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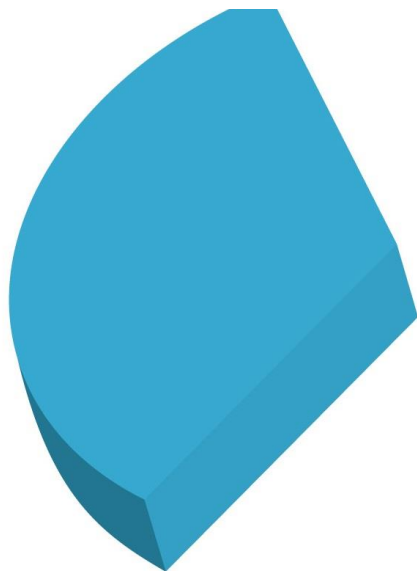
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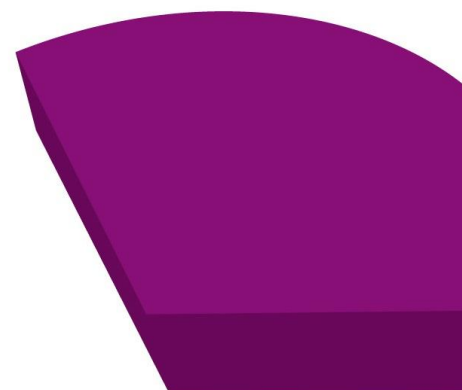
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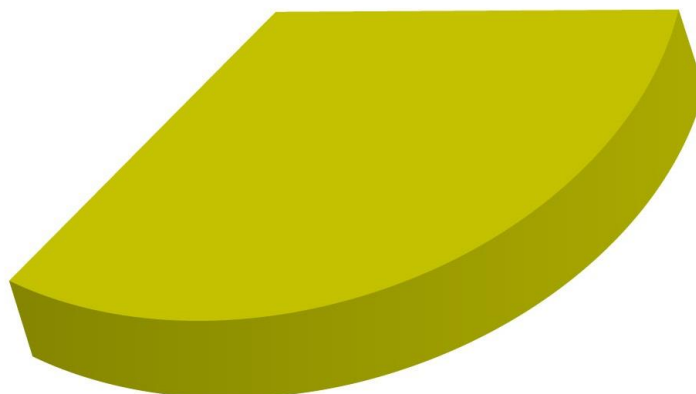
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**Cross-asset holdings and the resiliency  
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**Louis RAFFESTIN**



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# Cross-asset holdings and the resiliency of wholesale funding

Olessia Caillé \*      Louis Raffestin †

November 9, 2018

## Abstract

We provide a model in which financial institutions that share similar portfolios have an incentive to provide each other with favorable lending conditions, in order to avoid fire sales. This mechanism helps to explain the relatively mild reaction of interest rates to rising risk in short-term unsecured lending markets during the 2008-2009 crisis. Nevertheless, when similar institutions have privileged lending relationship, the financial network becomes clustered. This may increase systemic risk by fostering contagion.

*Keywords:* Wholesale Funding, Short-term Unsecured Lending, Fire Sales, Liquidity Crises.

*JEL classification:* E43, G21.

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# 1 Introduction

In the aftermath of the 2008-2009 crisis, economists and market participants rapidly agreed that the main catalyst had been a freeze in wholesale funding. However with more hindsight and data, the story behind the behavior of short-term liquidity markets proved to be more complex than expected. While the repo market completely shut down, the same is not true for unsecured lending. Interbank lending did not fall drastically in terms of volume, while rates rose only moderately and with limited dispersion among lenders (Angelini et al., 2011). The market for certificates of deposits also experienced stress but never ceased to function (Pérignon et al., 2018). The general message from the dedicated literature is that while troubled financial institutions typically had more difficulties in obtaining funding, they were rarely completely cut off from the market.

This relative moderation is harder to explain than a generalized market failure. A possibility is that the market simply downplayed risk, however this appears to be invalidated by the levels of risk aversion observed during the crisis. It is also conceivable that market participants adopted altruistic behaviors, but this seems unlikely considering the state of the financial sector at the time of the crisis. A more convincing theoretical explanation is linked to the existence of safety nets. If financial institutions are protected by an implicit guarantee from the government, they should obtain funding more easily than what their financial statement suggest. Consistent with this idea, Angelini et al. (2011) provide evidence that size appeared to facilitate the access to funding during the crisis, although somewhat paradoxically the effect grows stronger after the failure of Lehman brothers.

This paper offers a novel explanation for the observed robustness of wholesale funding, based on a new form of implicit guarantee, provided this time by the financial network instead of a government. The core idea is that if two financial institutions  $A$  and  $B$  are closely related in terms of asset holdings, a failure of  $A$  would impact negatively bank  $B$ . This gives  $B$  an incentive to provide funding to  $A$  at more favorable conditions than it would otherwise, effectively insuring it against liquidity shocks. Thus a guarantee for troubled institutions arises endogenously from the structure of the banking system, which is referred to as the "common interests subsidy".

More specifically, we provide a model in which two banks have overlapping portfolios and a liquidity shock leads one bank to be in need of short-term funds. The cash poor bank may obtain liquidity either by borrowing from the healthy bank, or by selling assets at a discount. The possibility of asset sales then enters the expected utility of the healthy bank, leading the utility maximizing interest rate to be a decreasing function of the level of commonality between both banks. This relationship between commonality and equilibrium interest is studied analytically and computationally.

We find that this simple set-up generates lending patterns that are consistent with the empirical features of wholesale funding markets observed during the crisis. In particular, the fact that the lender is willing to offer better conditions to the borrower - in order to ensure itself against sales - explains why rates were relatively insensitive to rising risk in unsecured lending markets. In the same vein, we uncover that the incentive to subsidize the borrower is stronger during episodes of stress, when liquidity risk is high and the price impact of fire sales is large.

Extending the analysis, we generalize the model to the  $N$  banks case, in order to study the

systemic implications of the common interests subsidy. First, we examine how the interbank lending market responds to changes in global liquidity conditions in the model. We find that the banking network becomes more clustered when liquidity risk rises. Second, we study whether this network is desirable from a systemic risk perspective. We find that it is not: borrower risk is indeed lower when lenders subsidize borrowers, but systemic risk rises because the clustered network fosters contagion compared to a network where banks diversify their sources of funding. In this way the impact of allowing banks to consider fire sales is somewhat counter-intuitive. On one hand it limits the manifestations of risk by making rates less sensitive to borrower risk. On the other hand, it nurtures underlying risk by increasing the scope for contagion.

This paper draws on three strands of literature.

First, through its focus, it is related to the empirical studies on wholesales funding market. The finding that interest rates and volumes -to a lesser extent- react relatively mildly to shocks is consistent with the papers mentioned at the start of the paper (Angelini et al., 2011; Pérignon et al., 2018). The results of Afonso et al. (2011) also fit particularly well in our setting. These authors show that, in the US, the activity on the market did not cease after Lehman's bankruptcy, despite episodes of stress. The same can be said of the Italian and Portuguese interbank markets (Affinito, 2012; Cocco et al., 2009). Outside the subprime crisis, Furfine (2002) documents that the Russian crisis did not cause any major disruption on the federal funds market. Another prediction of our model that bodes well with empirical work is that the financial network should become more clustered when risk rises (see for instance Craig et al. (2015) or Bolton et al. (2016)). In particular, Bräuning and Fecht (2016) focus on the German interbank shocks through market and find that "relationship lending has a negative effect on the bilateral interest rate when market conditions suffer from credit risk uncertainty" and a "positive effect on the access to interbank liquidity".

Second, through its defining idea that implicit guarantees explain the behavior of wholesale funding markets during times of stress, this paper can be linked to the literature on public safety nets and moral hazard in the banking sector. Allen et al. (2009) and Freixas et al. (2011) show that central bank's intervention acts as an insurance for the interbank market during stress periods. Allen et al. (2018) finds that the level of confidence in the banking system, which reflects the quality of institutions and legal regulations, plays a key role in the activity of the interbank market. The existence of public guarantees also changes the behaviors of agents outside times of crisis: Farhi and Tirole (2012) show that it may lead banks to choose similar leverage levels, Rochet and Tirole (1996) add that a TBTF policy impacts the incentives of banks to monitor each other. The difference between these papers and ours is that the guarantee is provided exogenously by the government, while in our case it arises endogenously through the structure of the banking system.

Third, through the modeling it uses, this study is related to the literature on contagion in the interbank market. Empirical contributions on this subject include Furfine (2003), who examine the exchange of interbank credits on the US federal funds market, and Upper and Worms (2004) who focus on the German interbank market. Both studies conclude that contagion exists and depends on the size of the losses. Theoretical work on contagion has globally identified two channels for contagion: direct balance sheet linkages (see for instance Allen and Gale (2000))

and common asset holdings (see for instance Raffestin (2014)). In this paper we consider both channels simultaneously, as in Cifuentes et al. (2005) who use a comparable set-up to show that mark-to-market rules have an accelerating effect of contagion. Importantly, in our paper both channels for contagion are not independent : interbank credit risk is decreased by the possibility of fire sales, as banks with similar asset structure provide each other with favorable lending conditions.

Overall this study makes three contributions. First we highlight a previously unexplored mechanism at play in wholesale funding market, that helps explaining some of its seemingly puzzling features. Second, we study the systemic implications of this mechanism. Finally the third contribution is of technical nature. We provide a flexible model that features contagion both on the asset side and the liability side. The model can be applied to a two banks case or the N-banks case, and has an analytically tractable solution under certain conditions.

The rest of the paper is organized as follows. Section 2 presents the model in the 2 banks case. Section 3 solves it in its analytically tractable version, and describes the algorithm used for simulations in the general version. Section 4 presents the results in the two banks set-up. Section 5 extends the analysis to a wider banking system. Section 6 concludes.

## 2 The model

### 2.1 At $t=0$

Consider an economy in which credit is intermediated by two banks  $B$  and  $L$ . There are three periods  $t = 0, 1, 2$ . At  $t = 0$ , both banks hold similar quantities of deposits  $D_0$ , and have the same amount of capital  $K_0$ . However the banks differ on the asset side.

Indeed three assets coexist in the economy:

- asset  $b$ , which is held exclusively by bank  $B$  at a quantity of  $q_b$
- asset  $l$ , which is held exclusively by bank  $L$  at a quantity of  $q_l$
- asset  $bl$ , which is held by both banks, at quantities of  $q_{bl}^B$  and  $q_{bl}^L$  for banks  $B$  and  $L$  respectively.

The  $t = 0$  balance sheet for bank  $I$  writes:

assets	liabilities
Asset held by $I$ only $q_{i,0}p_{i,0}$	Deposits $D_{i,0}$
Asset held by both banks $q_{bl,0}^I p_{bl,0}^I$	Capital $K_{i,0}$

Two variables in this balance sheet will move in stochastic fashion: deposits and asset prices. We set that both follow simple random walk processes:

$$D_{I,t} = D_{I,t-1} + \varepsilon_{I,t}^D$$

$$p_{i,t} = p_{i,t-1} + \varepsilon_{i,t}^p$$

where  $\varepsilon_{i,t}^p$  and  $\varepsilon_{I,t}^D$  will follow any symmetric distribution with moments  $\{\mu_p, \sigma_p^2\}$  and  $\{\mu_d, \sigma_d^2\}$  respectively. To capture the fact that the banks perform maturity transformation, the shock to deposits is set to occur at time  $t = 1$  while the shock to assets occurs at  $t = 2$ . This means a maturity mismatch is embedded in the balance sheet.

Before proceeding, note that the fact that the model considers "banks" is a simplification designed to ease presentation. Conceptually the mechanisms that are being modeled apply to any market where financial institutions have commonalities in their balance sheets. Thus we believe that the results can reasonably be generalized to any short run unsecured lending market, notably that for certificates of deposit.

## 2.2 At $t=1$

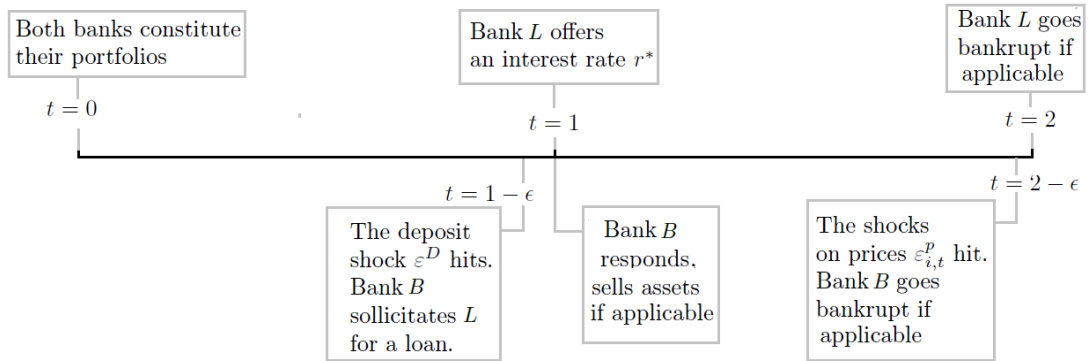
### 2.2.1 The market for loans

At  $t = 1$  the deposit shock occurs. This represents a liquidity shock for the banks, and will often be referred to in this way. To simplify exposition, we restrict our attention to a symmetric shock: an inflow of deposits of  $\varepsilon^D$  in one bank is matched by an outflow of  $\varepsilon^D$  in the other bank, which means that a market for liquidity can always exist. Bank  $B$  is defined as the bank that receives the negative shock in short-term liquidity, and bank  $L$  as the bank that receives the positive shock. Mathematically:  $\varepsilon_B^D = -\varepsilon^D$  and  $\varepsilon_L^D = \varepsilon^D$ .

When  $\varepsilon^D$  hits, bank  $B$  is in need of short-term funds. There are two ways for bank  $B$  to accommodate this need. First, it may borrow the short liquidity  $\varepsilon^D$  from bank  $L$  at a rate  $r$ . Second, it may quickly sell assets on the market. However such sales will have a depressive impact on prices. We set a simple linear relationship between sales  $\Delta q$  and price movements  $\Delta p$ :  $\Delta p_i = \lambda \Delta q_i$ , but a non linear form is considered in appendix D.

The option that bank  $B$  chooses depends on its expected utility in both cases. Past a given level for the interest rate, bank  $B$  will decide to sell assets. The negotiation process occurs as follows: at  $t = 1 - \epsilon$  the need for short term funds is revealed, and thus the bank  $B$  demands a cash loan of  $\varepsilon^D$ . Bank  $L$  then responds by offering a given interest rate  $r$  on the loan. Bank  $B$  immediately accepts or refuses the loan.

Figure 1 provides a timeline of the events.



**Figure 1: Sequence of events in the banking system**

This figure presents the chronology of occurrence for the events considered in this paper.

### 2.2.2 The balance sheets at $t=1$

This section studies the balance sheets of both banks at  $t = 1$ , following the liquidity shock  $\varepsilon^D$ . Two cases need to be considered separately:

- (i) bank  $B$  accepts the interest rate offered by bank  $L$ , and a loan occurs.

(ii) bank  $B$  refuses the rate offered by bank  $L$ , and sells assets.

*The loan case*

In the event of a loan, a debt (claim) of  $B_B = \varepsilon^D$  appears in the books of bank  $B$  (bank  $L$ ). The balance sheet for bank  $B$  at  $t = 1$  is then given by :

	assets	liabilities
bank $B$ :	$q_{b,0}p_{b,0}$	$D_{B,1} = D_{B,0} - \varepsilon^D$
	$q_{bl,0}^B p_{bl,0}$	$K_{B,1} = K_{B,0}$
		$B_B = \varepsilon^D$

While the balance sheet for bank  $L$  writes:

	assets	liabilities
bank $L$ :	$q_{l,0}p_{l,0}$	$D_{L,1} = D_{L,0} + \varepsilon^D$
	$q_{bl,0}^L p_{bl,0}$	$K_{L,1} = K_{L,0}$
	$B_B = \varepsilon^D$	

*The “no loan” case*

In the absence of a loan, bank  $B$  must sell an amount  $\Delta q_{b,1}$  and  $\Delta q_{bl,1}^B$  of both assets to obtain  $\varepsilon^D$ . These sales will have an impact on prices given by  $\Delta p_{b,1} = \lambda \Delta q_{b,1}$  and  $\Delta p_{bl,1} = \lambda \Delta q_{bl,1}^B$ . Appendix A shows that in this case the balance sheet for bank  $B$  writes:

	assets	liabilities
bank $B$ :	$(q_{b,0} - \Delta q_{b,1})(p_{b,0} - \lambda \Delta q_{b,1})$	$D_{B,1} = D_{B,0} - \varepsilon^D$
	$+(q_{bl,0}^B - \Delta q_{bl,1}^B)(p_{bl,0} - \lambda \Delta q_{bl,1}^B)$	$K_{B,2} = K_{B,0} +$
		$-\lambda q_{b,0} \Delta q_{b,1} - \lambda q_{bl,0}^B \Delta q_{bl,1}^B$

Turning to bank  $L$ , two new elements appear in the balance sheet at  $t = 1$ . The first element is a surplus of liquidity of  $\varepsilon^D$ , since it received a positive liquidity shock<sup>1</sup>. The second element is a fall in asset values due to the fact that the price of asset  $bl$  has fallen as a result of the sales from bank  $B$ . The balance sheet at  $t = 1$ , for bank  $L$  in the no loan case, is then given by :

	assets	liabilities
bank $L$ :	$q_{l,0}p_{l,0}$	$D_{L,1} = D_{L,0} + \varepsilon^D$
	$+q_{bl,0}^L(p_{bl,0} - \lambda \Delta q_{bl,1}^B)$	$K_{L,2} = K_{L,0} - \lambda q_{bl,0} \Delta q_{bl,1}^B$
	$L_1 = \varepsilon^D$	

The key point is that the sudden sales from bank  $B$  impacts the utility of bank  $L$  through the common asset  $bl$ .

## 2.3 At $t=2$

### 2.3.1 The modalities of default

A bank defaults if it loses more than its capital over the period, i.e. if we have  $\Pi = K_2 - K_0 < -K_0$ . When a bank enters bankruptcy, a share  $\delta$  of its assets is immediately sold, and any loan

<sup>1</sup>We simply set that this excess liquidity goes into a liquid short-term market at one period horizon that yields no interests  $L_1 = \varepsilon^D$



is defaulted on. This involves two types of cost for bank  $L$  if bank  $B$  fails: a direct one through default, and an indirect one through the sales of the common asset  $bl$ . The event of a failure of bank  $I$  is denoted by  $d_I$ , while survival is denoted  $\bar{d}_I$ .

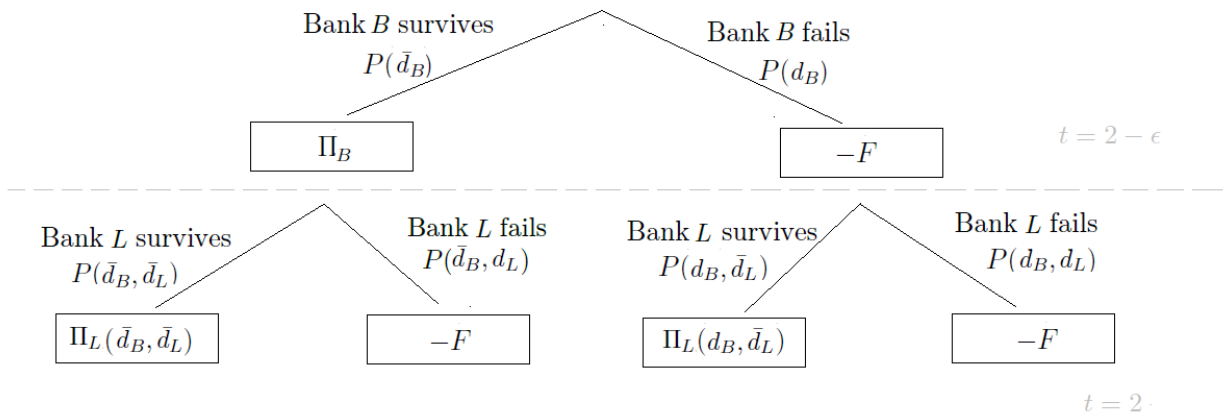
We set that bankruptcy is costly for the bank, assigning a lump sum cost of  $-F$  to it. Each bank thus has the following utility function  $U_I$  :

$$\begin{cases} U_I|\bar{d}_I = \Pi_I & \text{if } \Pi_I > -K_{I,0} \\ U_I|d_I = -F & \text{if } \Pi_I < -K_{I,0} \end{cases} \quad (1)$$

and the likelihood of a bankruptcy is:

$$P(\Pi_I < -K_{I,0}) \quad (2)$$

Shocks to asset prices are revealed at  $t = 2 - \epsilon$ , and both banks may be declared bankrupt at this point. However bank  $L$  may also declare bankruptcy later, at  $t = 2$ , if bank  $B$  failed earlier and contaminated it. Note that we only consider knock-on effects from the liquidity poor bank  $B$  to the liquidity rich bank  $L$ <sup>2</sup>. Figure 2 summarizes the profit for each bank in each possible case.



**Figure 2: Profit tree diagram**

This figure provides the profit for each bank in each possible case associated to its corresponding probability, at  $t = 2 - \epsilon$  and  $t = 2$ .

### 2.3.2 Profits at $t = 2$

This section derives the profits for both banks, obtained from the  $t = 2$  balance sheets. For conciseness we relegate the balance sheets to appendix B-a in the loan case, and appendix B-b in the no-loan case.

*The loan case*

At  $t = 2$ , the shocks on assets occur, and the interests on the  $t = 1$  loan must be repaid. This yields the following profit between  $t = 0$  and  $t = 2$ .

<sup>2</sup>Including the possibility of contagion from bank  $L$  to bank  $B$  would change the expression for the expected utility of bank  $B$  in the case of a loan. This would make the model much more complex, with little benefit because the quantitative impact of contagion on the expected utility of bank  $B$  is very small, as the probability that contagion occurs is near zero.

(i) for bank  $B$ :

$$\Pi_B = -r\varepsilon^D + q_{b,0}\varepsilon_b^p + q_{bl,0}^B\varepsilon_{bl}^p \quad (3)$$

(ii) for bank  $L$ , when bank  $B$  survives:

$$\Pi_{L,S} = r\varepsilon^D + q_{bl,0}^L\varepsilon_{bl}^p + q_{l,0}\varepsilon_l^p \quad (4)$$

(iii) for bank  $L$ , when bank  $B$  fails:

$$\Pi_{L,F} = q_{l,0}\varepsilon_l^p + q_{bl,0}^L\varepsilon_{bl}^p - \varepsilon^D + q_{bl,0}^L(-\lambda\delta q_{bl,0}^B) \quad (5)$$

*The no-loan case*

In this case the only  $t = 2$  change comes from the realization of the asset shocks. This yields the following profits, between  $t = 0$  and  $t = 2$ .

(i) for bank  $B$ :

$$\Pi_B^{nd} = q_{b,0}(-\lambda\Delta q_{b,1}) - \lambda q_{bl,0}\Delta q_{bl,1}^B + (q_{b,0} - \Delta q_{b,1})\varepsilon_b^p + (q_{bl,0}^B - \Delta q_{bl,1}^B)\varepsilon_{bl}^p \quad (6)$$

(ii) for bank  $L$ , when bank  $B$  survives:

$$\Pi_{L,S}^{nd} = q_{bl,0}^L(-\lambda\Delta q_{bl,1}^B) + q_{l,0}\varepsilon_l^p + q_{bl,0}^L\varepsilon_{bl}^p \quad (7)$$

(iii) for bank  $L$ , when bank  $B$  fails:

$$\Pi_{L,F}^{nd} = -\lambda q_{bl,0}^L q_{bl,0}^B + q_{l,0}\varepsilon_l^p + q_{bl,0}^L\varepsilon_{bl}^p \quad (8)$$

## 3 Solving the model

### 3.1 General method

The market equilibrium appears at the end of a three-stage process.

#### 3.1.1 Finding the optimal rate for bank $L$

The first step is to find the interest rate  $r^*$  that bank  $L$  would like to offer to bank  $B$  at  $t = 1$ . This  $r^*$  corresponds to the rate that maximizes its expected utility on the loan, given by:

$$E(U_L) = P(d_B, d_L)*(-F) + P(\bar{d}_B, d_L)*(-F) + P(d_B, \bar{d}_L)*E(U_L|d_B, \bar{d}_L) + P(\bar{d}_B, \bar{d}_L)*E(U_L|\bar{d}_B, \bar{d}_L) \quad (9)$$

where  $P(\bar{d}_B, \bar{d}_L)$  is the likelihood that both banks survive, and is equal to  $P(\bar{d}_B, \bar{d}_L) = P(\bar{d}_L|\bar{d}_B) * P(\bar{d}_B)$ .

All the probabilities and expected gains that feature in (9) are obtained from equations (3) to (8). For instance, applying (4) to expression (1) yields:

$$\left\{ \begin{array}{ll} U_L|\bar{d}_B, \bar{d}_L = \Pi_{L,S} = r\varepsilon^D + q_{bl,0}^L \varepsilon_{bl}^p + q_{l,0} \varepsilon_l^p & \text{if } r\varepsilon^D + q_{bl,0}^L \varepsilon_{bl}^p + q_{l,0} \varepsilon_l^p > -K_{L,0} \\ U_L|\bar{d}_B, d_L = -F & \text{if } r\varepsilon^D + q_{bl,0}^L \varepsilon_{bl}^p + q_{l,0} \varepsilon_l^p < -K_{L,0} \end{array} \right.$$

From this, we can draw the expected utility for bank  $L$  in the case where it survives  $E(U_L|\bar{d}_B, \bar{d}_L)$  and in the case where it fails  $E(U_L|\bar{d}_B, d_L) = -F$ . Furthermore, the probability of survival for bank  $L$  when  $B$  survived can be expressed as:

$$P(\bar{d}_L|\bar{d}_B) = P(r\varepsilon^D + q_{bl,0}^L \varepsilon_{bl}^p + q_{l,0} \varepsilon_l^p > -K_{L,0})$$

Proceeding in the same way for the other element then gives equation (9) its final form. Once this form is obtained,  $r^*$  appears as the interest rate  $r$  that satisfies the equality  $\frac{\partial E(U_L)}{\partial r} = 0$ , which is uniquely defined. Section 3.2 provides an analytically tractable version of the model that should make the procedure for finding  $r^*$  clearer for the reader.

### 3.1.2 Adding the participation constraint of bank $B$

The rate that maximizes the expected utility of bank  $L$  will not necessarily be the one that prevails. Indeed because bank  $B$  always has the alternative of selling assets, it will not be willing to accept any rate. Formally, there is a threshold interest rate  $\bar{r}$  above which the borrower refuses to get a loan. This rate  $\bar{r}$  is defined by the point where the expected utilities in the loan and no-loan cases are equivalent:

$$E(U_B) = E(U_B^{nl})$$

Both  $E(U_B)$  and  $E(U_B^{nl})$  are obtained using the same procedure as the one used to get  $E(U_L)$  in section 3.1.1. The reservation rate  $\bar{r}$  then appears naturally, and is also unique.

Once we have  $r^*$  and  $\bar{r}$ , two possibilities emerge. First, the rate that bank  $L$  wishes to implement  $r^*$  is lower than the threshold rate  $\bar{r}$ . In this case bank  $B$  will be happy to accept  $r = r^*$  since the borrower always benefits from a lower rate. In the second case, the rate that the lender would like to charge  $r^*$  is higher than the maximum rate  $\bar{r}$  that the borrower is willing to accept. Bank  $L$  is then constrained, and the best possible rate it can get is  $r = \bar{r}$ .

The utility maximizing rate for bank  $L$ , under the constraint that bank  $B$  participates, is thus given by:

$$r = \min(r^*, \bar{r})$$

### 3.1.3 Adding the participation constraint of bank $L$

$r$  is the rate that will prevail if a loan takes place, yet it is not clear that a loan will take place at this point. Indeed bank  $L$  has no obligation to lend, and may prefer to let bank  $B$  sell assets if the interest rate  $r$  is too low. To choose, bank  $L$  compares its expected utility in the case of a loan with a rate  $r$  to the same expected utility without a loan. If  $E(U_L(r)) > E(U_L^{nl})$ , the loan takes place, if  $E(U_L(r)) < E(U_L^{nl})$  it does not.

Expression (10) summarizes the possibilities for the market equilibrium:

$$\left\{ \begin{array}{ll} \varepsilon^D \text{ at a rate } r = \min(r^*, \bar{r}) & \text{if } -E(U_L(r)) > E(U_E^{nl}) \\ \text{no loan} & \text{if } -E(U_L(r)) > E(U_E^{nl}) \text{ with } r = \min(r^*, \bar{r}) \end{array} \right. \quad (10)$$

### 3.2 Analytical solution

This section presents a simplified version of the model that allows for a tractable solution for  $r^*$ . We simply provide an outline, the complete proof can be found in appendix C.

The analytical solution requires choosing specific distributions for the asset shocks. Here we set  $\varepsilon_b^p$  and  $\varepsilon_l^p$  to both be uniformly distributed over  $[\mu - V, \mu + V]$ , while  $\varepsilon_{bl}^p$  is set to zero. Uniform distributions are convenient because their cumulative distribution has a closed-form solution, and they fit into the general structure of the model which only requires asset shocks to follow a symmetric distribution. The choice of ruling out shocks on the common asset  $bl$  allows both banks to be independent in terms of asset returns, which simplifies the analysis while keeping the intuition intact.

We make two additional assumptions, that are not necessary to obtain a closed form solution but will ease presentation. First we set that both banks have the same structure in terms of asset composition, capital, asset variance, etc. This means for instance that  $K_{B,0} = K_{L,0} = K$ ,  $q_{b,0} = q_{l,0} = q_i$ , or  $q_{bl,0}^B = q_{bl,0}^L = q_{bl}$  in what follows. Second, the dispersion in asset returns  $V$  is set to be small enough that  $P(d_L|\bar{d}_B) = 0$ , meaning that the liquidity rich bank may not default if the liquidity poor bank has survived and paid back its loan<sup>3</sup>.

Applying the asset distributions to the profits in the loan scenario obtained in section 2.3.2, the profit for bank  $B$  simplifies to:

$$\Pi_B = -r\varepsilon^D + q_i\varepsilon_b^p$$

The uniform distribution then makes it easy to draw  $P(\bar{d}_B)$ , the likelihood that bank  $B$  survives at  $t = 2 - \varepsilon$ :

$$P(\bar{d}_B) = P(\Pi_B > -K) = \frac{q_i\mu + q_iV - r\varepsilon^D + K}{2q_iV}. \quad (11)$$

Straightforward calculus also yields the expression for the expected utility of bank  $B$  knowing that the bank has survived  $E(U_B|\bar{d}_B)$ :

$$E(U_B|\bar{d}_B) = \frac{1}{2}(-r\varepsilon^D + q_i\mu + q_iV - K).$$

We proceed in similar fashion for the situation of bank  $L$  when bank  $B$  defaulted on its loan. The profit of bank  $L$  writes:

$$\Pi_{L,F} = q_i\varepsilon_l^p - \varepsilon^D - \lambda\delta(q_{bl})^2.$$

The odds that bank  $L$  survives while  $B$  default are then given by:

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<sup>3</sup>Formally this translates in to the following condition:  $V_{pL} < \frac{r\varepsilon^D + K_{L,0} + q_l\mu_{pL}}{2q_{l,0}}$

$$P(\bar{d}_L, d_B) = \frac{q_i\mu + q_iV - \varepsilon^D - \lambda\delta(q_{bl})^2 + K}{2q_iV}.$$

And the expression for the expected utility of bank  $L$  knowing that it has survived while bank  $B$  failed, denoted by  $E(U_L|\bar{d}_L, d_B)$ , is:

$$E(U_L|\bar{d}_L, d_B) = \frac{1}{2}[q_i(\mu + V) - \varepsilon^D - \lambda\delta(q_{bl})^2 - K].$$

Finally we study the situation of bank  $L$  if bank  $B$  survives and consequently pays back its loan. In this case, by definition, we have:

$$P(\bar{d}_L, \bar{d}_B) = 1$$

and

$$E(U_L|\bar{d}_L, \bar{d}_B) = q_i\mu + r\varepsilon^D.$$

Plugging these expressions into (9) yields the expression for  $E(U_L)$ , the  $t = 2$  utility for bank  $L$  estimated at  $t = 1$ , which can be found in appendix C.

Maximising this expression then leads the following form for  $r^*$ , the expected utility maximizing interest rate for bank  $L$ :

$$r^* = \frac{1}{2 * \varepsilon^D} \left( q_iV + K - F + \left( \frac{1}{2q_iV} \right) * [q_i\mu + q_iV + K - \lambda\delta(q_{bl})^2 - \varepsilon^D] * \left[ F + \frac{1}{2}(q_i\mu + q_iV - \lambda\delta(q_{bl})^2 - \varepsilon^D - K) \right] \right)$$

This form shows that the rate that lenders are willing to offer depends in non monotonic fashion on all variables, particularly the amount of common exposure  $q_{bl}$ . To investigate in details we chose to abandon the limiting framework of the analytical solution, and resort to simulations.

### 3.3 Simulations

In the simulations normal distributions are used for all asset returns, including the common asset  $bl$ .

The code simply reflects the set-up defined in section 2. A slight technical challenge arises from the fact that both banks are not independent in terms of asset returns, which must be accounted for. In particular, a survival of bank  $B$  at  $t = 2 - \epsilon$  has implications for the expected value of the shock on asset  $bl$ , and thus implications on the expected utility for bank  $L$  in the ‘‘bank  $B$  survives’’ case. We have matlab recompute the new probability distribution of  $\varepsilon_{bl}^p$ , knowing that bank  $B$  did not fail, i.e.  $-r\varepsilon^D + q_{b,0}\varepsilon_b^p + q_{bl,0}^B\varepsilon_{bl}^p > -K_0$ . We then plug the revised expected value for  $\varepsilon_{bl}^p$  into  $U_L(\bar{d}_B, \bar{d}_L)$ . The equilibrium is then found following the three steps procedure described earlier.

We now turn to the calibration process.

Remember that at time  $t = 0$ , both banks hold a similar quantity of deposits and have the same amount of capital. The quantity of deposits is set to 9 (i.e.  $D_{B,0} = D_{L,0} = 9$ ) while the amount of capital is set to 1 (i.e.  $K_{B,0} = K_{L,0} = 1$ ). This choice implies a capital ratio of 10%, roughly consistent with the ratios observed in the US and in Europe<sup>4</sup>.

Asset returns are normally distributed with moments  $\mu = 0.1$  and  $\sigma = 0.2$ . This implies a Sharpe ratio of 0.5, which again is in line with casual observation of the financial markets<sup>5</sup>. Finally the prices of the three assets  $p_{b,0}$ ,  $p_{l,0}$  and  $p_{bl,0}$  is normalized to 1.

Three parameters remain to be calibrated: the impact of a sale on prices  $\lambda$ , the utility cost of failure  $F$ , and the share  $\delta$  of the bank's assets that are immediately sold following failure. These parameters are more difficult to calibrate, because they are unobservable and/or constantly changing. For all three we follow the same method: we take an obvious value, and consider alternatives in the robustness checks provided in section 4.2.

$\lambda$  is set to 0.1 which means that sales will have a small but significant effect on prices.  $F$  is set to 5, which represents half of the total exposure at period  $t = 0$ . Finally we specify  $\delta = 0.5$ , which is simply the average of the  $[0, 1]$  range in which this parameter is contained. This signifies that a bank that goes bankrupt immediately sells 50 percent of its remaining assets.

## 4 Equilibrium interest rates in the two bank model

### 4.1 Cross-asset holdings and the equilibrium interest rate

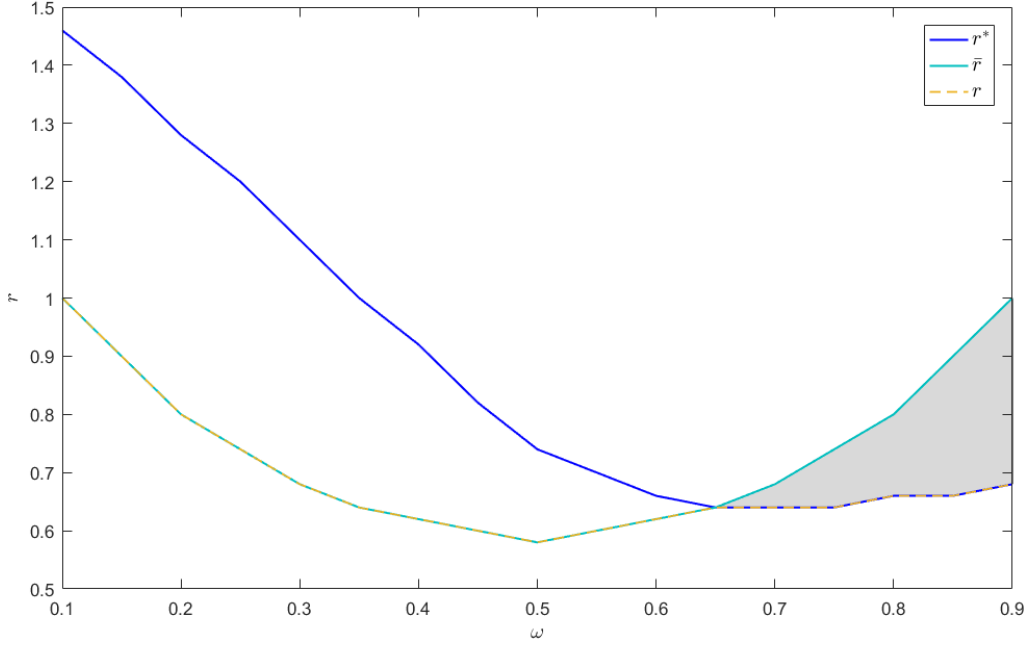
This section studies the market equilibrium on the interbank lending market at  $t = 1$  as a function of the level of commonality between banks. We present the result for a liquidity shock of  $\varepsilon^D = 1$ , and will consider  $\varepsilon^D = 0.75$  and  $\varepsilon^D = 1.25$  in section 4.2. Commonality is expressed through  $\omega$ , the share of the common asset in the banks' portfolios  $\omega = \frac{q_{bl}}{q_{bl} + q_i}$ .

Figure 3 plots the maximum rate that the lender is willing to accept  $\bar{r}$  (red line), the rate that the lender would like to charge  $r^*$  (blue line), and the equilibrium interest rate  $r = \min(\bar{r}, r^*)$  (dashed blue line).

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<sup>4</sup>According to the ECB, the average leverage ratio of European banks in the first quarter of 2018 is 5.11% and the tier 1 ratio is 15.32%

<sup>5</sup>The 10-years Sharpe ratio of the S&P500



**Figure 3: Equilibrium interest rate with  $\varepsilon^D = 1$**

This figure represents the interest rate that bank  $L$  would like to charge  $r^*$  (blue line), the maximum rate that bank  $B$  is willing to accept  $\bar{r}$  (red line) and the equilibrium interest rate  $r$  (dashed blue line) for different shares of common asset  $\omega$ . The equilibrium interest rate  $r$  is defined as  $\min(r^*, \bar{r})$ .

The first observation is that  $r^*$ , the profit-maximizing rate for the lender, is globally decreasing with the level of commonality  $\omega$ . This captures the key mechanism: when the portfolios of the two banks become similar, bank  $L$  becomes more exposed to a default of bank  $B$ , through the possibility of fire sales in the common asset  $bl$ . This gives bank  $L$  an incentive to charge low interest rates to keep the default risk of bank  $B$  at a low level, and thus indirectly protect itself. This falling interest rate can be viewed as a subvention from bank  $L$  to bank  $B$ , in the sense that the credit risk of bank  $B$  would suggest a higher rate.

Nevertheless, past a given level of similarity between both portfolios, the incentive to subsidize the borrower starts to decrease. The reason is that past a certain level of similarity in the asset structure, the fates of the banks become so closely tied together that bank  $L$  is bounded to fail if bank  $B$  fails, regardless of the loan. Bank  $L$  then becomes better off betting on survival, and  $r^*$  increases slightly.

Turning to  $\bar{r}$ , we note that the maximum rate that bank  $B$  may accept is U-shaped. This reflects the evolution of the probability of default for bank  $B$ , which finds its minimum when the bank holds both assets in equal measure. The intuition is as follows: when default is less likely, the borrowing bank is more willing to take the risk of selling assets, which translates into a lower threshold interest rate.

Let us now turn to the equilibrium rate  $r = \min(r^*, \bar{r})$ , represented by the blue dashed line. We observe that  $r = \bar{r}$  when  $\omega$  is low. However as  $r^*$  falls with  $\omega$  relative to  $\bar{r}$ , the situation reverses and we have  $r = r^*$ . In words, the lender accepts to offer a rate that is lower than the one it could obtain  $\bar{r}$ , in order to keep the risk of its borrower controlled. *The distance between the offered rate and the rate that bank  $B$  was prepared to pay,  $\bar{r} - r^*$ , can be viewed as the size*

of the subsidy from the lender to the borrower on the lending market. It is represented by the shaded zone in the figure.

The findings presented in this section echo those of the empirical literature on the subject. First, in the model  $r$  falls with the level of similarity  $\omega$  for reasonable values of  $\omega$ , which means that “neighboring” banks tend to lend to each other at more favorable conditions. This is consistent with the evidence from Bräuning and Fecht (2016). Second,  $r$  exhibits less variability than  $r^*$  and  $\bar{r}$ . *This means that rates on the interbank market are relatively insensitive to the fundamental characteristics of its participants*, due to the implicit guarantee that arises endogenously when banks are similar. This is in line with the empirical findings of Angelini et al. (2011) who find that rates have limited dispersion among lenders.

Note that in the parameter set chosen there is always a market, because the participation constraint for bank  $L$  is respected.

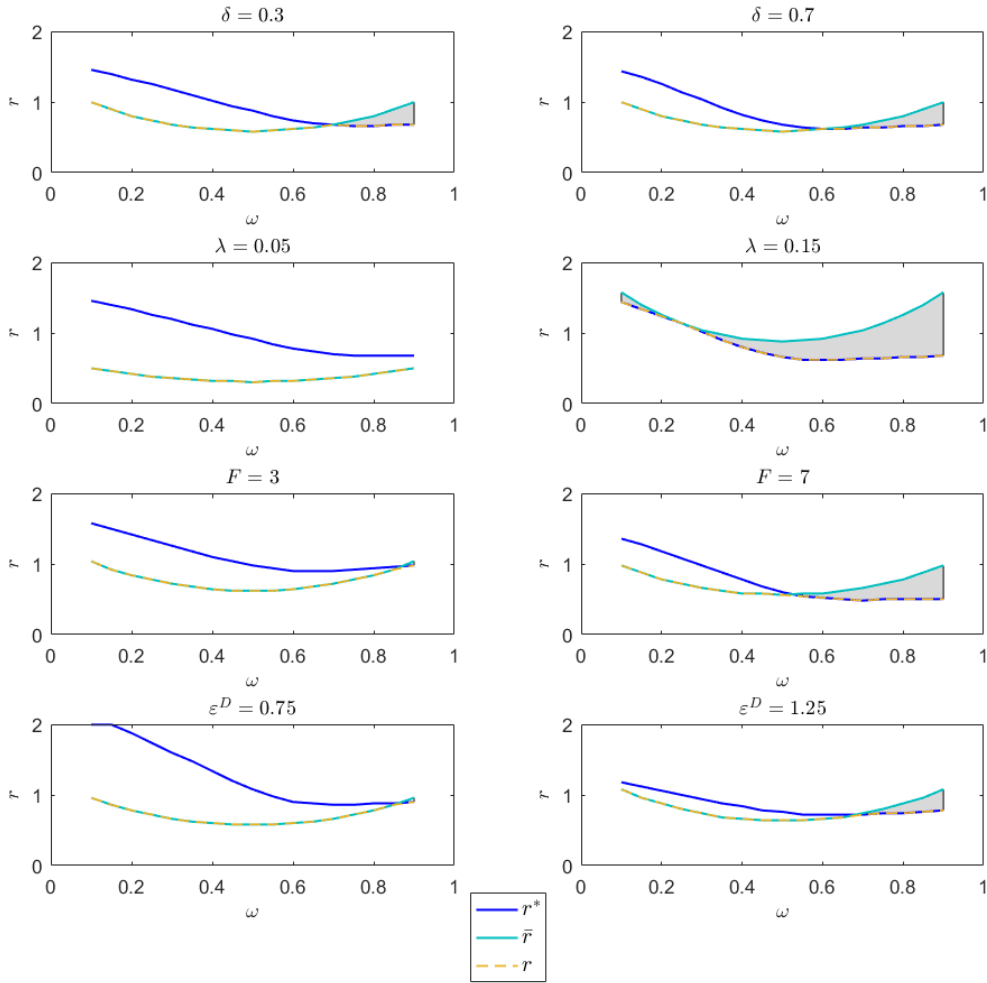
## 4.2 Robustness

We study how the equilibrium interest rate changes with the characteristics of the financial markets. In particular, we examine the impact of a change in the value of the following four parameters:

- the size of the liquidity shock  $\varepsilon^D$ .
- the impact of sales on prices  $\lambda$ .
- the disutility of bankruptcy  $F$ .
- the share of assets liquidated following bankruptcy  $\delta$ .

For each parameter we consider two values, on the low and high side of the default value. Figure 4 plots the results:





**Figure 4: Equilibrium interest rate for different values of  $\varepsilon^D$ ,  $\lambda$ ,  $F$  and  $\delta$ .**

This figure represents the behaviour of the three interest rates  $r^*$ ,  $\bar{r}$  and  $r$  for low and high values of the size of the liquidity shock  $\varepsilon^D$ , the impact of sales on prices  $\lambda$ , the disutility of bankruptcy  $F$  and the share of assets liquidated following bankruptcy  $\delta$ . This figure plots the interest rate that bank  $L$  would like to charge  $r^*$  (blue line), the maximum rate that bank  $B$  is willing to accept  $\bar{r}$  (red line) and the equilibrium interest rate  $r$  (dashed blue line) for different level of commonality  $\omega$ .

The first comment is that the behavior of both  $r^*$  and  $\bar{r}$  are very consistent across subplots. In particular, the interest rate that maximizes the utility of the lender initially falls with the level of asset commonality, before stabilizing. Different parameters values may lead the initial drop to be more or less pronounced, but do not change the overall pattern. What changes is the point of appearance of the common interest subsidy.

The parameter that appears to have the strongest effect is  $\lambda$ , the price impact of fire sales. When  $\lambda$  drops to 0.05, the equilibrium interest rate is always the maximum rate that the borrower is willing to pay,  $r = \bar{r}$ . In other words, it is always in the interest of the lender to offer the maximum rate he can get. However, when  $\lambda = 0.15$ , the subsidy point arrives as soon as  $\omega = 0.25$ , meaning that the lender is willing to forego interest rate payments to keep the borrower safe. This strong sensibility is not surprising: when the lender has more to fear from fire sales, his incentive to try avoiding them grows stronger.

$F$  and  $\varepsilon^D$  appear to have a qualitatively similar yet more moderate impact. When the cost

of failure increases, the lenders is more willing to accept the opportunity cost of lower rates in exchange for lower bankruptcy risk. When the liquidity needs of the borrower rise, the amount of sales required to face those needs also increases, which makes fire sales more costly, and gives the lender more reasons to subsidize. Note that the impact of  $\varepsilon^D$  on fire sales is non-linear: because the sales lowers the price of the asset, the amount of sales required to obtain the first unit of liquidity is lower than that required to obtain the second unit. The effect is accentuated when the impact of sales on prices is itself non-linear, a case considered in appendix D.

The last parameter is  $\delta$ , which represents the proportion of asset that gets liquidated following bankruptcy. Qualitatively, its effect is in line with expectations: when a failure involves more fire sales, the cost it imposes on banks with similar asset structure rises, which increases the incentive to subsidize. However it is interesting to note that this impact is very modest. The reason for this is that an immediate sale of 30% already has a strong negative impact on the expected profit of the lender, implying that there is little room for worsening.

Overall, all parameters appear to have the expected impact, which is encouraging from a robustness perspective.

Before turning to the systemic analysis, two unrelated points deserve to be mentioned.

First, if similar banks provide liquidity insurance, there may be an incentive for banks to choose correlated portfolios ex ante. Such moral hazard effects are studied in appendix E, which finds them to be limited. The reason is that having an asset structure that is similar to other banks limits liquidity risk, but also fosters asset-based contagion. Both effects approximately cancel each other out.

Second, and to end this section, we find it interesting to provide anecdotal evidence that a common interest subsidy was at play between the FED and the ECB during the 2008 crisis<sup>6</sup>. At the height of the financial crisis, the fed put in place the dollar swap line, a lending scheme designed to accommodate the need for dollars of the other central banks. The mechanism initially took the form all a market format auctioning which lead to ECB to borrow at rates as high as 12% on October 8,2008. However on October 13, the Fed changed the lending scheme to full-allotment format, in which central banks could borrow limitless amount at an previously defined (much lower) rate. Interestingly, the chairman of the Fed of New York, T. Geithner, justified the move in the following terms:

*"Not that we have an obligation in this sense, but we have an interest  
in helping these guys mitigate the problems they face"*  
(FOMC Transcripts, 2008: Geithner, Oct. 28-29, p. 21)

## 5 Systemic analysis

Section 4 showed that there is a single equilibrium rate between two banks, for any value of asset commonality  $\omega$  or any loan size  $\varepsilon^D$ . This implies that the 2 banks set-up can easily be generalized

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<sup>6</sup>The reader may find it controversial to use central banks as an example for a model that concerns private institutions. We agree, but taking the fed only bears a strong advantage: its motivation are made public in the transcripts. It is then possible to know with certainty that the low rate offered to the ECB was intended as a subsidy

to any bilateral lending relationship in the entire banking network. The present section studies the interbank lending market within a given network of asset holdings.

## 5.1 The evolution of the financial network during a crisis

One implication of the 2 banks model is that banks that are close to each other in terms of asset structures should also have privileged relationship in terms of funding. From a systemic perspective this means that the banking network should have a tendency to become clustered. We study this tendency, and how it is influenced by the market conditions.

The first step is to specify a network of  $N$  financial institutions with varying levels of commonalities in terms of asset holdings. We then introduce a single liquidity shock  $\varepsilon^D$  that will be positive for half of the banks and negative for the other half, leading an open market for liquidity at  $t = 1$ . The liquidity shock  $\varepsilon^D$  is drawn from a gaussian with a mean of zero and a variance of  $\sigma_d^2$ . It is then possible to observe how the lending network behaves for different values for the variance  $\sigma_d^2$ , and thus *how the banking network reacts to changes in liquidity risk*.

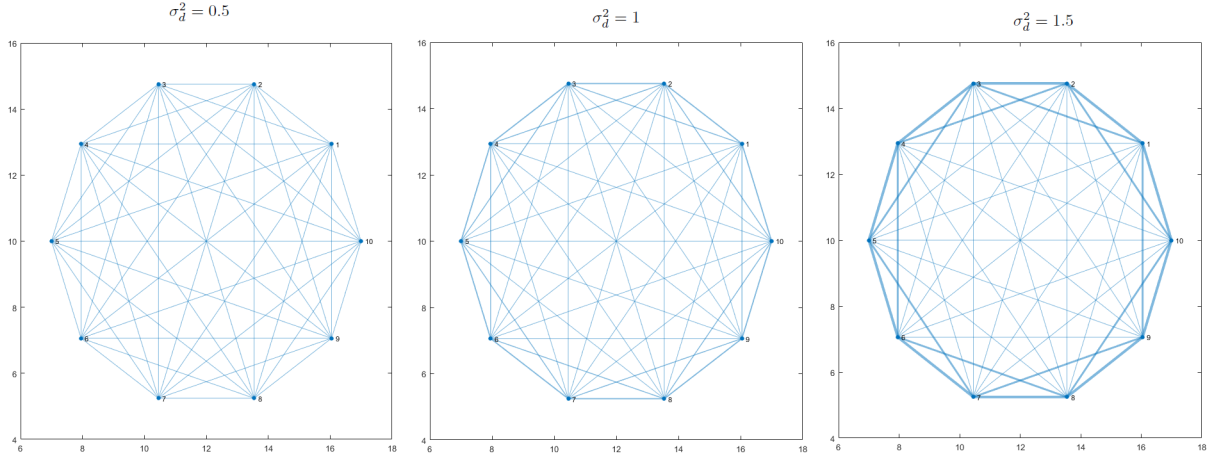
The asset holding network is set as follows: all banks hold 3 assets in total, and may share 0, 1, or 2 of these assets with other banks in the network<sup>7</sup>. Each bank will then have close neighbors (2 assets in common), distant neighbors (1 asset in common) and “outsiders” (no asset in common). More specifically, the financial network is modeled as a circulant network with 10 banks. The circulant structure is fairly classic in the network literature (see for instance Raffestin (2014)). In our context it simply implies that each banks will have 2 close neighbors and 2 distant neighbors. The choice of the ten banks is inconsequential: including more banks will simply increase the number of “outsiders”, which adds little to the discussion.

We run the model for many realizations of  $\varepsilon^D$  and all possible relationships between borrower and lender (close neighbor, distant neighbor, outsider). This gives a matrix of equilibrium interest rates, where each element corresponds to a given  $\{\varepsilon^D, \omega\}$  couple. For some shocks/lending relationship couples, there no lending equilibrium: in this case the corresponding element in the matrix is left empty.

For some realizations of  $\varepsilon^D$ , the close neighbor, the distant neighbor, and the outsider will offer the same rate  $r = \bar{r}$ . In this case the lender is chosen randomly by the borrower. For other realizations of  $\varepsilon^D$ , the rates offered to bank  $B$  will change with the distance in the network. In this case, the borrower chooses the lowest rate, as he benefits unambiguously from cheaper loans. This matching procedure produces the lending network represented in figure 4.

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<sup>7</sup>Simulations with more than 3 assets give comparable results and are available on request .



**Figure 5: The interbank lending network when liquidity risk rises**

This figure represents the behaviour of the  $N = 10$  interbank network for different levels of liquidity risk  $\sigma_d^2$ . The width of the lines represents the likelihood for entering a loan agreement between banks.

The figure shows that the banking lending network becomes increasingly clustered as liquidity risk rises. When the variance of the liquidity shock is of 0.5 and for a given borrower, a close neighbor is 1.5% more likely to enter a loan agreement compared to a distant neighbor, and 1.8% compared to an outsider. When the variance rises to 1 these two figures rise to 31% and 123% respectively. When the variance goes to 1.5, they reach 38% and 408%.

This difference comes from the fact that the loan is not very risky for the lender when the amount lent is small: both the likelihood of default and the loss if default are relatively low in this case. All lenders then offer the maximum rate that the borrower will accept, given by the threshold rate  $\bar{r}$ . However when the loan becomes larger, neighbors start to worry about the potential knock-on effect of a bankruptcy through fire sales, and thus offer cheaper money to lower this risk, which translates into more clustering. This evolution of the lending network is consistent with the small literature mentioned in the introduction (see for instance Bräuning and Fecht (2016)).

## 5.2 Is systemic risk lower when banks support each other?

The fact that cash rich banks may be willing to subsidize cash poor banks should enhance systemic stability. However the model also embeds a destabilizing mechanism, through the fact that the banking network is clustered. Indeed, intuitively a network where banks diversify their lending across all banks should be more robust than one in which banks only develop lending relationship with their neighbors. This section compares both effects to discuss the desirability of the common interests subsidy on systemic risk.

To do so, we compare the network described in the previous section to one where all banks offer the same conditions and have the same chance of being chosen as the lender. Comparison is based on three statistics: the likelihood that a bank who received a loan goes bankrupt, the likelihood that a lending bank who goes bankrupt if its borrower failed, and the total likelihood of a lender going bankrupt because of its borrower. The following exhibit presents the results:

	with subsidy	without subsidy
$P(d_B)$	0.0743	0.0781
$P(d_L d_B)$	0.4655	0.4328
$P_c(d_L)$	0.0385	0.0374

We first observe that borrower failure is 5% less likely when banks account for potential fire sales, which means that the subsidy reaches its goal of helping the borrower. Nevertheless, a second observation is that the probability that the bankruptcy of the borrower triggers the bankruptcy of the lender is higher in the network with the subsidy. This comes from the fact that, conditional on a default of their borrower, close neighbors have a lower chance of survival than outsiders because they are hit harder by fire sales. Because close neighbors are much more likely to lend than outsiders in the subsidy network, the possibility of contagion is higher there.

$P_c(d_L)$  represents the total chance of a knock-on effect occurring in the system, accounting both for the lower borrower risk and the higher contagion effect. It appears that the latter dominates: seeing a lending bank fail with its borrower is less likely when the banking network is diversified. Note that this does not imply that banks act irrationally. Each bank has a private incentive to subsidize its neighbors, which produces a sub-optimal social equilibrium.

## 6 Conclusion

In a banking model where fire sales may spread shocks throughout the banking system, similar banks are tempted to lend to each other at more favorable conditions than what the economy suggests. This explains why rates and volumes on the interbank lending market have showed strong resilience during the 2008 crisis, and why the lending network became more clustered at that time. However this practice is not necessarily welfare-enhancing, as it implies a clustered financial network.

The simple findings derived from our model have interesting implications. For instance, if banks of similar profiles exhibit solidarity during crises, international risk diversification becomes less desirable from the perspective of a social planner. Similarly, if banks support each other when they are at risk, there may be an unintended risk in having a given bank that is more solid than its neighbors, because this bank may then be tempted to switch to a predatory behavior. Such unintended effects do not call into questions the fact that diversification or stronger levels of capital have a positive impact on financial stability, but the regulator should keep them in mind when assessing the scope of the next banking crisis.

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## Appendix A

This appendix describes the steps involved in obtaining the balance sheet of bank  $B$  at  $t = 1$ , in the no-deal scenario.

At  $t = 1$ , if there has been no agreement on a loan, bank  $B$  sells a quantity  $\Delta q_{b,1}$  of asset  $b$  at a price  $p_{b,1} = p_{b,0} - \lambda \Delta q_{b,1}$ , and a quantity  $\Delta q_{bl,1}$  of asset  $bl$  at a price  $p_{bl,1} = p_{bl,0} - \lambda \Delta q_{bl,1}$ , in order to obtain a sum of  $\varepsilon^D$ . Mathematically this translates into the following condition:

$$\Delta q_{b,1}(p_{b,0} - \lambda \Delta q_{b,1}) + \Delta q_{bl,1}(p_{bl,0} - \lambda \Delta q_{bl,1}) = \varepsilon^D. \quad (12)$$

The total assets remaining, denoted by  $TA_b$ , have a value of  $TA_1 = (q_{b,0} - \Delta q_{b,1})(p_{b,0} - \lambda \Delta q_{b,1}) + (q_{bl,0} - \Delta q_{bl,1})(p_{bl,0} - \lambda \Delta q_{bl,1})$ . Using (12) above, this can be reexpressed as:

$$TA_1 = q_{b,0}(p_{b,0} - \lambda \Delta q_{b,1}) + q_{bl,0}(p_{bl,0} - \lambda \Delta q_{bl,1}) - \varepsilon^D$$

$$TA_1 = q_{b,0}p_{b,0} + q_{bl,0}p_{bl,0} - \lambda q_{b,0}\Delta q_{b,1} - \lambda q_{bl,0}\Delta q_{bl,1} - \varepsilon^D$$

$$TA_1 = TA_0 - \lambda q_{b,0}\Delta q_{b,1} - \lambda q_{bl,0}\Delta q_{bl,1} - \varepsilon^D$$

Taking the asset liability equality condition then yields:  $TA_0 - \lambda q_{b,0}\Delta q_{b,1} - \lambda q_{bl,0}\Delta q_{bl,1} - \varepsilon^D = D_{B,0} - \varepsilon^D + K_{B,1}$ . Denoting by  $\Pi_{B,1} = K_{B,1} - K_{B,0}$  the profit of bank  $B$  between  $t = 0$  and  $t = 1$  and rearranging, we obtain:

$$\Pi_{B,1} = -(\lambda q_{b,0}\Delta q_{b,1} + q_{bl,0}\Delta q_{bl,1})$$

which allows us to draw the the balance sheet at  $t = 1$ :

assets	liabilities
$(q_{b,0} - \Delta q_{b,1})(p_{b,0} - \lambda \Delta q_{b,1})$	$D_{B,1} = D_{B,0} - \varepsilon^D$
$+(q_{bl,0} - \Delta q_{bl,1})(p_{bl,0} - \lambda \Delta q_{bl,1})$	$K_{B,1} = K_{B,0} +$
	$-\lambda q_{b,0}\Delta q_{b,1} - \lambda q_{bl,0}\Delta q_{bl,1}$

We obtain the levels of sales for asset  $b$  and  $bl$  as follows. First, the price response is a linear function of sales, so that the selling price for asset  $i$  is given by  $p_{i,1} = p_{i,0} - \lambda \Delta q_{i,1}$ , with  $\Delta q_{i,1}$  the total sales in asset  $i$  at  $t = 1$ . Sales for bank  $B$  then satisfy the following condition:  $(p_{b,0} - \lambda \Delta q_{b,1})\Delta q_{b,1} + (p_{bl,0} - \lambda \Delta q_{bl,1})\Delta q_{bl,1} = \varepsilon^D$  Second, we assume that, because the intrinsic characteristics of assets  $b$  and  $bl$  are unchanged, the relative quantities of each asset in the portfolio of the bank stay constant. This yields:  $\frac{q_{b,0}}{q_{bl,0}} = \frac{q_{b,1}}{q_{bl,1}} = cst$ , which allows us to uniquely define  $\Delta q_{b,1}$  and  $\Delta q_{bl,1}$

## Appendix B

### B-a

#### Bank B

Because the loan must be repaid the loan now has a value of  $\varepsilon^D(1 + r)$  in the liabilities, and the shocks  $\varepsilon_b^p$  and  $\varepsilon_{bl}^p$  impact asset values. Capital adjusts to maintain the equality between assets and liabilities, which yields:



	assets	liabilities
<i>bank B:</i>	$q_{b,0}(p_{b,0} + \varepsilon_b^p)$	$D_{B,2} = D_{B,0} - \varepsilon^D$
	$q_{bl,0}(p_{bl,0} + \varepsilon_{bl}^p)$	$K_{B,2} = K_{B,0} - r\varepsilon^D + q_{b,0}\varepsilon_b^p + q_{bl,0}^B\varepsilon_{bl}^p$
		$B_{B,2} = \varepsilon^D(1 + r)$

*Bank L, given survival of bank B*

The same logic applies, except that the loan now appears on the asset side since bank *L* is the lender:

	assets	liabilities
<i>bank L:</i>	$q_{bl,0}^L(p_{bl,0} + \varepsilon_{bl}^p)$	$D_{L,2} = D_{L,0} + \varepsilon^D$
	$q_{l,0}(p_{l,0} + \varepsilon_l^p)$	$K_{L,2} = K_{L,0} + r\varepsilon^D + q_{bl,0}^L\varepsilon_{bl}^p + q_{l,0}\varepsilon_l^p$
	$B_{B,2} = \varepsilon^D(1 + r)$	

*Bank L, given failure of bank B*

If bank *B* has gone bankrupt, the loan is defaulted on, which implies a dead loss of  $\varepsilon^D$  for the lender. A share  $\delta$  of assets is also liquidated by bank *B* to manage its bankruptcy, which lowers the market value of asset *bl*. The balance sheet of bank *L* writes:

	assets	liabilities
<i>bank L:</i>	$q_{bl,0}^L(p_{bl,0} - \lambda\delta q_{bl,0}^B + \varepsilon_{bl}^p)$	$D_{L,1} = D_{L,0} + \varepsilon^D$
	$q_{l,0}(p_{l,0} + \varepsilon_l^p)$	$K_{L,2} = K_{L,0} + q_{l,0}\varepsilon_l^p + q_{bl,0}^L\varepsilon_{bl}^p$
		$-\varepsilon^D + q_{bl,0}^L(-\lambda\delta q_{bl,0}^B)$

## B-b

This appendix gives the expressions for the balance sheets of both banks at  $t = 2$ , in the no-deal case. At  $t = 2$  shocks on assets occur. One simply needs to add these shocks to the  $t = 1$  balance sheets described in section 2.2.2. This yields:

(i) For bank *B*:

	assets	liabilities
<i>bank B:</i>	$(q_{b,0} - \Delta q_{b,1})(p_{b,0} - \lambda\Delta q_{b,1} + \varepsilon_b^p)$	$D_{B,1} = D_{B,0} - \varepsilon^D$
	$+(q_{bl,0}^B - \Delta q_{bl,1}^B)(p_{bl,0} - \lambda\Delta q_{bl,1}^B + \varepsilon_{bl}^p)$	$K_{B,2} = K_{B,0} +$
		$-\lambda q_{b,0}\Delta q_{b,1} - \lambda q_{bl,0}^B\Delta q_{bl,1}^B$
		$+(q_{b,0} - \Delta q_{b,1})\varepsilon_b^p + (q_{bl,0}^B - \Delta q_{bl,1}^B)\varepsilon_{bl}^p$

(i) For bank *L*, when bank *B* survives:

	assets	liabilities
<i>bank L:</i>	$q_{l,0}(p_{l,0} + \varepsilon_l^p)$	$D_{B,1} = D_{B,0} + \varepsilon^D$
	$+q_{bl,0}^L(p_{bl,0} - \lambda\Delta q_{bl,1}^B + \varepsilon_{bl}^p)$	$K_{B,2} = K_{B,0} - \lambda q_{bl,0}^L\Delta q_{bl,1}^B$
	$L_1 = \varepsilon^D$	$+q_{l,0}\varepsilon_l^p + q_{bl,0}^L\varepsilon_{bl}^p$

(i) For bank *L*, when bank *B* fails:

	assets	liabilities
<i>bank L:</i>	$q_{l,0}(p_{l,0} + \varepsilon_l^p)$	$D_{B,1} = D_{B,0} + \varepsilon^D$
	$+q_{bl,0}^L(p_{bl,0} - \lambda q_{bl,0}^B + \varepsilon_{bl}^p)$	$K_{B,2} = K_{B,0} - \lambda q_{bl,0}^L q_{bl,0}^B$
	$L_1 = \varepsilon^D$	$+q_{l,0}\varepsilon_l^p + q_{bl,0}^L \varepsilon_{bl}^p$

## Appendix C

This appendix describes the steps involved in solving the analytically tractable version of the model in section 3.2.

### Reexpressing $E(U_L)$

The first step we reexpress equation (9). We use the fact that the odds attached to any outcome  $\{d_B, d_L\}$  can be decomposed as  $P(d_B, d_L) = P(d_L|d_B) * P(d_B)$ , and the fact that  $P(d_L|d_B) = 1 - P(\bar{d}_L|d_B)$ .

$$E(U_L) = P(d_L|d_B) * P(d_B) * (-F) + P(d_L|\bar{d}_B) * P(\bar{d}_B) * (-F) + P(\bar{d}_L|d_B) * P(d_B) * E(U_L|d_B, \bar{d}_L) + P(\bar{d}_L|\bar{d}_B) * P(\bar{d}_B) * E(U_L|\bar{d}_B, \bar{d}_L)$$

Using the fact that we study a version of the model where  $P(d_L|\bar{d}_B) = 0$  and consequently  $P(\bar{d}_L|\bar{d}_B) = 1$ :

$$E(U_L) = P(d_L|d_B) * P(d_B) * (-F) + P(\bar{d}_L|d_B) * P(d_B) * E(U_L|d_B, \bar{d}_L) + P(\bar{d}_B) * E(U_L|\bar{d}_B, \bar{d}_L)$$

Rearranging:

$$E(U_L) = \underbrace{[1 - P(\bar{d}_L|d_B)] * [1 - P(\bar{d}_B)] * (-F)}_{\text{Block A}} + \underbrace{P(\bar{d}_L|d_B) * [1 - P(\bar{d}_B)] * E(U_L|d_B, \bar{d}_L)}_{\text{Block B}} + \underbrace{P(\bar{d}_B) * E(U_L|\bar{d}_B, \bar{d}_L)}_{\text{Block C}} \quad (13)$$

This breaks  $E(U_L)$  into terms that can easily be obtained.

### Obtaining each term in $E(U_L)$

As mentioned in the body of the paper the profit for bank  $B$  is:

$$\Pi_B = -r\varepsilon^D + q_i \varepsilon_b^p$$

Since  $\varepsilon_b^p$  is uniformly distributed between  $[\mu - V, \mu + V]$ , the maximum value for  $\Pi_B$  is  $\Pi_B^{max} = -r\varepsilon^D + q_i(\mu + V)$  and the minimum one is  $\Pi_B^{min} = -r\varepsilon^D + q_i(\mu - V)$ . Bank  $B$  survives if  $\Pi_B$  stands between  $-K$  and  $\Pi_B^{max}$ . The probability of survival is then :

$$P(\bar{d}_B) = P(\Pi_B > -K) = \frac{\Pi_B^{max} - (-K)}{\Pi_B^{max} - \Pi_B^{min}}$$

which yields:

$$P(\bar{d}_B) = \frac{q_i \mu + q_i V - r\varepsilon^D + K}{2q_i V}$$

$$\text{and } 1 - P(\bar{d}_B) = \frac{-q_i \mu + q_i V + r\varepsilon^D - K}{2q_i V}$$

We proceed in similar fashion for the situation of bank  $L$  when bank  $B$  defaulted on its loan.

The profit of bank  $L$  writes:

$$\Pi_{L,F} = q_i \varepsilon_l^p - \varepsilon^D - \lambda \delta(q_{bl})^2$$

The odds that bank  $L$  survives while  $B$  defaults are then given by:

$$P(\bar{d}_L|d_B) = \frac{q_i \mu + q_i V - \lambda \delta(q_{bl})^2 - \varepsilon^D + K}{2q_i V}$$

$$\text{and } 1 - P(\bar{d}_L|d_B) = \frac{-q_i \mu + q_i V + \lambda \delta(q_{bl})^2 + \varepsilon^D - K}{2q_i V}$$

We now turn to  $E(U_L|d_B, \bar{d}_L)$  the expected utility for bank  $L$  knowing that it is not bankrupt, though bank  $B$  has failed. Survival of bank  $L$  when  $B$  failed implies that  $\Pi_{L,F} > -K$ , which means:

$$\begin{aligned} \Pi_{L,F} &= q_i \varepsilon_l^p - \varepsilon^D - \lambda \delta(q_{bl})^2 > -K \\ \Rightarrow \varepsilon_l^p &> \frac{-K + \varepsilon^D + \lambda \delta(q_{bl})^2}{q_i} \end{aligned}$$

Because  $\varepsilon_l^p$  is uniformly distributed between  $[\mu_L - V_L, \mu_L + V_L]$ , its expected value knowing  $\varepsilon_l^p > \frac{-K + \varepsilon^D + \lambda \delta(q_{bl})^2}{q_i}$  writes:

$$\begin{aligned} E(\varepsilon_l^p|d_B, \bar{d}_L) &= \frac{1}{2} \left( \frac{-K + \varepsilon^D + \lambda \delta(q_{bl})^2}{q_i} + \mu + V \right) \\ E(\varepsilon_l^p|d_B, \bar{d}_L) &= \frac{1}{2q_{i,0}} (-K + \varepsilon^D + \lambda \delta(q_{bl})^2 + q_i(\mu + V)) \end{aligned}$$

Plugging back into  $E(U_L|d_B, \bar{d}_L)$  yields :

$$\begin{aligned} E(U_L|d_B, \bar{d}_L) &= q_i E(\varepsilon_l^p|d_B, \bar{d}_L) - \varepsilon^D - \lambda \delta(q_{bl})^2 \\ E(U_L|d_B, \bar{d}_L) &= \frac{1}{2} [-K + \varepsilon^D + \lambda \delta(q_{bl})^2 + q_i(\mu + V)] - \varepsilon^D - \lambda \delta(q_{bl})^2 \end{aligned}$$

$$E(U_L|d_B, \bar{d}_L) = \frac{1}{2} (-K - \varepsilon^D - \lambda \delta(q_{bl})^2 + q_i \mu + q_i V)$$

Finally we turn  $E(U_L|\bar{d}_B, \bar{d}_L)$ . This term is simple: because  $P(\bar{d}_L|\bar{d}_B) = 1$  there is no condition on  $\varepsilon_l^p$ , we take the unconditional expected value:

$$E(\Delta U_L|\bar{d}_B, \bar{d}_L) = r \varepsilon^D + q_i \mu$$

We now plug the expressions obtained in this subsection in equation (13), proceeding block by block.

## block A

We start from:

$$A = [1 - P(\bar{d}_L|d_B)] * [1 - P(\bar{d}_B)] * (-F)$$

Plugging the expressions for  $1 - P(\bar{d}_L|d_B)$  and  $1 - P(\bar{d}_B)$  gives:

$$\begin{aligned} A &= (-F) * \left( \frac{q_i V - q_i \mu + r \varepsilon^D - K}{2q_i V} \right) * \left( \frac{-q_i \mu + q_i V + \lambda \delta(q_{bl})^2 + \varepsilon^D - K}{2q_i V} \right) \\ A &= (-F) \left( \frac{1}{2q_i V} \right)^2 * r \varepsilon^D * (q_i V - q_i \mu - K + b(q_{bl})^2 + \varepsilon^D) + cst \end{aligned}$$

where  $cst$  gathers all the terms that are unrelated to  $r$ . This means that the derivative of  $A$  with respect to  $r$  is given by:

$$\frac{\partial A}{\partial r} = -F \left( \frac{1}{2q_i V} \right)^2 * \varepsilon^D * (q_i V - q_i \mu - K + b(q_{bl})^2 + \varepsilon^D) \quad (14)$$

Rearranging:

$$\frac{\partial A}{\partial r} = -F \left( \frac{1}{2q_i V} \right)^2 * \varepsilon^D * [2q_i V - (q_i V + q_i \mu + K - b(q_{bl})^2 - \varepsilon^D)]$$

$$\frac{\partial A}{\partial r} = -F\left(\frac{1}{2q_i V}\right)^2 * \varepsilon^D * 2q_i V + F\left(\frac{1}{2q_i V}\right)^2 * \varepsilon^D * (q_i V + q_i \mu + K - b(q_{bl})^2 - \varepsilon^D)]$$

### block B

Moving to B

$$B = P(\bar{d}_L|d_B) * [1 - P(\bar{d}_B)] * E(U_L|d_B, \bar{d}_L)$$

Replacing

$$B = \frac{1}{2}(-K - \varepsilon^D - \lambda\delta(q_{bl})^2 + q_i \mu + q_i V) * \left(\frac{q_i V - q_i \mu + r\varepsilon^D - K}{2q_i V}\right) * \left(\frac{q_i \mu + q_i V - \lambda\delta(q_{bl})^2 - \varepsilon^D + K}{2q_i V}\right)$$

$$B = \frac{1}{2}\left(\frac{1}{2q_i V}\right)^2 * (-K - \varepsilon^D - \lambda\delta(q_{bl})^2 + q_i \mu + q_i V) * (q_i V - q_i \mu + r\varepsilon^D - K) * (q_i \mu + q_i V - \lambda\delta(q_{bl})^2 - \varepsilon^D + K)$$

$$B = \frac{1}{2}\left(\frac{1}{2q_i V}\right)^2 * (q_i \mu + q_i V - \lambda\delta(q_{bl})^2 - \varepsilon^D - K) * (q_i \mu + q_i V - \lambda\delta(q_{bl})^2 - \varepsilon^D + K) * r\varepsilon^D + cst$$

Then we take the derivative with respect to  $r$

$$\frac{\partial B}{\partial r} = \frac{1}{2}\left(\frac{1}{2q_i V}\right)^2 * (q_i \mu + q_i V - \lambda\delta(q_{bl})^2 - \varepsilon^D - K) * (q_i \mu + q_i V - \lambda\delta(q_{bl})^2 - \varepsilon^D + K) * \varepsilon^D \quad (15)$$

$$\frac{\partial A}{\partial r} + \frac{\partial B}{\partial r} = \left(\frac{1}{2q_i V}\right)^2 * \varepsilon^D * (-F * 2q_i V) + \left(\frac{1}{2q_i V}\right)^2 * \varepsilon^D * F * (q_i \mu + q_i V + K - \lambda\delta(q_{bl})^2 - \varepsilon^D) + \frac{1}{2}\left(\frac{1}{2q_i V}\right)^2 * \varepsilon^D * (q_i \mu + q_i V - \lambda\delta(q_{bl})^2 - \varepsilon^D - K_{2,0}) * (q_i \mu + q_i V + K - \lambda\delta(q_{bl})^2 - \varepsilon^D)$$

$$\frac{\partial A}{\partial r} + \frac{\partial B}{\partial r} = \left(\frac{1}{2q_i V}\right)^2 * \varepsilon^D * (-F * 2q_i V) + \left(\frac{1}{2q_i V}\right)^2 * \varepsilon^D * (q_i \mu + q_i V + K - \lambda\delta(q_{bl})^2 - \varepsilon^D) * [F + \frac{1}{2}(q_i \mu + q_i V - \lambda\delta(q_{bl})^2 - \varepsilon^D - K)]$$

$$\frac{\partial A}{\partial r} + \frac{\partial B}{\partial r} = -F * \left(\frac{1}{2q_i V}\right) * \varepsilon^D + \left(\frac{1}{2q_i V}\right)^2 * \varepsilon^D * (q_i \mu + q_i V + K - \lambda\delta(q_{bl})^2 - \varepsilon^D) * [F + \frac{1}{2}(q_i \mu + q_i V - \lambda\delta(q_{bl})^2 - \varepsilon^D - K)]$$

$A + B$  equals the same times  $r$  plus a  $cst$

### block C

$$C = P(\bar{d}_B) * E(U_L|\bar{d}_B, \bar{d}_L)$$

$$C = \left(\frac{q_i \mu + q_i V - r\varepsilon^D + K}{2q_i V}\right) * (q_i \mu + r\varepsilon^D)$$

$$C = \left(\frac{1}{2q_i V}\right) * (q_i \mu + q_i V + K - r\varepsilon^D) * (q_i \mu + r\varepsilon^D)$$

$$C = \left(\frac{1}{2q_i V}\right) * [(q_i \mu + q_i V + K) * r\varepsilon^D - (r\varepsilon^D)^2 - r\varepsilon^D * q_i \mu + cst]$$

$$C = \left(\frac{1}{2q_i V}\right) * [(q_i \mu + q_i V + K - q_i \mu) * r\varepsilon^D - (r\varepsilon^D)^2] + cst$$

Then we take the derivative wrt  $r$

$$\frac{\partial C}{\partial r} = \left(\frac{1}{2q_i V}\right)\varepsilon^D * [q_i V + K - 2r\varepsilon^D]$$

$$\text{Solving } \frac{\partial A}{\partial r} + \frac{\partial B}{\partial r} + \frac{\partial C}{\partial r} = -F * \left(\frac{1}{2q_i V}\right) * \varepsilon^D + \left(\frac{1}{2q_i V}\right)^2 * \varepsilon^D * (q_i \mu + q_i V + K - \lambda\delta(q_{bl})^2 - \varepsilon^D) * [F + \frac{1}{2}(q_i \mu + q_i V - \lambda\delta(q_{bl})^2 - \varepsilon^D - K)] + \left(\frac{1}{2q_i V}\right)\varepsilon^D * [q_i V + K - 2r\varepsilon^D]$$

$$\frac{\partial A}{\partial r} + \frac{\partial B}{\partial r} + \frac{\partial C}{\partial r} = 0$$

$$\Rightarrow -F + \left(\frac{1}{2q_i V}\right) * (q_i \mu + q_i V + K - \lambda\delta(q_{bl})^2 - \varepsilon^D) * [F + \frac{1}{2}(q_i \mu + q_i V - \lambda\delta(q_{bl})^2 - \varepsilon^D - K)] + q_i V + K - 2r\varepsilon^D = 0$$

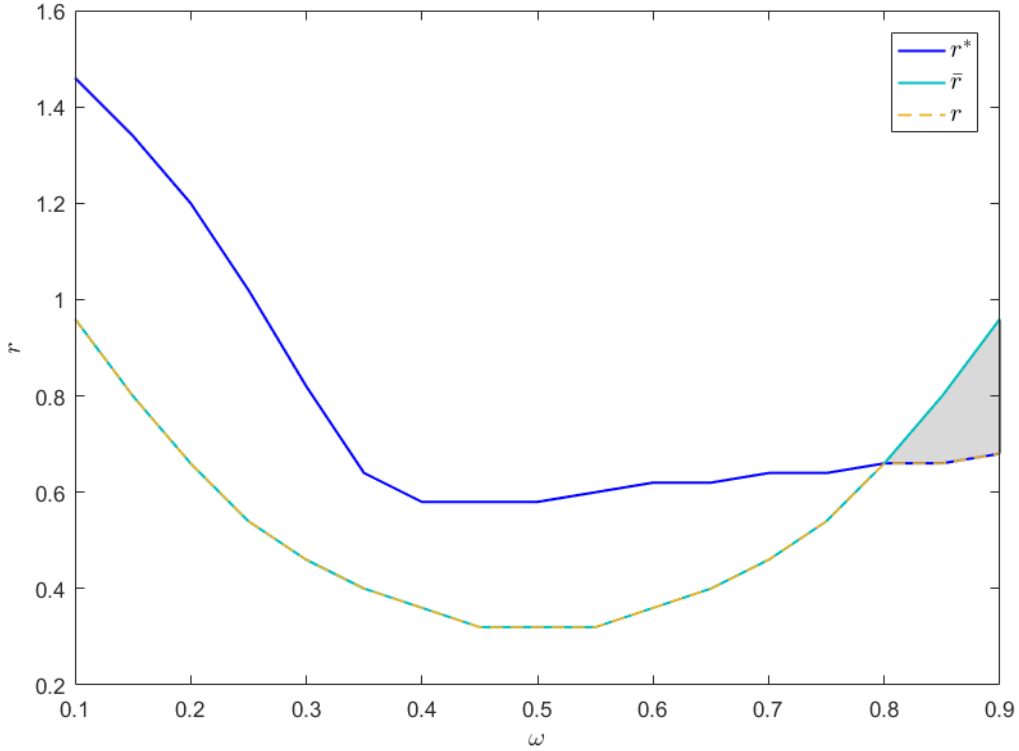
$$\Rightarrow q_i V + K - F + \left(\frac{1}{2q_i V}\right) * (q_i \mu + q_i V + K - \lambda \delta (q_{bl})^2 - \varepsilon^D) * \left[F + \frac{1}{2}(q_i \mu + q_i V - \lambda \delta (q_{bl})^2 - \varepsilon^D - K)\right] = 2r\varepsilon^D$$

This yields

$$r^* = \frac{1}{2 * \varepsilon^D} \left( q_i V + K - F + \left(\frac{1}{2q_i V}\right) * [q_i \mu + q_i V + K - \lambda \delta (q_{bl})^2 - \varepsilon^D] * \left[F + \frac{1}{2}(q_i \mu + q_i V - \lambda \delta (q_{bl})^2 - \varepsilon^D - K)\right] \right)$$

## Appendix D

This section considers the possibility that sales have a non-linear impact on prices. Specifically, for a given asset  $i$ , we specify the following quadratic relationship between sales and prices:  $\Delta p_i = \lambda \Delta q_i^2$ .



**Figure 6: Equilibrium interest rate with non-linear impact of sales on prices**

This figure represents the interest rate that bank  $L$  would like to charge  $r^*$  (blue line), the maximum rate that bank  $B$  is willing to accept  $\bar{r}$  (red line) and the equilibrium interest rate  $r$  (dashed blue line) for different shares of common asset  $\omega$  and in the case of non-linear effect of fire sales on prices. The equilibrium interest rate  $r$  is defined as  $\min(r^*, \bar{r})$ .

The change seems to have limited effect on the dynamics of  $r$ . The global behavior of both  $r^*$  and  $\bar{r}$  remains the same.

Nevertheless the subsidy appears only around  $\omega = 0.8$ , which represents a very high level of commonality between the borrower and the lender. This means that non-linear sales lower the incentive for a bank to help similar banks. This surprising finding comes from the fact that

when sales are below one, the quadratic form in fact downplays the impact of the sales. Thus the potential lender worries less about sales.

This changes when the variance of liquidity shock increases and sales often exceed unity. In this case (available on request) the overall pattern is unchanged but the common interest subsidy appears earlier than in the baseline case. In short, allowing sales to have a non-linear impact on prices accentuates (lessens) the dynamics of the model when sales are large (low),

## Appendix E

So far we have taken the portfolios of the banks as given. However the existence of an implicit guarantee in the funding conditions at  $t = 1$  should change the problem of each bank at  $t = 0$ .

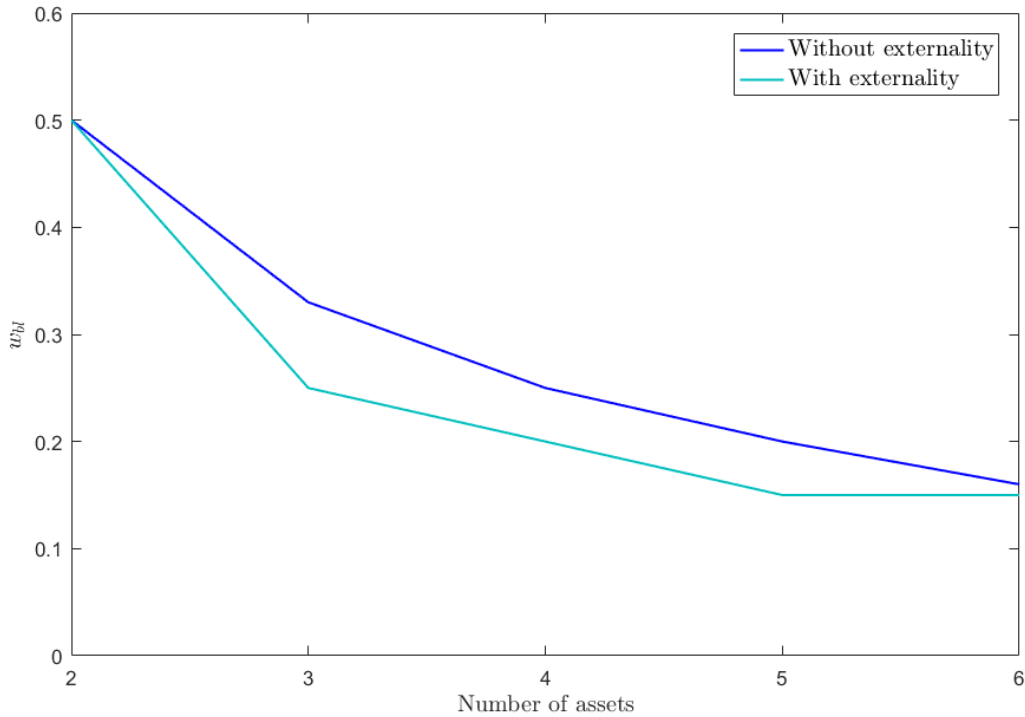
To study this we consider the following situation: a bank that needs to constitute a portfolio chooses between a number  $n$  of assets, one of which is owned in great quantities by another bank. The bank chooses its portfolio at  $t = 0$  in order to maximize a classic mean-variance utility  $U_{mv}$ :

$$U_{mv} = E(U_i) - \frac{1}{\tau} V(U_i)$$

where  $\frac{1}{\tau}$  represents the level of risk aversion, and is set to 5. As in section 3 returns are harmlessly assumed to be normally distributed, have the same moments, and be independent.

If common holdings had no impact on the interest rate charged at  $t = 1$ , the utility maximizing allocation would be the one that minimizes portfolio risk, which assigns the same weight of  $w = 1/n$  to all assets. However, allowing for common holdings to contain fire sales risk at  $t = 1$ , asset  $bl$  takes a new role. On the upside, it provides bank  $B$  with an insurance against  $t = 1$  liquidity shocks. On the downside, it exposes it to fire sales by bank  $L$ . The difference between the weight  $w_{bl}$  in this case and  $w = 1/n$  may thus be seen as an indicator of distortive power of common asset holdings on ex-ante portfolio choices.

More specifically, we run the model for a number of assets  $n = 1 : 1 : 6$ , for weights in the common asset  $w_{bl} = 0.1 : 0.05 : 0.9$ , and for liquidity shocks  $\varepsilon^D = 0 : 0.25 : 2.5$ . This yields a 3-dimensional matrix of equilibrium interest rates, which in turns defines a 3-dimensional matrix of expected utilities for the borrower. We then integrate these expected utilities over  $\varepsilon^D$  to obtain the expected value for utilities estimated at  $t = 0$  when the realization of  $\varepsilon^D$  is uncertain. The weight in the common asset  $w_{bl}$  that maximizes this  $t = 0$  expected utility is kept and compared with the prior of  $w_{bl} = 1/n$ . Figure 7 plots the results.



**Figure 7: Share of the common asset in optimal portfolio, including or excluding fire sales**

This figure represents the banks' optimal weight for the common asset  $w_{bl}$  as a function of the number of assets  $n$ . The blue line is the case where the externalities induced by potential fire sales are not taken into account (i.e.  $w_{bl} = 1/n$ ). The red line represents the weight chosen by the bank while taking into account externalities.

The figure shows that banks typically invest less in the common asset when they take into account its potential to be sold by comparable banks. This finding echoes those of papers on "crowded trades" such as Pojarliev and Levich (2011), which show that certain assets are subject to tail risk due to the possibility of a sudden exit by other owners. In our case, this negative selling externality is mitigated by the fact that common holdings provide an insurance against liquidity shock, but the overall impact remains. The banks are not tempted to become more similar to shelter themselves from liquidity crises, as postulated by Acharya and Yorulmazer (2007). Nevertheless, as more assets are included in the portfolio, the cost attached to the selling externality decreases, and the optimal share of asset  $bl$  then converges towards  $w_{bl} = 1/n$ .