Thick composite structures under a load pinch

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Université de Toulouse, INPT, ENIT, LGP , 47 Av. Azereix, BP 1629, 65016 Tarbes, France

moussa.karama@enit.fr

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Abstract. Thick composites are increasingly used in the design of mechanical structures. Combined with low weight, they are generally resistant structures, which can support importante loads. In addition, depending on the number and nature of the materials used, it is possible to adapt properties for specific applications (damping structures).This work proposes the establishment of a new theoretical model of multilayer beam. The model, which is simple and easy handling, is intended for the subsequent establishment of a finite element. The goals are:

- improve the refinement of the transverse displacement and transverse shear, avoiding the calculation of transverse shear, the use of correction factors,
- keep only the usual displacement,
- test the accuracy of the model compared with models from the literature (for an equivalent single-layer approach).

The proposed approach is of the kinematics, the form adopted for the displacement field is justified from a dimensional point of view, by the equations of elasticity. The equations of motion and boundary conditions are obtained by applying the principle of virtual power.
The validity of the model is tested on problems for which solutions (obtained by previous theories) exist.

Introduction

Indeed, there appears a discontinuity stress at interfaces, values a beam is a three-dimensional environment in which one dimension predominates over other side. Solve the three-dimensional elastic problems with boundary conditions leads to painful and heavy calculations, often difficult to use. When the beam is more multilayer composite, the boundary conditions of continuity of the motion vector and the constraint vector to the crossing of interfaces additional difficulties. It is therefore desirable, wherever possible, to reduce to three-dimensional theory, to obtain a formulation of a reference surface.

The first theories developed were those type Kirchhoff - Love. They are first-order theories (so described because their linear dependence following the variable thickness). Normals are assumed not undergo rotation relative to the average surface. These theories bending, essentially applicable to thin structures, to which the effects of deflection (corresponding to the rotations around the fibers of the tangents to the reference surface) predominate over those due to shearing. Such theories are limited by the result and lead to sufficiently accurate in the following case results:

- the height / length ratio is large,
- the material is only weakly anisotropic.

The application of such theories to multilayer composite beams can lead to errors of at least 30% for the calculation of strain and effort. It is therefore essential to refine. The various possibilities for the treatment of multilayer composite beams approaches can be grouped as follows:

- monolayer equivalent approach,
- continuity approach to interfaces,
- three-dimensional approach.

Monolayer equivalent approach. The multilayer here is homogenized. Is approximated in the first fields of displacement by means of a series expansion (as polynomial functions) following the variable thickness. These developments are often cubic. As outlined in its classification Whitney [1], this approach can be understood as a formulation where the displacement field has at least one C1 continuity across the thickness, covering the conventional beam models that appear when as special cases of this theory.
However, the results of such calculations are not always the most accurate. Correction coefficients for transverse shear stresses in particular, tend to consider.
The work of Love [2], Idlbi [3] using this type of method for anisotropic multilayered plates and shells, static.

Continuity approach to interfaces. The overall structure is divided into sub-structures (actually corresponding to each layer). Is applied to each sub-structure theory Reissner-Mindlin type [4] [5], imposing a displacement field checking continuity at the interfaces between the different layers. Models of this type are relatively expensive, but possible to obtain more accurate results, especially as regards the calculation of the transverse shear stresses.
We can cite the work of Di Sciuva [6] Touratier et al [7] He [8] Ossadzow et al [9] and most recently Abu Harb et al. [10].

Three-dimensional approach. The three-dimensional approach is to obtain accurate three-dimensional results, particularly useful as references. We can cite the work of Pagano [11] for plates, Ren [12] for symmetric multi-hulls and Srinivas [13] for sandwich structures.
Adopting a three-dimensional approach, however, has utility in so far as the differential equations finally obtained, can be resolved. The use of such theories generally leads to complex systems where the high number of unknowns and potential linkages between the various variables, make resolution impossible.

Theories of multilayer beams

The ultimate goal of this work is to be able to treat problems involving thick structures, it was necessary to establish a general model of beam (which can be used to treat the case of thin or thick beams), while allowing accurate calculations. The original idea was to continue the work of Touratier, Idlbi, Karama and [14]. It considers possible refinements (terms of deflection, transverse shear). In this first approach, the cross σ_{33} normal stress is introduced to take into account the behavior of thick structures. Continuity for the transverse shear and σ_{33} is not guaranteed. A summary of the various models are listed below**.**

Theories	Displacement field
Kirchhof-Love assumptions: • neglected shear • thin structures	$\begin{cases} U_\alpha = u_\alpha^0 - z w_{,\alpha} \\ U_3 = w \end{cases}$ u_α^0 membrane displacement, w transverse displacement
Reissner Mindlin assumptions: • constant shear • thin structures	$\begin{cases} U_\alpha = u_\alpha^0 - z w_{,\alpha} + z\gamma_\alpha^0 \\ U_3 = w \end{cases}$ u_α^0 and a displacement point mean surface γ_α^0 rotation shear in the planes (α,z)
Reddy assumptions: • semi thick structures • symmetric	$\begin{cases} U_\alpha = u_\alpha^0 - z w_{,\alpha} + z(1 - \frac{4}{3}\frac{z^2}{h^2})\gamma_\alpha^0 \\ U_3 = w \end{cases}$ u_α^0 and a displacement point mean surface γ_α^0 rotation shear in the planes (α,z
Di Sciuva assumptions: • anisotropic multilayer beam • piecewise linearity of the tangential displacement field	$\begin{cases} U_\alpha = u_\alpha^0 + z\left(\gamma_\alpha^0 - w_{,\alpha}\right) + \sum_{k=1}^{n-1} U_{k\alpha}(z - z_k)H(z - z_k) \\ U_3 = w \end{cases}$ u_α^0 and w a displacement point mean surface γ_α^0 rotation shear in the planes (α, z) $U_{k\alpha}$: determined by the continuity of transverse shear interfaces functions H : Heaviside function
He assumptions: • piecewise linearity of the field tangential displacement • continuity of transverse shear interfaces verification of the boundary conditions on the transverse shear (nullity on the upper and lower surfaces)	$\begin{cases} U_\alpha = u_\alpha^0 - z w_{,\alpha} + h_\alpha \varphi_\alpha \\ U_3 = w \end{cases}$ u_α^0 and w a displacement point mean surface h_α coefficient depending on the geometry (thickness parameter) and the material.
Touratier assumptions: • Continuity of the transverse shear uninsured interfaces	$\begin{cases} U_\alpha = u_\alpha^0 - z w_{,\alpha} + \frac{h}{\pi}\sin\frac{\pi z}{h}\gamma_\alpha^0 \\ U_3 = w \end{cases}$ u_α^0 and w a displacement point mean surface γ_α^0 shear deformation on the measured average area

Variational Formulation

The kinematic. Consider a composite beam, comprising a stack of N layers, assumed perfectly glued together. The total thickness of the structure is h. The behavior is considered by an orthotropic layer behavior. The beam in question is subject on its upper and lower surfaces, a transverse load (Pi and Ps, respectively).

Kinematic restraint is in the following form:

$$\begin{aligned} U_1(x_1,x_3,t) &= u_1^0(x_1,t) - x_3{}^1\ \ w_{,1}(x_1,t) + f(x_3)\gamma_1(x_1,t) \\ U_2(x_1,x_3,t) &= 0 \\ U_3(x_1,x_3,t) &= w(x_1,t) + f(x_3)f_{,3}(x_3)\delta(x_1) + \frac{h}{p}f_{,3}(x_3)\psi(x_1) \end{aligned} \quad (1)$$

with u_1^0 membrane the displacement in one direction 1, $w_{,1}$ the rotation due to bending, w the deflection of the beam, γ_1 the shear strain measured on the mean transverse plane, $f(x_3) = \frac{h}{p}\sin(\frac{\pi x_3}{h})$ stress distribution of transverse shear, $\delta(x_1,x_2)$ and $\psi(x_1,x_2)$ are functions to determine the characterizing nip. Direction 2 is assumed to be infinite.

Operating boundary conditions. The U_α is assumed to be known, δ and ψ only remain to be determined. Our kinematics, it is assumed that the membrane displacement u_1^0 is zero. The conditions on the normal stress can be written:

$$\sigma_{33}(x_3 = -h/2) = P_i \ \text{ et } \ \sigma_{33}(x_3 = +h/2) = P_s \quad (2)$$

where Pi and Ps are the normal loads applied to the upper and lower skins of the multilayer structure. We have:

$$\begin{aligned} U_1(x_1,x_3,t) &= -x_3 w_{,1} + f.\gamma_1 \\ U_2 &= 0 \\ U_3(x_1,x_3,t) &= w + f.f_{,3}.\delta + \frac{h}{\pi}f_{,3}.\psi \end{aligned} \quad (3)$$

The introduction of terms δ and ψ provides a non-zero normal stress distribution σ_{33}.

With $f = \frac{h}{\pi}\sin\frac{\pi x_3}{h}$; $f_{,3} = \cos\frac{\pi x_3}{h}$; $f_{,33} = -\frac{\pi}{h}\sin\frac{\pi x_3}{h}$ (4)

The deformation field is:

$$\begin{aligned} \varepsilon_1 &= -x_3 w_{,11} + f(x_3).\gamma_{1,1} \\ \varepsilon_2 &= 0 \\ \varepsilon_3 &= f_{,3}^2(x_3)\delta + f(x_3)f_{,33}(x_3)\delta + \frac{h}{\pi}f_{,33}(x_3)\psi \end{aligned} \quad \text{and} \quad \begin{aligned} \gamma_6 &= 0 \\ \gamma_5 &= f_{,3}(x_3)\gamma_1 + f.f_{,3}\,\delta_{,1} + \frac{h}{\pi}f_{,3}\,\psi_{,1} \\ \gamma_4 &= 0 \end{aligned} \quad (5)$$

The transverse normal stress is given by:

$$\sigma_3 = C_{13}\varepsilon_1 + C_{33}\varepsilon_3$$

$$\sigma_3 = -C_{13}x_3 w_{,11} + C_{13}f(x_3)\gamma_{1,1} + C_{33}(f_{,3}^2(x_3) + f(x_3)f_{,33}(x_3))\delta + C_{33}\frac{h}{\pi}f_{,3}(x_3)\psi \quad (6)$$

The conditions on the top and bottom of the beam on the faces normal stress are written:

on the lower face: $\sigma_3(-\frac{h}{2}) = P_i = C_{13}\frac{h}{2}w_{,11} - C_{13}\frac{h}{\pi}\gamma_{1,1} - C_{33}(\delta - \psi)$ (7)

and on the upper face: $\sigma_3(\frac{h}{2}) = P_S = -C_{13}\frac{h}{2}w_{,11} + C_{13}\frac{h}{\pi}\gamma_{1,1} - C_{33}(\delta + \psi)$ (8)

by adding (7) and (8), we obtain: $P^+ = -2C_{33}\delta$ where $\delta = -\frac{P^+}{2C_{33}}$ with $P^+ = P_S + P_i$ (9)

subtracting (7) and (8), we obtain: $\psi = \frac{C_{13}hw_{,11} + 2C_{13}\frac{h}{\pi}\gamma_{1,1} - P^-}{2C_{33}}$ with $P^- = P_S - P_i$ (10)

Final kinematic. Using the loading conditions on the upper and lower surfaces is defined for sandwiches or multi-beams kinematics follows:

$$\begin{aligned} &U_1(x_1, x_3, t) = -x_3 w_{,1} + f(x_3)\gamma_1 \\ &U_2 = 0 \\ &U_3(x_1, x_3, t) = w - f(x_3)f_{,3}(x_3)\frac{P^+}{2C_{33}} - \frac{P^-}{2C_{33}}\frac{h}{\pi}f_{,3}(x_3) + .\frac{C_{13}}{C_{33}}f_{,3}(x_3)\frac{h^2}{\pi}\left(\frac{\gamma_{1,1}}{\pi} - \frac{w_{,11}}{2}\right) \end{aligned} \quad (11)$$

$P_S = -q_{x_3=\frac{h}{2}}$ (pressure at $x_3 = h/2$) and $P_i = -q_{x_3=-\frac{h}{2}}$ (pressure $x_3 = -h/2$)

To simplify the writing of the principle of virtual power, we ask:

$$\alpha(x_3) = \frac{C_{13}}{C_{33}}f_{,3}(x_3)\frac{h^2}{\pi} \quad \text{and} \quad g(x_1, x_3) = -\frac{f_{,3}(x_3)Ps(x_1)}{2C_{33}}\left(f(x_3) + \frac{h}{\pi}\right)$$

Kinematics becomes:

$$\begin{aligned} &U_1(x_1, x_3, t) = -x_3 w_{,1}(x_1, t) + f(x_3)\gamma_1(x_1, t) \\ &U_2 = 0 \\ &U_3(x_1, x_3, t) = w(x_1, t) + g(x_1, x_3) + \alpha(x_3)\left(\frac{\gamma_{1,1}(x_1, t)}{\pi} - \frac{w_{,11}(x_1, t)}{2}\right) \end{aligned} \quad (12)$$

Principle of virtual power. Are Hilbert spaces which represent all kinematically admissible displacements and all virtual velocities. To determine the equilibrium equations and natural boundary conditions of the problem studied, the principle of virtual power is applied:

- The equilibrium equations $\forall\, w^*$, $\forall\, \gamma^*$:

$$\begin{aligned} &\Gamma^{(w)} = M_{11,11} + \frac{R_{33,1}}{2} - \frac{L_{13,111}}{2} + q_z - \frac{v_{3,11}}{2} + n_{1,1} \\ &\Gamma^{(\gamma)} = -\tilde{Q}_1 + \tilde{M}_{11,1} + \frac{R_{33,1}}{\pi} - \frac{L_{13,11}}{\pi} + \tilde{m}_1 - \frac{v_{3,1}}{\pi} \end{aligned} \quad (13)$$

The natural boundary conditions $\forall\, w^*$, $\forall\, \gamma^*$, $\forall\, w^*_{,1}$, $\forall\, w^*_{,11}$, $\forall\ \gamma^*_{1,1}$:

$$\begin{aligned} &\overline{\Gamma}^{(w)} = -M_{11,1} - \frac{R_{33,1}}{2} + \frac{L_{13,11}}{2} - n_1 + \frac{v_{3,1}}{2} + T_z \\ &\overline{\Gamma}^{(w_{,1})} = M_{11} + \frac{R_{33}}{2} - \frac{L_{13,1}}{2} - M_f - \frac{v_3}{2} \end{aligned} \quad (14)$$

$$0 = \frac{L_{13}}{2} - \frac{V_3}{2}$$

$$\overline{\Gamma}^{(\gamma)} = -\widetilde{M}_{11} - \frac{R_{33}}{\pi} + \frac{L_{13,1}}{\pi} + \frac{v_3}{\pi} + C_1$$

$$0 = -\frac{L_{13}}{\pi} + \frac{V_3}{\pi}$$

Numerical results

Bending of a simply supported beam thick under sinusoidal distributed load. The study is done in static; therefore, the virtual power of the amounts of acceleration is zero, so the first member of equation (13) disappears. For simple terms of support, the unknowns are deducted directly from equilibrium equations. The study consisted of analytical resolution, a comparison with existing models (Euler Bernoulli, Timoshenko and Reddy), a numerical solution is then performed on the software Abaqus finite element. The components of the forces of surface and volume are zero except f_3 (Fig. 1). Value is then deducted $\overline{n}_3 = \int_0^h f_3 dx_3 = q_z = q_0 \sin\frac{\pi x_1}{L}$ with $q_0 = -10^6$ Pa.

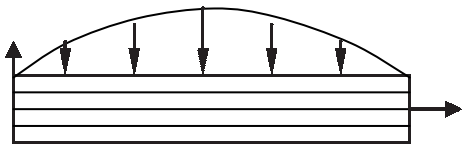


Figure 1: Thick beam simply supported under a sinusoidal loading

Levy-type solutions are suitable for this problem:

$$w = w_0 \sin\frac{\pi x_1}{L} \quad \text{and} \quad \gamma_1 = \gamma_0 \cos\frac{\pi x_1}{L} \tag{15}$$

Levy-type solutions are adapted to this aim is therefore to determine the value of w_0 et γ_0 by solving the system of Eq.25:

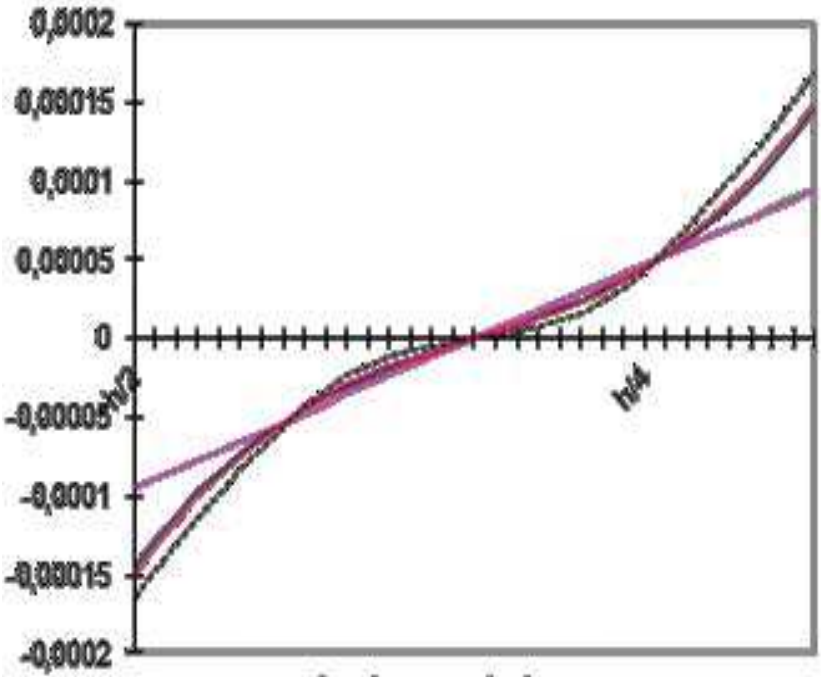
$$0 = \left(-D_{11} - B_{13} - \frac{\widetilde{B}_{33}}{4}\right) w_{,1111} + \frac{\widetilde{S}}{4} w_{,111111} + \left(d_{11} + \frac{B_{13}}{\pi} + \frac{b_{13}}{2} + \frac{\widetilde{B}_{33}}{2\pi} - \frac{\widetilde{T}}{2}\right)\gamma_{1,111} - \frac{\widetilde{S}}{2\pi}\gamma_{1,11111} + q_z - \frac{v_{3,11}}{2}$$

$$0 = \left(\frac{\widetilde{T}}{2} - d_{11} - \frac{b_{13}}{2} - \frac{B_{13}}{\pi} - \frac{\widetilde{B}_{33}}{2\pi}\right) w_{,111} + \frac{\widetilde{S}}{2\pi} w_{,11111} - \widetilde{A}_{55}\gamma_1 + \left(-\frac{\widetilde{T}}{2} + \frac{2b_{13}}{\pi} + \widetilde{D}_{11} + \frac{\widetilde{B}_{33}}{\pi^2}\right)\gamma_{1,11} - \frac{\widetilde{S}}{\pi^2}\gamma_{1,1111} - \frac{v_{3,1}}{\pi} \tag{16}$$

Analytical results. The error is calculated between the value determined by the analytically model studied and the value provided by Abaqus. The mesh size for the laminated beam is 32 elements 32 along the length and across the width of elements. These meshes enable us to obtain a convergence quite decent.

Table 1. $\gamma 0$ values w0 and in the case of a laminated beam

	w_0 (m)	γ_0 (m)
Euler Bernouilli	$-1{,}935167.10^{-4}$	/
Timoshenko	$-5{,}04081964.10^{-4}$	$-1{,}5378936.10^{-4}$
Reddy	$-6{,}039314.10^{-4}$	$-2{,}2728823.10^{-4}$
Present model	$-6{,}099273.10^{-4}$	$-2{,}3506862.10^{-4}$



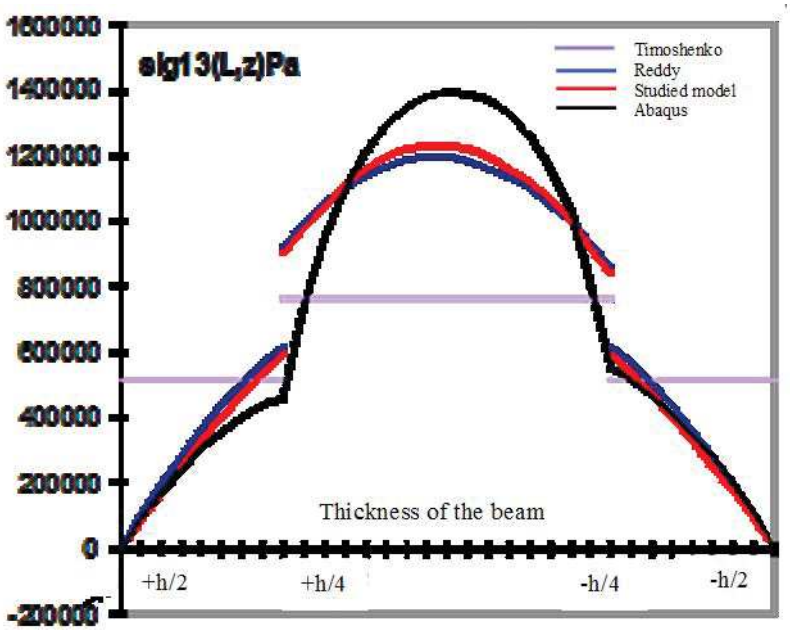


Table 2. Results for a laminated beam

	Euler Bernouilli	Timoshenko	Reddy	Present model	Abaqus	error
U1(L,h/2)	$1{,}337.10^{-4}$	$1{,}3355.10^{-4}$	$2{,}.573.10^{-4}$	$2{,}125.10^{-4}$	$2{,}018.10^{-4}$	5%
U3(L/2,0)	$-1{,}935.10^{-4}$	$-5{,}157.10^{-4}$	$-6{,}110.10^{6}$	$-6{,}166.10^{-4}$	$-5{,}978.10^{-4}$	3,1%
σ11(L/2,0)	0	159168,1	159168	159168	127220	20%
σ11(L/2,h/4)	$-8{,}087.10^{6}$ $-6{,}3895.10^{5}$	$-8{,}235.10^{6}$ -502977,2	$-7{,}592.10^{6}$ -608113,9	$-7{,}195.10^{6}$ -536269	$-7{,}3.10^{6}$ -568420	1,4% 6%
σ13(L,h/4)	/ /	767448,75 514084,5	864853,44 612601,56	843713,87 595790,13	553590 547030	/ 8,8%

Conclusion

The model studied improves the refinement of the transverse displacement and the transverse shear. Indeed, results from the model study are closer to those found numerically from the results from the previous models; and this, without using correction factors.
Indeed, it appears a discontinuity stresses at the interfaces, the values found do not have a big gap with those given by Abaqus.
The values from the model for cross σ_{33} normal stress are quite comparable to those found numerically by finite elements, particularly in the case of the laminated beam.

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