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# HYBRID PORT-HAMILTONIAN SYSTEMS: FROM PARAMETERIZED INCIDENCE MATRICES TO HYBRID AUTOMATA

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**Abstract:** This paper shows how to formally design a hybrid automaton model for a wide class of dissipative physical systems with sources and switching topology. This method is based on a mathematical representation of the dynamic network graph and of its dual graph, using the hybrid incidence matrix, and on a constructive method to analyze admissible and constrained configurations. The port-Hamiltonian representation associated with the set of hybrid system configurations, parameterized by the discrete state of the switches, is synthesized to be part of the hybrid automaton of the system. This is a further step to a generic control synthesis for physical switching systems.

**Keywords:** Hybrid dynamical systems, Modeling, Hybrid automata, Dynamic network graph, Incidence matrix, Analysis, Admissible configurations, Constrained configurations.

## 1. INTRODUCTION

In a physical switching system (PSS), the topology may change instantaneously depending on certain discrete parameters [33,34]. The aim of this paper is to formally synthesize a hybrid automaton model [1,14] for a wide class of physical switching systems (PSS) with ideal controlled or autonomous switches. Our constructive method is based on mathematical representations of network graphs and the analysis of the admissible and constrained configurations of the system. A family of hybrid parameterized incidence matrices is developed representing a primal dynamic network graph and a dual graph associated with the PSS. The algebro-differential equations associated with the set of system configurations are deduced and written in a non-minimal parameterized implicit port-Hamiltonian formulation using a kernel representation. Within the hybrid automaton, the transitions from one discrete state of the system,  $W$ , (or discrete dynamic, configuration, topology) to another are determined by the conditions producing autonomous switches, and the control actions on the controlled switches. The method for modeling the complete autonomous PSS presented in this paper does not include how to control the switches. The work presented here is an extension of both, [19], which explained how to obtain the parameterized incidence matrices for a class of physical switching system and [31] which analyzed admissible and constrained configurations. The paper [32] proved a theorem about the formulation of the parameterized incidence matrix for a wider class of physical switching systems with varying constraints.

The energy-based approach used here, is related to other works. First of all, it provides a generic method for synthesizing a structured autonomous model and analyzing a class of hybrid port Hamiltonian systems that includes the models of various power converters presented in [9]. Haddad, Nersesov and Chellaboina [13] expressed the model of a class of physical switching systems (PSS) as an explicit port-controlled Hamiltonian system, including both the dynamics of each configuration and the discontinuities in the real valued state variables at the transitions from one configuration to another. Buisson, Cormerais, Leirens, Richard [2,3,6] developed hybrid models from another energy-based graphical model, bond graphs, where the switches are modeled by effort or flow sources; a set of algebro-differential equations associated with the PSS is written as a minimal implicit state space model after dealing with causality and a rank condition determines if a given configuration is acceptable or not. Buisson, Cormerais, Zainea, Guéguen, Godoy [4] also proposed an approach for computing hybrid automata models for linear PSS with four types of switches: ideal, transistors, diodes and thyristors. This is a bottom-up approach concerning the discrete locations of the PSS but a global approach concerning the computation of the set of algebraic and differential equations associated with each configuration of the PSS. The method presented in this paper is based on network graphs and a port-Hamiltonian formulation and is entirely global. Jeltsema, Scherpen [16] provided valuable insight by establishing a dual relation between the energy-based port Hamiltonian formulation and the power-based Brayton-Moser equations.

The method presented in this paper is illustrated only on electrical power converters. However, since many mechanical systems or hydraulic systems have an equivalent network representation, the networks treated here could be seen as equivalent physical systems arising from another field, through analogy [17,27].

Section 2 provides some background on dynamic network graphs and our modeling formalism based on parameterized incidence matrices. Section 3 defines the non-admissible and constrained configurations of the class of physical switching system (PSS) under study. Section 4 recalls the parameterized port-Hamiltonian formulation. Section 5 presents the main result of this paper, which is a generic and constructive approach to design a hybrid automaton model of a class of autonomous PSS. This approach is based on the port-Hamiltonian formulations of the configurations of the PSS presented in the previous sections and the analysis of the hybrid incidence matrices for the system.

## **2. GRAPHS AND MATHEMATICAL BACKGROUND**

We shall consider a class of physical switching systems (PSS) with dissipation, sources and ideal controlled or autonomous switches. They can be viewed as physical systems with switching topology or multiconfiguration systems. The autonomous part of such systems can be mathematically represented by a hybrid automaton [1,14]. We shall recall in this section the formal calculation of the piecewise continuous dynamics associated with the various configurations of the system (discrete states of the automaton) defined in [18,19,32]. This calculation is based on graph theory and a port-Hamiltonian representation and it leads to a family of models parameterized by the discrete state of the switches. This generic approach based on network graphs is an interesting way of understanding and analyzing energy exchanges and element interactions in physical switching systems, independently of the field and in a constructive way which is not only based on experience.

Network graphs [25,29] have been used successfully to model physical switching systems in various domains [5,8,11,22]. In order to represent equipment failures, various transformations of the graph have been defined, such as edges or vertices removal. Turning on or off ideal switches is a more complicated operation since edges must be disconnected from one vertex and reconnected to another. Therefore an original graph transformation in a dynamic network graph has been defined in [18,19,32]. The ideal switches are considered as elements whose function is to change the interconnection of the functional elements of the system.

**Definition 1:** A dynamic network graph  $G_w$  is defined as an oriented graph  $G_w = (V, E, E_w)$  in which:

- \*  $V$  is a nonempty finite set of  $n_v$  vertices ( $v_x \in V$ ),
- \*  $E$  is a nonempty finite set of  $n_{ef}$  pairs of elements of  $V$  called functional edges ( $e_i \in E / e_i = (v_x, v_y)$ ,  $v_x$  being the starting vertex and  $v_y$  being the ending vertex). The port of a functional element is associated with every  $n_{ef}$  oriented functional edge of this graph. If  $v_x = v_y$ , the edge is a self-loop.
- \*  $E_w$  is a nonempty finite set of  $n_s$  pairs of elements of  $V$  called virtual edges ( $e_{wi} \in E_w / e_{wi} = (v_x, v_y)$ ,  $(v_x, v_y) \in V^2$ ). The port of a switching element is associated with every  $n_s$  oriented virtual edge of this dynamic graph. The set of switches is split into two separate sets: the set of  $n_{as}$  ideal autonomous switches, indexed from 1 to  $n_{as}$ , and a set of  $n_{cs}$  ( $n_{cs} = n_s - n_{as}$ ) ideal controlled switches, indexed from  $n_{as}+1$  to  $n_s$ .

A network graph is said to be *cyclically connected* if and only if there is a circuit sub-graph (connected sub-graph with each vertex having degree two) containing any given pair of vertices. This concept is independent of any orientation. A pair ( $pf$ ,  $pe$ ) of conjugated variables, called *power variables* because their product has the unit of power [17,27], is associated with every functional or virtual edge of the dynamic network graph. The variable  $pf$  denotes the *cocycle variable* (also called *flow variable*) and the variable  $pe$  denotes the *cycle variable* (also called *effort variable*). Edges are oriented as flows. The orientation of virtual edges associated with autonomous switches is chosen in order that the flow variable is positive when the switch is closed (for example, current through a diode).

The parameterized incidence matrix defined in [19] is a highly appropriate mathematical representation of dynamic network graphs in order to model and analyze physical systems with a variable structure. The parameterized incidence matrix gives the geometric interconnection structure for all the configurations of the system in a single matrix parameterized by the discrete state of the switches. A discrete variable  $w_k \in \{0, 1\}$  is associated with each switch  $S_{w_k}$ , so that:  $w_k=1$  if the switch is closed and  $w_k=0$  if the switch is open. Thus, the discrete state of the model is given by:  $W = [w_1, w_2, \dots, w_{n_s}]^T$ . After instantiation of  $W$ , each row of the incidence matrix gives the edges connected to the corresponding vertex and each column gives the two vertices connected to an edge associated with a functional element.

Let us recall here the definition of a hybrid incidence matrix, following [18,19]. It models the various topologies of the physical switching systems defined by proposition 1, as a single representation parameterized by  $W$ .

**Proposition 1:** The physical switching systems (PSS) studied in this paper satisfy the following four assumptions:

- Assumption 1. The dynamic network graph associated with the PSS is cyclically connected.
- Assumption 2. The dynamic network graph associated with the PSS has no self-loop.

*Assumption 3. The outdegree of each vertex of the sub-graph composed of all virtual edges with their incident vertices is below or equal to 1.*

*Assumption 4. All virtual edges are indexed so that an oriented sequence of virtual edges is decreasing.*

Proposition 1 defines the class of physical switching systems (PSS) studied in this paper. In the sequel, the PSS belong to this class. Moreover, it should be noted that the orientation of the edges in the network graphs is neither unique nor predefined by the structure or the logic of the system as in Petri net models for example, and that a PSS is modeled by various parameterized hybrid incidence matrices, depending on the orientation of the edges in its dynamic network graphs.

**Definition 2:** Consider a physical switching system  $\Sigma_w$ , represented by a dynamic oriented network graph  $G_w$ .

\* The term  $(I_{n_v} + M_{T_k}(G_w)(w_k))$  gives the transformations of the geometric interconnections between the functional elements which are produced by closing the switch  $Sw_k$  associated with the oriented virtual edge,  $e_{Gwk} = (v_i, v_j)$ ,  $(i, j) \in \{1, \dots, n_v\}^2$ . For  $k \in \{1, \dots, n_s\}$ ,  $M_{T_k}(G_w)(w_k)$  is defined by:

$$\text{For } (m, n) \in \{1, \dots, n_v\}^2, M_{DRk}(G_w)(w_k)_{m,n} = \begin{cases} w_k & \text{if } m = j, n = i \text{ and } i \neq j \\ -w_k & \text{if } m = n = i \text{ and } i \neq j \\ 0 & \text{Otherwise} \end{cases}$$

\* The transformation matrix associated with the network graph,  $M_T(G_w)(W)$ , represents the graph transformation from the reference configuration  $G_r$  (network graph without the virtual edges i.e. with all switches open) to another configuration,  $W$ , given by the discrete state of the  $n_s$  switches.  $M_T(G_w)(W)$  is the following ordered product (from  $k=1$  to  $k=n_s$ ):

$$\begin{aligned} M_T(G_w)(W) &= \prod_{k=1}^{n_s} (I_{n_v} + M_{T_k}(G_w)(w_k)) \\ &= (I_{n_v} + M_{T_1}(G_w)(w_1)) \dots (I_{n_v} + M_{T_{n_s}}(G_w)(w_{n_s})) \end{aligned} \quad (1)$$

\* The hybrid parameterized incidence matrix  $IM(G_w)(W)$  of the family of geometric interconnection structures of the  $2^{n_s}$  configurations of the PSS depends on  $W$  and is given by:

$$IM(G_w)(W) = M_T(G_w)(W) IM(G_r) \text{ with } W \in \{0, 1\}^{n_s} \quad (2)$$

To illustrate these definitions, let us give the hybrid incidence matrices for the dynamic network graph and a dynamic dual graph (figure 2) associated with the simplified Buck power converter schemed in figure 1.

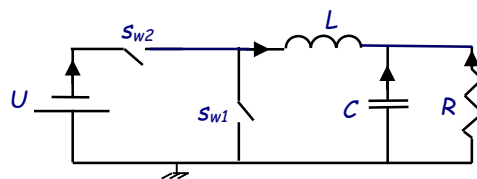


Fig. 1. simplified Buck power converter

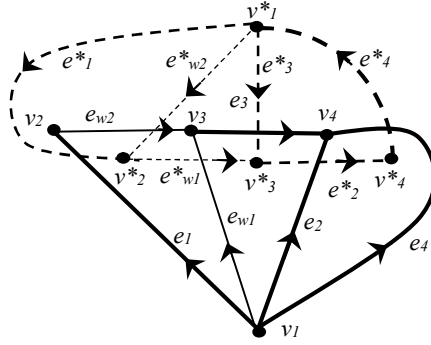


Fig. 2. a primal and a dual dynamic network graphs for the Buck converter:  $G_{wa}$  and  $G_{wa}^*$

The functional edges of the primal network graph  $G_{wa}$  are represented with thick lines and the virtual edges with thin lines. The functional edges of the dual network graph  $G_{wa}^*$  are represented with dotted lines and dual virtual edges with thin dotted lines. The orientation of the edges is chosen in accordance with the proposition 1 and is not unique.

The parameterized incidence matrices for the simplified Buck converter are:

$$IM(G_{wa})(W) = \begin{bmatrix} 1-w_1 & 1-w_1 & 0 & 1-w_1 \\ w_2-1 & 0 & 0 & 0 \\ w_1-w_2 & w_1 & 1 & w_1 \\ 0 & -1 & -1 & -1 \end{bmatrix} \quad (3)$$

$$IM(G_{wa}^*)(W) = \begin{bmatrix} w_2 & 0 & w_2 & -w_2 \\ -w_1w_2 & 0 & w_1(1-w_2) & w_1(w_2-1) \\ w_2(w_1-1) & 1 & (1-w_1)(1-w_2)-1 & (1-w_1)(w_2-1) \\ 0 & -1 & 0 & 1 \end{bmatrix} \quad (4)$$

with  $(w_1, w_2) \in \{0, 1\}^2$ .

Some states of the switches may lead to non-admissible configurations for the physical system with variable topology. Indeed, a problem arises if generalized Kirchhoff's laws and independent non zero effort or flow sources are in conflict. It is therefore of prime importance to remove them for the control synthesis procedure.

### 3. ADMISSIBLE AND CONSTRAINED CONFIGURATIONS ANALYSIS

#### 3.1 Admissible configurations

Naturally, electrical, mechanical or thermal specialists analyze the non admissible configurations according to their experience. In this section, we propose a generic and constructive method so that the modeling and control specialist can carry out this analysis, even if he is unfamiliar with the field.

**Proposition 2:** a non-admissible configuration corresponds to:

- i) An effort source in short-circuit or several independent effort sources connected in parallel or in reversed parallel.

ii) A flow source connected in an open-circuit or several independent flow sources connected in series or in reversed series.

A main advantage of the incidence matrix  $IM(G_w)(W)$  is that its direct analysis gives all the admissible configurations dealing with effort sources, because the generalized Kirchhoff's flow laws can be expressed from  $IM(G_w)(W)$ . Non-admissible configurations dealing with flow sources may be deduced from the direct analysis of the dual dynamic network graph because the generalized Kirchhoff's effort laws can be expressed from  $IM(G_w^*)(W)$ . If a flow source is connected in an open-circuit, its dual edge shows a short-circuit. If several independent flow sources are connected in series, their dual edges are connected in parallel.

Therefore, in order to allow the systematic analysis of the hybrid incidence matrices, the edges in the reference network graph  $G_r$  are indexed by gathering the elements of each type together. The following edges indexation is arbitrarily chosen: effort sources, elastic energy storage elements, kinetic energy storage elements, dissipative elements and flow sources. Thus, the characterization of non-admissible configurations is:

**Definition 3:** If the physical switching system  $\Sigma_w$ , the geometric structure of which is modeled by the hybrid incidence matrices  $IM(G_w)(W)$  and  $IM(G_w^*)(W)$ , includes  $n_{es}$  effort sources and  $n_{fs}$  flow sources, a non-admissible configuration defined by a vector  $W \in \{0, 1\}^{ns}$  fulfills one of the four following conditions:

- 1) If  $n_{es} \neq 0$ ,  $\exists j \in \{1, \dots, n_{es}\} / IM(G_w)(W)_{\bullet j} = 0$ .
- 2) If  $n_{es} > 1$ ,  $\exists (i, j) \in \{1, \dots, n_{es}\}^2$  with  $i \neq j / |IM(G_w)(W)_{\bullet j}| = |IM(G_w)(W)_{\bullet i}|$ .
- 3) If  $n_{fs} \neq 0$ ,  $\exists j \in \{n_{ef}-n_{fs}+1, \dots, n_{ef}\} / IM(G_w^*)(W)_{\bullet j} = 0$ .
- 4) If  $n_{fs} > 1$ ,  $\exists (i, j) \in \{n_{ef}-n_{fs}+1, \dots, n_{ef}\}^2$  with  $i \neq j / |IM(G_w^*)(W)_{\bullet j}| = |IM(G_w^*)(W)_{\bullet i}|$ .

$|V|$  represents the vector where each component is the absolute value of each component of  $V$ . Condition 1) detects effort sources in short-circuit, condition 2) independent effort sources connected in parallel or reversed parallel, condition 3) flow sources connected in open circuit and condition 4) independent flow sources connected in series or reversed series, in the configuration characterized by  $W$ . The set of admissible configurations of the physical switching system  $\Sigma_w$ , according to proposition 2, is denoted  $A(\Sigma_w)$ . Note that:  $A(\Sigma_w) \subset \{0, 1\}^{ns}$ .

Let us consider the simplified Buck converter presented in figure 1. Its primal and dual dynamic network graphs  $G_{wa}$  and  $G_{wa}^*$ , given in figure 2, respect the index order of the functional elements proposed in this section and there is only one effort source. This source is represented in  $IM(G_{wa})(W)$  by the first column (equation (3)).

$$IM(G_{wa})(W)_{\bullet 1} = 0 \Leftrightarrow \begin{bmatrix} 1-w_1 \\ w_2-1 \\ w_1-w_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Thus, the only non-admissible configuration among the four possible is:  $W_a = [1 \ 1]^T$ . Therefore, as there are no flow sources in the circuit,  $A(\Sigma_{wa}) = \{0, 1\}^2 - \{[1 \ 1]^T\}$ . The simplified Buck power converter is a good educational example which is easy to understand.

This result is therefore highly expectable. However, for larger size applications, this generic analysis method is an interesting support for modeling and control specialists.

In the Bond graphs context, Buisson, Cormerais and Richard [3] proposed a rank condition on part of an implicit state space standard form (equation (5)) to determine if a given configuration is admissible (or acceptable). Definition 3 could be written as a rank condition too, but with the loss of information about the nature of the non-admissibility.

### 3.2 Constrained configurations

It should be noted that some configurations of a physical switching system are represented by implicit state space representations with algebraic modes that introduce constraints. As a consequence, discontinuities may occur in the real valued state variables if the constraints configurations are reached when the constraints are not satisfied. It is therefore interesting to be aware of these constrained configurations for the control synthesis procedure.

**Proposition 3:** *a constrained configuration corresponds to elastic energy storage elements connected in short-circuit or in parallel (with the same orientation or not), or kinetic energy storage elements connected in open-circuit or in series (with the same orientation or not).*

**Definition 4:** *If the physical switching system,  $\Sigma_w$ , includes  $n_{es}$  effort sources,  $n_c$  elastic energy storage elements and  $n_L$  kinetic energy storage elements, a constrained configuration is defined by vector  $W \in \{0, 1\}^{ns}$  if one of the four following conditions is satisfied:*

- 1) *If  $n_c \neq 0, \exists j \in \{n_{es}+1, \dots, n_{es}+n_c\} / IM(G_w)(W)_{\bullet j} = 0$ .*
- 2) *If  $n_c > 1, \exists (i, j) \in \{n_{es}+1, \dots, n_{es}+n_c\}^2$  with  $i \neq j / |IM(G_w)(W)_{\bullet j}| = |IM(G_w)(W)_{\bullet i}|$ .*
- 3) *If  $n_L \neq 0, \exists j \in \{n_{es}+n_c+1, \dots, n_{es}+n_c+n_L\} / IM(G_w^*)(W)_{\bullet j} = 0$ .*
- 4) *If  $n_L > 1, \exists (i, j) \in \{n_{es}+n_c+1, \dots, n_{es}+n_c+n_L\}^2$  with  $i \neq j / |IM(G_w^*)(W)_{\bullet j}| = |IM(G_w^*)(W)_{\bullet i}|$ .*

Condition 1) detects elastic energy storage elements in short-circuit, condition 2) elastic energy storage elements connected in parallel condition 3) kinetic energy storage elements connected in open-circuit and condition 4) kinetic energy storage elements connected in series in the configuration characterized by  $W$ . The set of constrained configurations of the physical switching system  $\Sigma_w$  is denoted  $C(\Sigma_w)$ . A constrained configuration is admissible, thus  $C(\Sigma_w) \subset A(\Sigma_w)$ .

For example, take all the cases of potential discontinuities in real valued state variables for the simplified Buck converter. If the only capacitor is in short-circuit, the second column in  $IM(G_{wa})(W)$  (equation (3)) satisfies the following equations:

$$IM(G_{wa})(W)_{\bullet 2} = 0 \Leftrightarrow \begin{bmatrix} 1 - w_I \\ 0 \\ w_I \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \text{no solution}$$

If the only inductor is in open-circuit, the third column in  $IM(G_{wa}^*)(W)$  (equation (4)) satisfies the following equations:



$$IM(G_{wa}^*)(W) \bullet 3 = 0 \Leftrightarrow \begin{bmatrix} w_2 \\ w_1(1-w_2) \\ (1-w_1)(1-w_2)-1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Then, one configuration among the three admissible is constrained. Therefore:

$$C(\Sigma_{wa}) = \{[0 \ 0]^T\}.$$

#### 4. PARAMETERIZED PORT-HAMILTONIAN FORMULATION

In this section 4, the family of models of the piecewise continuous dynamics associated with the configurations of the system (discrete states of the hybrid automaton modeling the autonomous physical switching system (PSS)) is calculated in a generic and constructive way. Indeed, a non-minimal implicit parameterized port-Hamiltonian formulation of a PSS can be deduced from the hybrid incidence matrices for the primal and dual dynamic network graphs [19] of the system if these graphs are cyclically connected with no self-loop. As the network graph has some ports (edges) to which sources are connected with power variables being  $(i_S, u_S)$ , and others to which dissipative elements are connected with power variables being  $(i_R, u_R)$ , the so-called kernel representation [7] has been extended to admissible configurations of a physical switching system,  $\Sigma_w$ , as follows:

$$\begin{bmatrix} IM(G_w)(W) \\ 0 \end{bmatrix} \underline{pf} + \begin{bmatrix} 0 \\ IM(G_w^*)(W) \end{bmatrix} \underline{pe} = 0 \quad (5)$$

with:  $W \in A(\Sigma_w)$ ,  $\underline{pf} = \begin{bmatrix} i_S & \dot{q} & \frac{\partial H}{\partial \phi} & i_R \end{bmatrix}^T$ ,  $\underline{pe} = \begin{bmatrix} u_S & \frac{\partial H}{\partial q} & \dot{\phi} & u_R \end{bmatrix}^T$ ,  $H$  the total energy of the system and  $(q, \phi)$  the energy variables of the circuit. For circuit's models with linear elements,  $H = \sum_{i=1}^{nc} \frac{1}{2} \frac{q_i^2}{C_i} + \sum_{j=1}^{nl} \frac{1}{2} \frac{\phi_j^2}{L_j}$ , with  $n_c$  the number of elastic energy storage elements in the circuit and  $n_l$  the number of kinetic energy storage elements in the circuit.

This implicit port-Hamiltonian system is defined on the space of the energy variables. This representation is non-minimal in the sense that it is a set of  $(n_{ef}+n_s+2)$  equations while a minimal representation would have  $(n_s+2)$  less. This is a special case of a non-minimal representation of a Dirac structure corresponding to a terminal formulation of generalized Kirchhoff's laws. It is parameterized by the discrete state of the switches and defines a family of models.

In the Bond graphs context, Buisson, Cormerais and Richard [3] proposed an implicit state space standard form (equation (25)) for physical systems with switches. It depends on the discrete state of the switches in comparison with a reference configuration defined with the most elements in integral causality. Therefore the meaning of the discrete state is not immediately the state of the switches, but it changes from one model to another, depending on the reference configuration. Moreover, in the non-minimal implicit parameterized port-Hamiltonian formulation of a PSS presented in equation (5) in this section, power variables associated with the switches are reduced. Thus, the state of the switches appears immediately as a parameter of the structure matrices.

Note that in equation (5), flow and effort vectors are composed of both derivatives of the real valued state variables with respect to time, and real valued state variables. This non-minimal kernel representation of the parameterized implicit port-Hamiltonian formulation of a PSS may be rewritten, after a simple permutation of the variables and relations and the reduction of variables related to dissipative elements, to express the following non-minimal set of parameterized algebraic and differential equations.

$$P(W) \begin{bmatrix} \dot{q} \\ \dot{\phi} \end{bmatrix} = Q(W) \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial \phi} \end{bmatrix} + S(W) u_s \quad (6)$$

$$B(W) i_s = D(W) \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial \phi} \end{bmatrix}, \text{ with } W \in A(\Sigma_w) \quad (7)$$

This set of algebraic and differential equations (6) and (7) can be written as a single parameterized representation common to all the admissible configurations of the system, after having taken into account the constitutive equations of the dissipative elements and only if the associated power variables ( $i_R, u_R$ ) can be expressed by explicit equations depending on the real valued energy variables,  $q$  and  $\phi$ , and on the discrete state of the switches,  $W$ . The equations of each configuration must be calculated individually by taking into account the constitutive equations of the dissipative elements and instantiation of,  $W$ , in the set of equations (5). In the sequel, the dissipative elements have a constant positive resistance value.

For the simplified Buck converter, a non-minimal parameterized implicit port-Hamiltonian formulation, with ( $n_{ef}+n_s+2=8$ ) equations, is:

$$\begin{bmatrix} 1-w_1 & 1-w_1 & 0 & 1-w_1 \\ w_2-1 & 0 & 0 & 0 \\ w_1-w_2 & w_1 & 1 & w_1 \\ 0 & -1 & -1 & -1 \\ & & & 0_{4 \times 4} \end{bmatrix} \begin{bmatrix} i_s \\ \dot{q} \\ \frac{\partial H}{\partial \phi} \\ i_R \end{bmatrix} + \begin{bmatrix} & & 0_{4 \times 4} & \\ w_2 & 0 & w_2 & -w_2 \\ -w_1 w_2 & 0 & w_1(1-w_2) & w_1(w_2-1) \\ w_2(w_1-1) & 1 & (1-w_1)(1-w_2)-1 & (1-w_1)(w_2-1) \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_s \\ \frac{\partial H}{\partial q} \\ \dot{\phi} \\ u_R \end{bmatrix} = 0$$

$$\text{with } H = \frac{q^2}{2C} + \frac{\phi^2}{2L} \text{ and } W \in \{[0 \ 0]^T, [0 \ 1]^T, [1 \ 0]^T\}.$$

Furthermore, after a simple permutation of the variables and relations and a reduction of variables related to resistors, it is possible to obtain the following single parameterized representation with ( $n_{ef}+n_s+2- n_R=7$ ) equations:

$$\begin{bmatrix} w_2 & & \frac{1}{R} w_2 \\ w_1(1-w_2) & & \frac{1}{R} w_1(1-w_2) \\ (1-w_1)(1-w_2) & & \frac{1}{R} ((1-w_1)(1-w_2)-1) \\ 1 & & 0 \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & -w_2 \\ 0 & w_1(w_2-1) \\ -\frac{1}{R} & (1-w_1)(w_2-1) \\ -\frac{1}{R} & -1 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial \phi} \end{bmatrix} + \begin{bmatrix} -\frac{1}{R} w_2 \\ \frac{1}{R} w_1 w_2 \\ \frac{1}{R} (1-w_1) w_2 \\ 0 \end{bmatrix} u_s \quad (8)$$

$$\begin{bmatrix} I - w_1 \\ w_2 - I \\ w_1 - w_2 \end{bmatrix} i_s = \begin{bmatrix} 0 & I - w_1 \\ 0 & 0 \\ 0 & w_1 - I \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial \phi} \end{bmatrix} \quad (9)$$

$$\text{with } H = \frac{q^2}{2C} + \frac{\phi^2}{2L} \text{ and } W \in \{[0 \ 0]^T, [0 \ 1]^T, [1 \ 0]^T\}$$

A minimal implicit port-Hamiltonian formulation of each physical switching system's configuration is obtained from this non-minimal single representation, after instantiation of  $W$ .

Let us now suggest a new generic method for modeling the complete autonomous PSS as a hybrid automaton, based on the parameterized port-Hamiltonian formulation presented in the section 4 and the analysis of the hybrid incidence matrices for the system presented in the section 3. It does not include the control of the switches that can be carried out using several approaches as [10,13,15,20,23,24,26,28,30,34] etc. and therefore is not relevant in a generic method. Two kinds of switches are considered in this paper: controlled switches as transistors, and autonomous switches as diodes. These two switching elements are therefore viewed as ideal. The transitions from one discrete state of the system,  $W$ , to another are determined by the conditions producing autonomous switches, and the control actions on the controlled switches.

## 5. SYNTHESIS OF A HYBRID AUTOMATON MODEL

### 5.1 Synthesis of a model of the autonomous hybrid system

Hybrid automata were defined ten years ago in [1,14]. In this section, a new generic and constructive method to synthesize such a model for an autonomous physical switching system (PSS) with only ideal controlled or autonomous switches is proposed, based on the parameterized port-Hamiltonian formulation and the analysis of the parameterized hybrid incidence matrices for the system. Two various control objectives are considered with two possible behavior patterns for the inferred controlled system:

Control objective 1: Keep the maximum number of admissible configurations (as defined in proposition 2) in the control sequence, but discontinuities may occur along the real valued state variables trajectory if the constraints are not satisfied when reaching a constrained configuration (as defined in proposition 3),

Control objective 2: Guarantee that no real valued state variable discontinuities occur keeping only admissible and non-constrained configurations in the model of the autonomous system.

**Proposition 4:** *To obtain a hybrid automaton, HA, for the autonomous physical switching system with only ideal controlled or autonomous switches,  $\Sigma_w$ , in a generic and constructive way, each element of the 6-tuple  $HA=(X, L, \Sigma, Act, T, Inv)$  is defined as follows.*

- $X$  is the real valued state space,  $X \subseteq \mathfrak{R}^{nc+nl}$ . Real valued state variables are energy variables associated with functional elements storing energy in the PSS.

- $L$  is a finite set of configurations of the PSS (also called modes, locations, discrete phases or discrete valued states variables):

- If control objective is 1,  $L$  is the set of admissible configurations of the PSS defined in subsection 3.1,  $L = A(\Sigma_w)$ .

- If control objective is 2,  $L$  is the set of admissible and non-constrained configurations of the PSS as defined in subsections 3.1 and 3.2,  $L = A(\Sigma_w) - C(\Sigma_w)$ .

$L = \{l_j / l_j = [w_{j1}, w_{j2}, \dots, w_{jns}]^T \in \{0, 1\}^{ns}, j \in \{1, \dots, m\}\}$ .  $L$  is the set of  $m$  vertices of the graph of the hybrid automaton. Then, the hybrid state of the system is given by the pair  $(x, l_j) \in X \times L$ .

- $\Sigma$  is the synchronization (control) events set.  $\Sigma = \{\sigma_{ij}, (i, j) \in \{1, \dots, m\}^2\}$ . The events are synthesized by the control system (not given in this autonomous model).  $\varepsilon$  is the continually-occurring event.

- $Act$  is a mapping that assigns to each location  $l_i \in L$ , a set of equations which define the dynamics of real valued state variables:

- If the control objective is 1, it is the mixed set of differential and algebraic equations synthesized in section 4 (equations (6) and (7) reiterated below):

$$P(W) \dot{x} = Q(W) \frac{\partial H}{\partial x} + S(W) u_s$$

$$B(W) i_s = D(W) \frac{\partial H}{\partial x}$$

- If the control objective is 2, it is a set of explicit differential equations written from the parameterized port-Hamiltonian formulation synthesized in section 4 (equation (5)):

$$\dot{x} = [J(W) - S(W)] \frac{\partial H}{\partial x} + g(W) u_s$$

$$i_s = g(W)^T \frac{\partial H}{\partial x}$$

with  $W = [w_{j1} \ w_{j2} \ \dots \ w_{jns}]^T$ .  $(i_s, u_s)$  are the real valued external conjugated variables and  $H$  is the Hamiltonian function of the PSS. The solutions of the port-Hamiltonian system are called the activities of the location  $l_i$ .

- $T$  is a finite set of transitions. They are the edges of the graph of the hybrid automaton. Each transition,  $tr_{ij} = (l_i, \sigma_{ij}, guard_{ij}, \mu_{ij}, l_j)$ , consists of a source location  $l_i \in L$ , a target location  $l_j \in L$ , a  $guard_{ij}(x)$  which is a subset of  $X$ , a synchronization event  $\sigma_{ij}$ , and an initialization function  $\mu_{ij}$ , so that:

1)  $\sigma_{ij}$  is defined so that:

- If  $n_s = n_{as}$ , thus  $\forall (i, j) \in \{1, \dots, m\}^2$ ,  $\sigma_{ij} = \varepsilon$ . All switches are autonomous switches.

- If  $n_{as} < n_s$ :

- If  $\forall k \in \{n_{as}+1, \dots, n_s\} \ w_{ik} = w_{jk}$ , thus  $\sigma_{ij} = \varepsilon$ . The transition  $tr_{ij}$  does not affect any controlled switches.

- If  $\exists k \in \{n_{as}+1, \dots, n_s\} / w_{ik} \neq w_{jk}$ , thus  $\sigma_{ij}$  is given by the control system.

2)  $guard_{ij}(x)$  is defined so that the transition from the discrete state  $l_i$  to the discrete state  $l_j$  is made possible when the real valued state variable  $x$  fulfils the  $guard_{ij}(x)$ . It is described by  $c_{ij}$  linear equalities (at most  $n_{as}$ , because the system has  $n_{as}$  ideal autonomous switches):  $\forall k \in \{1, \dots, c_{ij}\}, \underline{C}_k^T x = \underline{d}_k$  with  $\underline{C}_k$  being a  $(n_c + n_l)$ -length vector and  $\underline{d}_k$  a constant.  $guard_{ij}(x) \neq \mathfrak{R}^{nc+nl}$  only if  $\sigma_{ij} = \varepsilon$  i.e. if the transition is autonomous (not controlled). Thus, if  $\sigma_{ij} = \varepsilon$ :

- $\forall k \in \{1, \dots, n_{as}\} / (w_{ik} = 0) \wedge (w_{jk} = 1), (p_{e_{wik}} = 0) \in \text{guard}_{ij}(x)$ . If the relation  $(p_{e_{wik}} = 0)$  does not make sense, it means that the transition  $tr_{ij}$  is not possible.
- $\forall k \in \{1, \dots, n_{as}\} / (w_{ik} = 1) \wedge (w_{jk} = 0), (p_{f_{wik}} = 0) \in \text{guard}_{ij}(x)$ . As before, if the relation  $(p_{f_{wik}} = 0)$  does not make sense, it means that the autonomous transition  $tr_{ij}$  is not possible.

If  $\sigma_{ij} \neq \varepsilon$ ,  $\text{guard}_{ij}(x) = \mathfrak{R}^{n_c+n_l}$ . In this case, it is not written on the model.

$p_{e_{wik}}$  (respectively  $p_{f_{wik}}$ ) are expressed by writing the generalized Kirchhoff's law of a dynamic network graph's cycle (respectively cocycle) including the virtual edge  $e_{wk}$  and only functional edges.  $p_{e_{wik}}$  (respectively  $p_{f_{wik}}$ ) is a function of the Hamiltonian,  $H$  (which depends on the real valued state variable,  $x$ ) and of the real valued external conjugated variables ( $i_s, u_s$ ).

3)  $\mu_{ij}$  is defined so that:  $\mu_{ij} : X \rightarrow X / x(t^+) = \mu_{ij}(x(t^-))$

- If the control objective is 1, a discontinuity occurs in the real valued state variables after that a transition  $tr_{ij}$  towards a constraint configuration occurs, if the constraints in  $l_j$  are not fulfilled at that moment. In this case,  $\mu_{ij}(x(t^-)) \neq x(t^-)$ .  $\mu_{ij}(x(t^-))$  is calculated so that the constraints in  $l_j$  (given in  $\text{Act}(l_j)$ ) and the physical properties of the PSS are satisfied. In (Haddad, et al., 2003) the initialization function (also called the resetting law in equation (2)) is written as an explicit port-controlled Hamiltonian system. In this objective 1, pulses may occur in some power variables associated with some switches,  $p_{e_{wik}}, p_{f_{wik}}$ , as a consequence of the discontinuities in the real valued state variables,  $x$ , which are energy variables. They meet the physical properties of the PSS.
- If the control objective is 2,  $\mu_{ij}(x(t^-)) = x(t^-)$  and no discontinuity occurs in the real valued state variables when the transition  $tr_{ij}$  happens. In this objective 2, discontinuities may occur in some power variables associated with some switches,  $p_{e_{wik}}$  and  $p_{f_{wik}}$  depending if the constitutive equations of the switches are satisfied when a transition occurs.

- $\text{Inv}$  is a mapping from the set of discrete states  $L$  to the set of subsets of  $X$ , that is  $\text{Inv}(l_j) \subseteq X$  for all  $l_j \in L$ . For each location  $l_j = [w_{j1} \ w_{j2} \ \dots \ w_{jns}]^T$ , the real valued state variable  $x$  must satisfy  $x \in \text{Inv}(l_j)$ .  $\text{Inv}(l_j)$  cannot be violated, which means that the phase  $l_j$  is left as soon as an inequality of  $\text{Inv}(l_j)$  is revealed to be false and that a transition to  $l_j$  can not occur if  $\text{Inv}(l_j)$  is false. It is described by  $c_j$  linear inequalities (at least,  $n_{as}$  which take into account the behavioral constraints of the ideal autonomous switches):  $\forall k \in \{1, \dots, c_j\}, C_{jk}^T x \leq d_{jk}$  with  $C_{jk}$  a  $(n_c+n_l)$ -length vector and  $d_{jk}$  a constant. The set of invariants of the hybrid automaton is not necessarily a state-space partition.

The set of  $c_j$  linear inequalities brings together various kinds of invariants:

- 1) Concerning ideal autonomous switches ( $n_{as}$  linear inequalities):
  - $\forall k \in \{1, \dots, n_{as}\} / w_{jk} = 0, (p_{e_{wjk}} \leq 0) \in \text{Inv}(l_j)$ ,
  - $\forall k \in \{1, \dots, n_{as}\} / w_{jk} = 1, (p_{f_{wjk}} \geq 0) \in \text{Inv}(l_j)$ ,
- 2) Physical constraints relating to functional elements, if they are considered,
- 3) It contains no invariants concerning ideal controlled switches.

$p_{e_{wjk}}$  (respectively  $p_{f_{wjk}}$ ) is expressed by writing the generalized Kirchhoff's law of a dynamic network graph cycle (respectively cocycle) including the virtual edge  $e_{wk}$  and only functional edges.  $p_{e_{wjk}}$  (respectively  $p_{f_{wjk}}$ ) is a function of the Hamiltonian,  $H$  (which depends on the real valued state variable,  $x$ ) and the real valued external conjugated variables ( $i_s, u_s$ ).

## 5.2 Synthesis of the hybrid automaton of the Buck converter

To illustrate proposition 4, a hybrid automaton is synthesized for the autonomous part of the Buck power converter. A more realistic scheme than in figure 1 is considered in figure 3, with switch  $S_{w1}$  being an ideal autonomous switch and switch  $S_{w2}$  an ideal controlled switch.

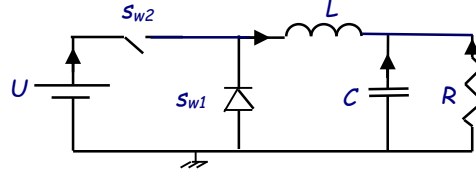


Fig. 3. More realistic Buck converter's scheme

### 5.2.1 Synthesis in the context of the control objective 2

The specifications given by the control objective 2 imply that no constrained configurations appear during the control of the system. It means that the control is supposed to ensure that the current through the inductance is never equal to 0. Thus,  $Act$  is a set of explicit differential equations expressed by the parameterized port-Hamiltonian formulation.

According to proposition 4:

- $L = \{l_1, l_2\} = \{[1 \ 0]^T, [0 \ 1]^T\}$  is the set of admissible and non constraints configurations.
- $x = [q \ \phi]^T \in \mathfrak{R}^2$
- $Act(l_i)$  comes from instantiation of  $W$  in equations (8) and (9) in section 4.
- $Inv(l_1): (pf_{w11} \geq 0)$  i.e.  $(\frac{\partial H}{\partial \phi} \geq 0)$
- $\sigma_{12} \neq \varepsilon$  because the transition  $tr_{12}$  affects the controlled switch  $S_{w2}$ .
- Thus,  $guard_{12}(x) = \mathfrak{R}^2$ . It is not written in the model.
- $Inv(l_2): (pe_{w21} \leq 0)$  i.e.  $(-U \leq 0)$ , which is always true, thus,  $Inv(l_2): \mathfrak{R}^2$
- $\sigma_{21} \neq \varepsilon$  because the transition  $tr_{21}$  affects the controlled switch  $S_{w2}$ .
- Thus,  $guard_{21}(x) = \mathfrak{R}^2$ . It is not written in the model.

The hybrid automaton of the autonomous part of the Buck power converter, in the context of the control objective 2, is:

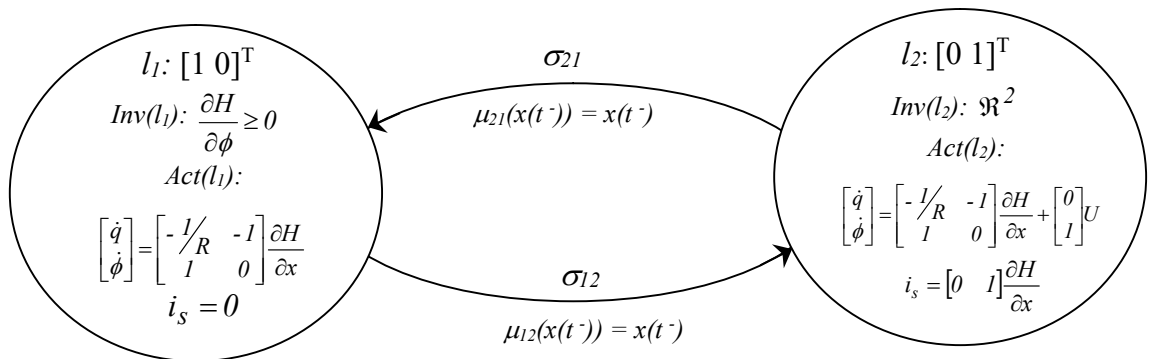


Fig. 4. Autonomous Buck converter's hybrid automaton within the control objective 2.

Discussion: Within this objective 2, there is no discontinuity in the state variables and therefore there is no pulse in the power variables associated with the switches. But, there are discontinuities in the effort variables associated with the switches and there may be discontinuities in the flow variables associated with the switches, depending on when transitions take place;

- After transition  $tr_{12}$ :
  - $pe_{w21} = pe_{w11} - U$  and  $pf_{w21} = pf_{w11} - \frac{\phi}{L} = 0$ .
  - $pe_{w22} = pe_{w12} + U$  and  $pf_{w22} = pf_{w12} + \frac{\phi}{L}$ .
- After transition  $tr_{21}$ :
  - $pe_{w11} = pe_{w21} + U$  and  $pf_{w11} = pf_{w21} + \frac{\phi}{L}$ .
  - $pe_{w12} = pe_{w22} - U$  and  $pf_{w12} = pf_{w22} - \frac{\phi}{L} = 0$ .

### 5.2.2 Synthesis in the context of the control objective 1

The specifications given by the control objective 1 imply that constrained configurations may appear during the control of the system. Thus,  $Act$  is a set of implicit or explicit differential equations expressed by the parameterized port-Hamiltonian formulation.  $Act(li)$  comes from instantiation of  $W$  in equations (8) and (9) in section 4. It may not be written as we would have written it immediately from experience, but the complete set of equations is nevertheless valid.

According to proposition 4:

- $L = \{l_0, l_1, l_2\} = \{ [0 \ 0]^T, [0 \ 1]^T, [1 \ 0]^T \}$  is the set of admissible configurations.
- The locations  $l_1$  and  $l_2$  are defined as in 5.2.1. as well as transitions  $tr_{12}$  and  $tr_{21}$ .
- $Inv(l_0)$ : ( $pe_{w01} \leq 0$ ) i.e. ( $\frac{\partial H}{\partial q} \leq 0$ )
- The autonomous transition  $tr_{10}$  is defined by:
  - $\sigma_{10} = \varepsilon$  because  $w_{12} = w_{02}$ . Indeed, the transition  $tr_{10}$  does not affect any controlled switches.
  - As  $(w_{11} = 1) \wedge (w_{01} = 0)$ , thus,  $(pf_{w11} = 0) \in guard_{10}(x)$  i.e.  $\frac{\partial H}{\partial \phi} = 0 \in guard_{10}(x)$ .
  - $\mu_{10}(x(t^-)) = [q(t^-) \ 0]^T = x(t^-)$  because the constraint  $\frac{\partial H}{\partial \phi} = 0$  is included in the  $guard_{10}(x)$ .
- The autonomous transition  $tr_{01}$  is defined by:
  - $\sigma_{01} = \varepsilon$  because  $w_{02} = w_{12}$ . Indeed, the transition  $tr_{10}$  does not affect any controlled switches.
  - As  $(w_{01} = 0) \wedge (w_{11} = 1)$ , thus,  $(pe_{w01} = 0) \in guard_{01}(x)$  i.e.  $\frac{\partial H}{\partial q} = 0 \in guard_{01}(x)$ .
  - $\mu_{10}(x(t^-)) = x(t^-)$  because  $Act(l_1)$  is not a constrained configuration.

Remark: in reality, this transition will occur when the capacitor C is fully discharged. Then it creates a zeno behavior with transitions  $tr_{01}$  and  $tr_{10}$  because, ( $pe_{w01} = 0$ ) and ( $pf_{w11} = 0$ ). Therefore the control has to ensure that it does not happen with the occurrence of  $\sigma_{02}$ .

The hybrid automaton of the autonomous part of the Buck power converter, in the context of control objective 1, is (each element is explained after the figure):

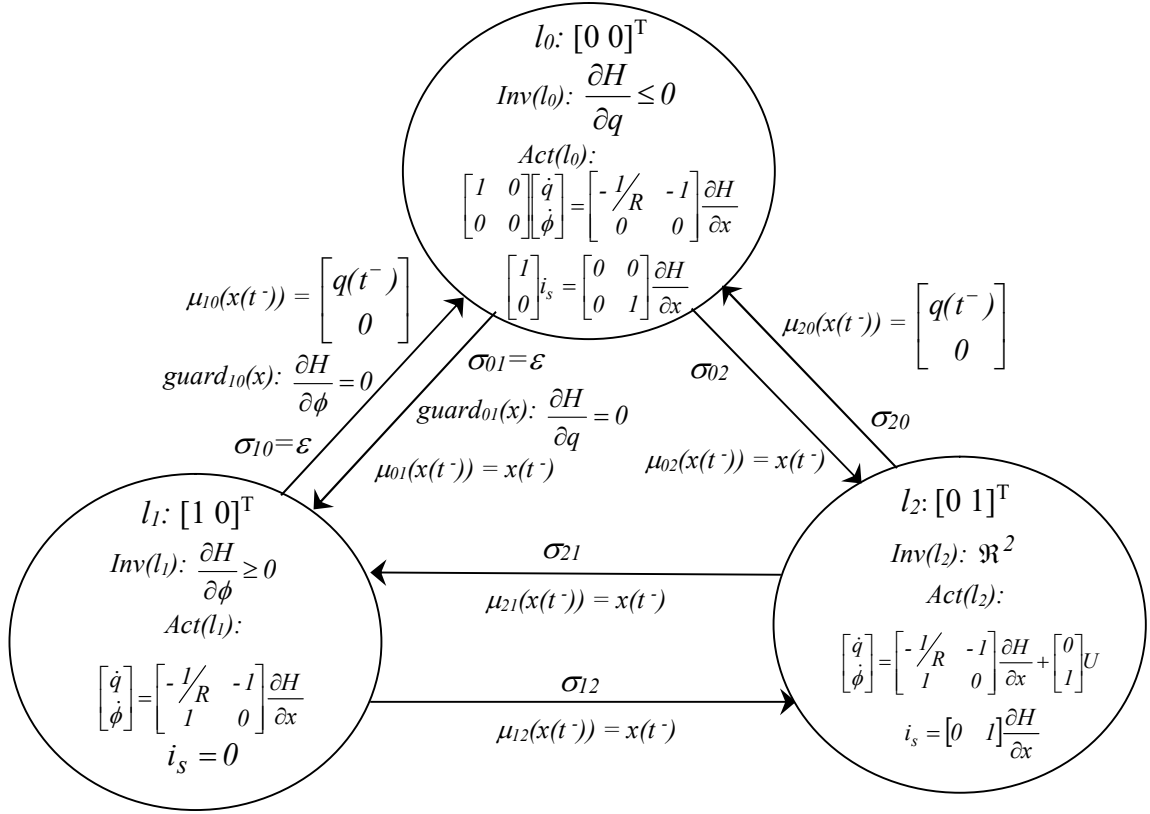


Fig. 5. Autonomous Buck converter's hybrid automaton within the control objective 1.

- The controlled transition  $tr_{02}$  is defined by:
  - $\sigma_{02} \neq \varepsilon$  because the transition  $tr_{02}$  affects the controlled switch  $S_{w2}$ .
  - Thus,  $guard_{02}(x) = \mathfrak{R}^2$ . It is not written on the model.
  - $\mu_{02}(x(t^-)) = x(t^-)$  because  $Act(l_2)$  is not a constrained configuration.
- The controlled transition  $tr_{20}$  is defined by:
  - $\sigma_{20} \neq \varepsilon$  because the transition  $tr_{20}$  affects the controlled switch  $S_{w2}$ .
  - Thus,  $guard_{20}(x) = \mathfrak{R}^2$ . It is not written on the model.
  - $\mu_{20}(x(t^-)) = \begin{bmatrix} q(t^-) \\ 0 \end{bmatrix}^T$  because the constraint  $\frac{\partial H}{\partial \phi} = 0$  in  $Act(l_0)$  must be satisfied.

Remark: From a control point of view, this transition  $tr_{20}$  does not make any sense, but it is nevertheless part of the autonomous model of the Buck power converter



because there is no generic rule to take it out. Moreover, if the invariant  $Inv(l_0)$  is false, at the time when  $\sigma_{20}$  occurs, this transition  $tr_{20}$  will not happen.

In the Bond graphs formalism, Buisson, Cormerais, Zainea, Guéguen, Godoy [4] proposed also an approach for computing hybrid automata models for linear physical systems with four types: of switches ideal, transistors, diodes and thyristors. This approach is bottom-up concerning the discrete locations of the PSS. It means that one elementary hybrid automaton is designed for each switch of the system. The complete automaton of the PSS is deduced from these elementary automata. On the other hand, the calculation of the set of algebraic and differential equations associated with each configuration of the PSS is global i.e. it is performed for the global system without previous decomposition. The method presented in this section 5 is based on network graphs and a port-Hamiltonian formulation and is entirely global which makes it possible to take utmost advantage of the interconnection structure of the various topologies of the PSS and of the control objectives.

## 6. CONCLUSION AND PERSPECTIVES

A generic and constructive method to synthesize a hybrid automaton model for a wide class of dissipative physical systems with sources and switching topology has been proposed. It is based on graph theory and on a parameterized port-Hamiltonian formulation of the piecewise continuous dynamics of the various configurations of the hybrid dynamical system and on the analysis of the admissible and the non-constrained configurations of the physical switching system (PSS). Indeed, it is of prime importance to remove the non-admissible configurations from the control synthesis procedure and to be aware of the constrained configurations which may lead to state discontinuities in the trajectory of the real valued state variables. This generic approach based on network graphs is an interesting way of understanding and analyzing energy exchanges and element interactions in physical switching systems, independently of the field and in a constructive way which is not only based on experience. This method has been illustrated in the educational example of the Buck converter.

The approach presented here is particularly well-suited to the context of a modular analysis of complex non-regular systems. Indeed, the hybrid (parameterized) incidence matrices can be calculated for all the subsystems (regular or not) and then connected through the ports.

A continuation of this work is to extend control synthesis methods based on continuous Hamiltonian systems such as Interconnection Damping Assignment Passivity Based Control [21,26] and continuous control synthesis method for hybrid port-controlled Hamiltonian systems affecting autonomous switching [13] to dissipative physical switching systems with sources and controlled switches. Some studies are already exploring this area as [24]. Another perspective of this paper would be the extension of solution concepts and the trajectory calculation developed in [12] to dissipative physical switching systems with sources.

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