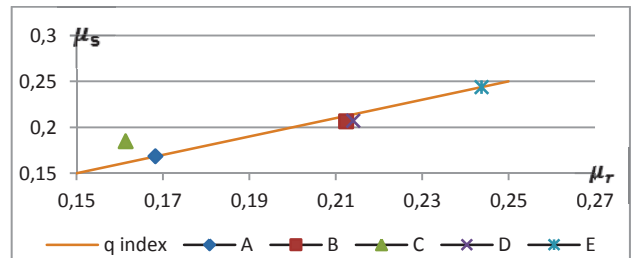


Figure 6: Graphic representation of alternatives in the plane  $(\mu_r, \mu_s)$  ( $\delta = 0.1, q = 1$ ).

## 7 CONCLUSION

This paper is concerned with the uncertain aspects of decision making problems, namely risk and opportunity. We have proposed a method of evaluation and recommendation to identify and quantify these uncertain factors through a BOCR analysis. In this context, the definition and measurement of risk and opportunity have been proposed. Taking into account bipolar nature of attributes that characterize alternatives, satisficing game theory has been proposed as a basic tool for final evaluation and recommendation and the Choquet integral has been used to aggregate data. Our approach also takes into account the risk aversion introduced by an index of the risk aversion that considers position of decision makers in relation to risk, this index permits to consider an important external factor that can have a significant impact to the final decision. Example of application was presented to test our model and the results when similar to that of literature, show rich analysis possibilities in terms of equilibrium and dominance. Finally, adapting our model to a problem with multi- decision makers which take into account the various factors influencing the assessment of decision makers can stand for a perspective. Bayesian networks could then serve as an assessment tool.

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