AN INTRODUCTION TO QUANTIFIED VARIATIONS OF ABSOLUTE TIME IN THE PRESENCE OF A GRAVITATIONAL FIELD IN A PLATONICIAN QUADRIDIMENSIONAL SPACE

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Abstract

Following a comparative examination of the results presented in articles hal-01584918 and hal-01756323 proposing a quantification of Keplerian orbits, it appears ***that the Platonic velocity of objects moving in a gravitational field*** is not uniform, but ***is subject to variations***.

These variations are related to the ***De Broglie waves*** generated by the massive bodies in interaction. The consequences of these ***quantified variations*** are numerous.

Within the Platonic framework of this modeling we can thus find, by way of illustration and according to the results proposed by the theory of general relativity (and verified experimentally), the variations of the measurement of time in a satellite in orbit around the Earth, the gravitational redshift and the advance of the perihelion of the planets.

UNE INTRODUCTION AUX VARIATIONS QUANTIFIÉES DU TEMPS ABSOLU
EN PRÉSENCE D'UN CHAMP GRAVITATIONNEL
DANS UN ESPACE QUADRIDIMENSIONNEL PLATONICIEN

Résumé

Suite à un examen comparatif des résultats exposés dans les articles hal-01577669 et hal-01739986 proposant une quantification des orbites képlériennes, il apparaît que la ***vitesse platonicienne des objets se déplaçant dans un champ de gravitation*** n'est pas uniforme, mais ***est soumise à des variations***.

Ces variations sont liées aux ***ondes de phase de De Broglie*** générées par les corps massifs en interaction. Les conséquences de ces ***variations quantifiées*** sont nombreuses.

Dans le cadre platonicien de cette modélisation, on retrouvera par exemple dans cet article, conformément aux résultats proposés par la théorie de la relativité générale (et vérifiés expérimentalement), les variations de la mesure du temps dans un satellite en orbite autour de la Terre, le redshift gravitationnel et l'avance du périhélie des planètes.

Introductory

One of the postulates of the Platonic model is that the objects move in a uniform way, that is to say that between two observations, whatever their trajectory, they all travel the same distance (see article hal-01165196).

From this postulate follows the ***notion of absolute time*** introduced in the article hal-01378215:

"the absolute time ΔT between two events is the distance traveled by (all) the moving objects between these two events".

It should be noted that this absolute time is measured in meters.

These definitions imply that the modulus of the velocity of any moving object in the Platonic space is equal to 1; this speed, called "Platonic", being without unity (see articles cited with reference to paragraph 1 below).

However, after a comparative examination of the results exposed in the articles hal-01584918 and hal-01756323, it appears in fact that ***the modulus of the Platonic speed of a mobile is subjected to variations in the presence of a gravitational field***.

These variations, related to the absolute frequency of the mass waves generated by the body at the origin of the gravitational field, lead to the formula:

$$v_2 = 1 - \frac{Gh}{c^3 \rho \Delta w_{01} \sin \beta_1}$$

in which (see diagram in paragraph 2):

v_2 denotes the modulus of the Platonic velocity of the point object C_2 observed in the gravitational field generated by the massive point body C_1;

$\Delta w_{01} = \frac{h}{m_{01} c}$ is the stratification distance of the sequence of hyperplanes of the Platonic space generating the De Broglie wave of the massive body C_1;

β_1 is a measure of the angle formed by the Platonic speed $\vec{v_1}$ of this body with these hyperplanes;

ρ is the projected distance $C_1'C_2'$.

The reasoning leading to this formula is described in paragraph 3.
The consequences of these variations are numerous and lead rapidly to results obtained in the framework of the theory of general relativity, as we will see in this introductory article.

1. The geometrical framework

This modeling is based on the Platonic space outlined in the following articles:

« UN MODÈLE PLATONICIEN (EUCLIDIEN-PROJECTIF) POUR LA THÉORIE DE LA RELATIVITÉ RESTREINTE » (pré-publication hal-01081576, version 1).

« A PLATONIC (EUCLIDEAN-PROJECTIVE) MODEL FOR THE SPECIAL THEORY OF RELATIVITY » (pre-publication hal-01165196, version 1).

$\left(O,\vec{i},\vec{j},\vec{k},\vec{h}\right)$ is a frame for the four-dimensional Euclidean space whose axes are denoted (OX), (OY), (OZ), (Ow); the direction of the projection is that of the vector $\vec{h}$.
Following the hal-01207447 v1 and hal-01213447 v1 articles, the notion of relativistic mass of a particle is described here as a result of its interaction with a stratification of the four-dimensional Platonic space by a sequence of hyperplanes $H^{(n)}$ which are orthogonal to the direction of the projection $\vec{h}$, regularly spaced by a distance $\Delta w_0 > 0$.

This distance Δw_0 is equal to the ***Compton wavelength*** of the particle in question (for example, for an electron, $\Delta w_e = \frac{h}{m_e c} \approx 2,426.10^{-12}$, where m_e denotes the rest mass of the electron).

These concepts are detailed in the Hal articles below:

hal-01165196, v1 : A platonic (euclidean-projective) model for the special theory of relativity.

hal-01207447, v1 : Towards a modeling of De Broglie waves in a platonic quadridimensional space.

hal-01213447, v1 : An idea of the mass of a particle in a platonic quadridimensional space.

hal-01247385, v1 : A modeling of Michelson-Morley interferometer in a platonic quadri-dimensional space.

hal-01340134, v1 : One-dimensional elastic collisions in a platonic quadridimensional space.

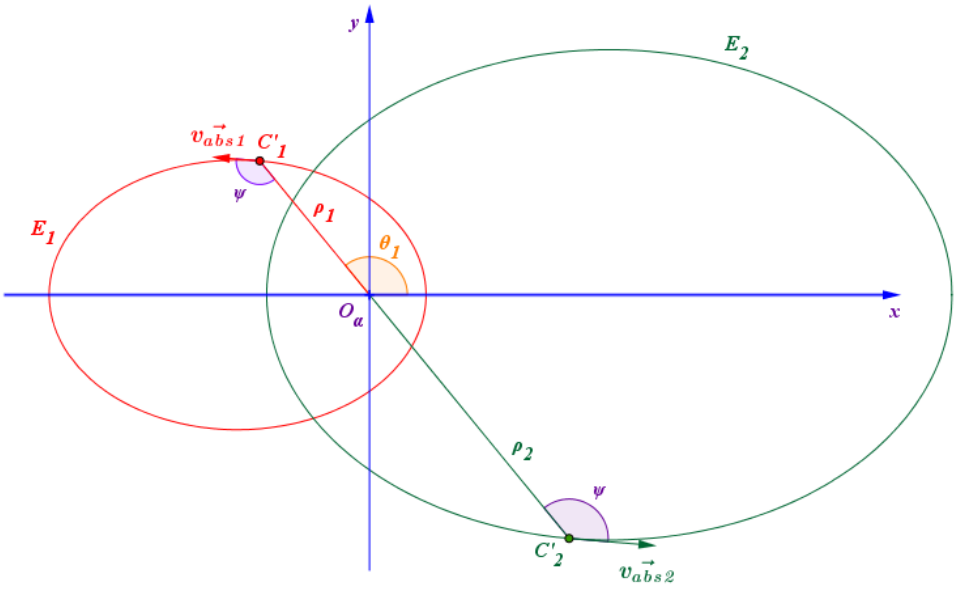
hal-01378215, v1 : About time measurement in a platonic quadridimensional space.

hal-01584918, v1 : A quantified approach to the laws of gravitation in a platonic quadri-dimensional space.

hal-01756323, v1 : De Broglie waves and quantification of Keplerian orbits in a platonic quadridimensional space

2. Diagrams illustrating this study

2.1. Trajectories observed in the reference frame R_α



Observers of a reference frame R_α study the movement of two bodies C'_1 and C'_2 in gravitational interaction.
These two objects are in fact the projections of two bodies C_1 and C_2 moving in the Platonic space (see also the diagram in paragraph 3).

Let us denote by β_1 and β_2 the measures of the angles formed by the velocities $\vec{v_1}$ and $\vec{v_2}$ of these two bodies with the hyperplanes of respective equations $W = W(C_1)$ and $W = W(C_2)$; by ε_1 and ε_2 the measures of the angles formed by the vector $\vec{i}$ and the absolute velocities $\overrightarrow{v_{abs1}}$ and $\overrightarrow{v_{abs2}}$ of the observed bodies C'_1 and C'_2 : $\varepsilon_1 = \left(\vec{i}, \overrightarrow{v_{abs1}}\right)$ and $\varepsilon_2 = \left(\vec{i}, \overrightarrow{v_{abs2}}\right)$.

The elliptic trajectories of C'_1 and C'_2 here observed in the reference frame R_α are the projections of the trajectories of the bodies C_1 and C_2 into the platonic space.

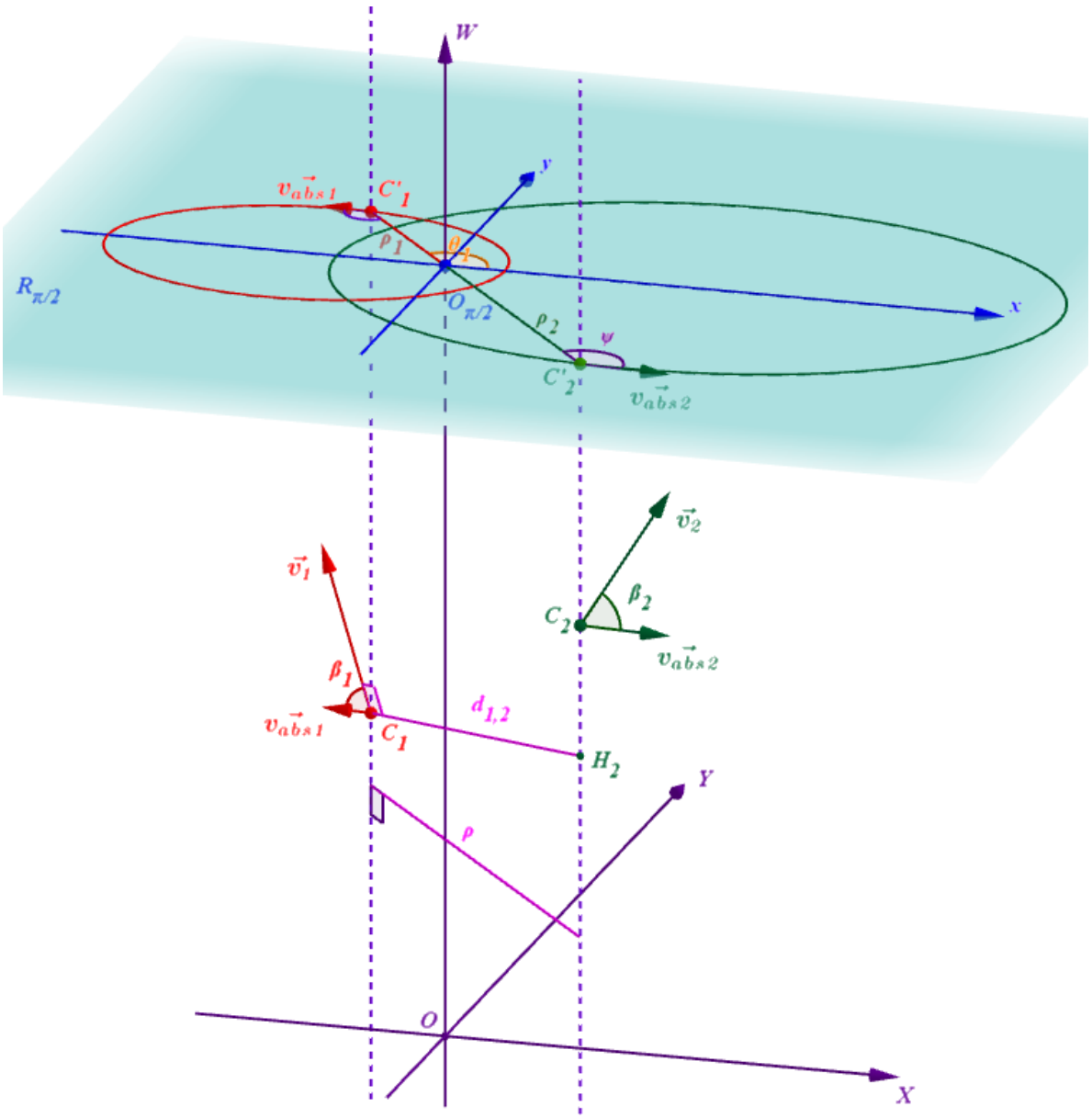
To simplify the calculations and the presentation of this quantum gravitational approach, the observation frame chosen is $R_{\pi/2}$, the orbits of C'_1 and C'_2 in this reference frame are in a plane whose equation is $z = z_0$, the axes of $R_{\pi/2}$ are positioned so as to have their origin $O_{\pi/2}$ on the axis (OW) and their guiding vectors $\overrightarrow{i_{\pi/2}}$, $\overrightarrow{j_{\pi/2}}$, $\overrightarrow{k_{\pi/2}}$ coincide with the guinding vectors $\vec{i}$, $\vec{j}$, $\vec{k}$ of the axes (OX), (OY), (OZ).

The orbits chosen for C'_1 and C'_2 in this reference frame are ellipses (the absolute velocities considered being small with respect to the speed of light).
Axes $(O_{\pi/2}z)$ and (OZ) are not shown.

2.2. Corresponding trajectories in the Platonic space

The notation used in this diagram illustrates the quantities mentioned in this article.

They have already been adopted and commented on in the articles referred to in paragraph 1.



3. Quantified variation of absolute time for a moving object in a gravitational field

In the article hal-01584918, the absolute energy of the gravitational interaction system leads to the following formula **[1]** (page 8):

$$\frac{d\rho}{dT}=\left[\frac{\cos\beta_1\sin\beta_2\left(\sin\beta_2-\frac{Gh}{c^3\rho\Delta w_{02}}\right)}{\sin^2\beta_2+\cos^2\beta_2\cos^2\psi}+\frac{\cos\beta_2\sin\beta_1\left(\sin\beta_1-\frac{Gh}{c^3\rho\Delta w_{01}}\right)}{\sin^2\beta_1+\cos^2\beta_1\cos^2\psi}\right]\left(1+\cos\beta_1\cos\beta_2\right)\cos\psi$$

while article hal-01756323 offers very similar results from the following relation **[2]** (obtained from purely geometrical considerations):

$$\frac{d\rho}{dT}=\left(\cos\beta_1+\cos\beta_2\right)\cos\psi\,.$$

Let's look at these two formulas in a particular case:
suppose that the rest mass m_{02} of the body C_2 is small in relation to the rest mass m_{01} of the body C_1.

The direction angle β_1 corresponding to the Platonic trajectory of C_1 is then little disturbed and can be chosen close to the angle $\frac{\pi}{2}$ of the observation frame (see paragraph 2, where C_1 would then be considered as being almost immobile into $R_{\pi/2}$).

We have, in this particular case, $\cos\beta_1\approx 0$.

Formulas **[1]** and **[2]** then lead to:

[1]bis :
$$d\rho=\frac{\cos\beta_2\sin\beta_1\left(\sin\beta_1-\frac{Gh}{c^3\rho\Delta w_{01}}\right)}{\sin^2\beta_1}\cos\psi\,dT$$

i.e.
$$d\rho=\cos\beta_2\cos\psi\left(1-\frac{Gh}{c^3\rho\Delta w_{01}\sin\beta_1}\right)dT\ ;$$

and **[2]bis** :
$$d\rho=\cos\beta_2\cos\psi\,dT\,.$$

The comparison of these two expressions and the results which derive from the theory of general relativity direct the thought towards ***a variation of the flow of absolute time for the body*** C_2***, caused by the massive body*** C_1*; to know :*

$$dT_2=\left(1-\frac{Gh}{c^3\rho\Delta w_{01}\sin\beta_1}\right)dT\,,$$

where dT_2 *measures the absolute time travelled by the body* C_2 *when* dT *measures the absolute time travelled by any other body not subject to a gravitational field.*

An interpretation of this result (supported in paragraph 5) could be:

in the gravitational field generated by the body C_1*, the* ***distance travelled***

by any moving object C_2 ***is reduced by the quantity*** $\frac{Gh}{c^3\rho}$

at each perception of an occurrence of the mass waves *generated by* C_1.

*According to the article hal-*01340134*, the absolute frequency of the phase wave generated by* C_1 *is:* $$\nu_{abs1} = \frac{1}{\Delta w_{01} \sin \beta_1} .$$ (*)

Thus, for an absolute time interval dT *measured out of any gravitational influence, the body* C_2 *will have traveled a distance diminished by* $\nu_{abs1} \frac{Gh}{c^3\rho} dT$,

which leads to the relation: $dT_2 = \left(1 - \frac{Gh}{c^3 \rho \Delta w_{01} \sin \beta_1}\right) dT$ *evoked previously.*

This interpretation corresponds to a ***quantified variation of the absolute time*** traveled by the body C_2 ***in the presence of the gravitation field*** generated by the body C_1.

This result has multiple consequences, some of which are developed in paragraph 5; in particular ***a quantified variation of the time measured by our clocks*** *in the presence of a gravitational field*.

(*) See also formula **[5]** in paragraph 6, with $\cos \beta_1 \approx 0$ and $\frac{Gh}{c^3\rho} \frac{1}{\Delta w_{0j} \sin \beta_j} \approx 0$.

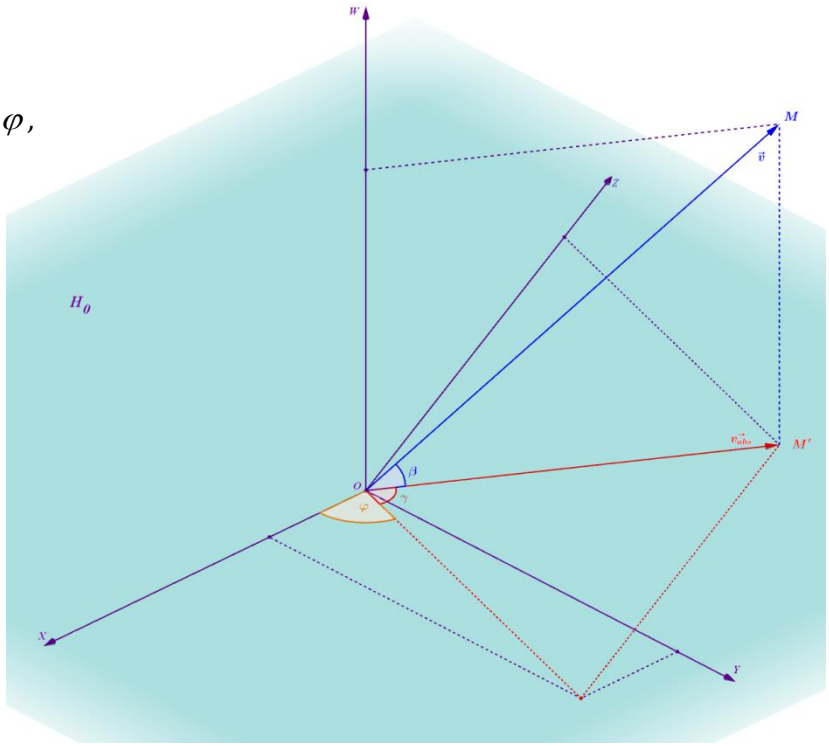
4. Modulation of the Platonic velocity in a gravitational field

In the article hal-01584918 the following properties have been emphasized regarding to the concept of ***velocity*** of a moving object M in the Platonic space (so-called "***platonic***" velocity):

*given the definition of the absolute time T (in m), the **norm of the speed vector** of all the moving objects **is equal to 1** and the **velocity vector** $\vec{v}=\dfrac{\overrightarrow{dM}}{dT}$ of any object M is given by:*

$$\vec{v}=\begin{pmatrix}\cos\beta\cos\gamma\cos\varphi\\ \cos\beta\cos\gamma\sin\varphi\\ \cos\beta\sin\gamma\\ \sin\beta\end{pmatrix}, \text{ with any } \varphi,$$

$$\gamma\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right] \text{ and } \beta\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right].$$



*Its **absolute speed** is given by:*

$$\overrightarrow{v_{abs}}=\begin{pmatrix}\cos\beta\cos\gamma\cos\varphi\\ \cos\beta\cos\gamma\sin\varphi\\ \cos\beta\sin\gamma\\ 0\end{pmatrix},$$

*whose **norm is equal to** $\cos\beta$.*

For the vector $\overrightarrow{v_2}$ in this study, we have $\gamma=0$ and $\varphi=\theta+\pi-\psi$.

Taking into account the assumptions considered in paragraph 3, these results now deserve to be refined.

Thus, ***the Platonic speed of a body C_2 in a gravitational field*** becomes:

$$\overrightarrow{v_2}=\frac{\overrightarrow{dC_2}}{dT}=\left(1-\frac{Gh}{c^3\rho\Delta w_{01}\sin\beta_1}\right)\begin{pmatrix}\cos\beta_2\cos\gamma\cos\varphi\\ \cos\beta_2\cos\gamma\sin\varphi\\ \cos\beta_2\sin\gamma\\ \sin\beta_2\end{pmatrix}.$$

Its ***platonic norm*** being equal to: $\left\|\overrightarrow{v_2}\right\|=1-\dfrac{Gh}{c^3\rho\Delta w_{01}\sin\beta_1}$.

Its ***absolute velocity*** becomes: $\overrightarrow{v_{abs2}}=\dfrac{\overrightarrow{dC_2}'}{dT}=\left(1-\dfrac{Gh}{c^3\rho\Delta w_{01}\sin\beta_1}\right)\begin{pmatrix}\cos\beta_2\cos\gamma\cos\varphi\\ \cos\beta_2\cos\gamma\sin\varphi\\ \cos\beta_2\sin\gamma\\ 0\end{pmatrix}$;

and its norm is:

$$v_{abs2}=\left(1-\frac{Gh}{c^3\rho\Delta w_{01}\sin\beta_1}\right)\cos\beta_2. \qquad \textbf{[3]}$$

5. First consequences

5.1. Quantified variation of time and time advance measured by a satellite

To repeat the interpretation proposed in paragraph 3:

« in the gravitational field generated by the body C_1*, the* ***distance travelled***

by any moving object C_2 ***is reduced by the quantity*** $\frac{Gh}{c^3\rho}$

at each perception of an occurrence of the mass waves *generated by* C_1*.»*

This interpretation results in ***a quantified variation of the time measured by the clocks of the reference frame linked to*** C_2' ***in the presence of the gravitational field generated by*** C_1.

Indeed, at each ***perception of an occurrence of the mass waves***, considering the absolute shift $\Delta T_2 = \frac{Gh}{c^3\rho}$, the formula $c\Delta t_2 = \Delta T_2 \sin\beta_2$ (see article hal-01378215) implies the following ***time delay*** (***independent of the mass of the object*** C_1):

$$\delta t_2 = \frac{Gh}{c^4\rho}\sin\beta_2 .$$

An order of magnitude of this delay, for a distance $\rho = 1\ m$ and $\beta_2 \approx \frac{\pi}{2}$, is:

$$\delta t_2 \approx 5,48.10^{-78}\ s$$

(see section 5.5 for some notes about Planck units).

As an example, this result leads ***to the advance measured by a clock located in a satellite orbiting the Earth***.

Indeed, the rest mass m_{02} of the body C_2 being small in relation to the mass at rest m_{01} of the body C_1, the directing angles β_1 and β_2 corresponding to the Platonic trajectories of these two bodies are undisturbed and can be chosen close to the angle $\frac{\pi}{2}$ of the observation frame (see paragraph 2, where C_1 and C_2 are then considered to be almost immobile in $R_{\pi/2}$).

In this case, with $\sin\beta_1 = \sin\beta_2 \approx 1$ and $\nu_{abs1} = \frac{1}{\Delta w_{01}\sin\beta_1}$, we have, for a duration Δt that would be measured in the center of the Earth (and considering the mass of the Earth applied at its center - see note ② below -):

$$\delta t_i = \nu_{abs1}\frac{Gh}{c^4\rho_i}c\Delta t = \frac{1}{\Delta w_{01}}\frac{Gh}{c^3\rho_i}\Delta t, \quad \text{with } i \in \{1;2\} ;$$

that becomes, with $m_{abs1} = \frac{h}{c\Delta w_{01}}$:

$$\delta t_i = \frac{Gm_{abs1}}{c^2\rho_i}\Delta t .$$

Let ρ_1 denote the radius of the Earth (considered almost spherical).

For a satellite that would be in orbit at an altitude ρ, the measured advance δt (here, the satellite being farther from the surface of the Earth than the Earth's clock, we have $(\rho_2 = \rho_1 + \rho) > \rho_1$ and so $\delta t_2 < \delta t_1$) is thus:

$$\delta t = \delta t_2 - \delta t_1 = \frac{Gm_{abs1}}{c^2 \rho_2}\Delta t - \frac{Gm_{abs1}}{c^2 \rho_1}\Delta t = \frac{Gm_{abs1}}{c^2}\Delta t\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right).$$

With : $$\frac{1}{\rho_2} - \frac{1}{\rho_1} = \frac{1}{\rho_1 + \rho} - \frac{1}{\rho_1} = -\frac{\rho}{(\rho_1)^2\left(1 + \frac{\rho}{\rho_1}\right)},$$

we obtain : $$\delta t = -\frac{Gm_{abs1}}{c^2}\frac{\rho}{(\rho_1)^2\left(1 + \frac{\rho}{\rho_1}\right)}\Delta t,$$ **[4]**

and finally, with $g = \frac{Gm_{abs1}}{(\rho_1)^2}$, the well-known formula: $$\delta t = -\frac{g\rho}{c^2\left(1 + \frac{\rho}{\rho_1}\right)}\Delta t\ ;$$ **[4]bis**

that becomes, with $\rho \ll \rho_1$: $\delta t \approx -\frac{g\rho}{c^2}\Delta t.$ **[4]ter**

Remarks :

① more rigorously, our clocks being located rather on the surface of the Earth than in its center, this advance should be measured by referring to the duration Δt_1.

With $\Delta t_1 = \left(1 - \frac{Gh}{c^3 \rho_1 \Delta w_{01} \sin\beta_1}\right)\Delta t$, that would lead, under the assumptions of this paragraph, to: $$\delta t \approx -\frac{g\rho}{c^2\left(1 + \frac{\rho}{\rho_1}\right)}\left(1 + \frac{g\rho_1}{c^2}\right)\Delta t_1.$$

As $\frac{g\rho_1}{c^2} \approx 6{,}96.10^{-10}$, we find the approximation used: $\delta t \approx -\frac{g\rho}{c^2\left(1 + \frac{\rho}{\rho_1}\right)}\Delta t_1.$

② The masses of the bodies C_i and C_j are as a first approximation equal to the sums of the masses of the particles which constitute them.

The stratification distances Δw_{0i} and Δw_{0j} taken into account in the calculations on the basis of the relation $\Delta w_0 = \dfrac{h}{cm_{abs}\sin\beta}$ are fictitious distances which make it possible to generate the occurrences of the mass waves corresponding to the accumulations of the occurrences generated by all of these particles.
For the record, a natural limit for the mass of an "elementary particle" is the mass of Planck, of which its resting value is: $m_P = \sqrt{\dfrac{hc}{2\pi G}} \approx 2{,}176.10^{-8}\ kg$ and its corresponding stratification distance Δw_{0P} (i.e. its Compton wavelength) is:

$$\Delta w_{0P} = \lambda_C = \frac{h}{m_P c} = 2\pi\sqrt{\frac{Gh}{2\pi c^3}} = 2\pi l_P \approx 1{,}015.10^{-34}\ m\,.$$

③ As an illustration, for a satellite of the GPS constellation in orbit at an altitude $\rho = 20200\ km$, the relation **[4]** leads to an acceleration of $45\,\mu s$ time per day, ***in agreement with the theory of general relativity***.

5.2. Gravitational Redshift

The theory of general relativity predicts that light moving in a non-homogeneous gravitational field is shifted towards red or blue. According to this theory, a frequency produced in a gravitational field is seen to be red-shifted (i.e., diminished) when viewed from a place where gravitation is less. This prediction was successfully confirmed in 1959 by Pound-Rebka's experiment and more recently (thanks to the two Galileo satellites accidentally placed in an eccentric orbit in 2014) by physicists of the Paris Observatory - PSL of the department. SYRTE (Observatory of Paris - PSL / CNRS / Sorbonne University).

Let us take again the relation **[4]** obtained in the preceding paragraph, with the following notations:

ρ_0 denotes here the radius of the considered planet, m_{abs0} its absolute mass;

δt_1 is the advance measured by an observer O_1 at an altitude ρ_1 ;

Δt_0 measures a time interval on the surface of the planet, Δt_1 is the corresponding measurement made by the observer O_1.

So we have : $\delta t_1 = -\dfrac{Gm_{abs0}}{c^2}\dfrac{\rho_1}{(\rho_0)^2\left(1+\dfrac{\rho_1}{\rho_0}\right)}\Delta t_0$ and $\Delta t_1 = \Delta t_0 + \delta t_1$.

By applying these relations to a luminous signal, of period Δt_0, emitted from the surface of this planet in the direction of the observer O_1, we have:

$$\Delta t_1 = \Delta t_0 - \frac{Gm_{abs0}}{c^2}\frac{\rho_1}{(\rho_0)^2\left(1+\frac{\rho_1}{\rho_0}\right)}\Delta t_0,$$

i.e. $$\Delta t_1 = \Delta t_0\left(1-\frac{Gm_{abs0}}{c^2}\frac{\rho_1}{\rho_0(\rho_0+\rho_1)}\right).$$

The corresponding frequencies are $\nu_0 = \frac{1}{\Delta t_0}$ and $\nu_1 = \frac{1}{\Delta t_1}$.

Hence the relation (considering, given the orders of magnitude encountered here, that we have $\frac{1}{1-\alpha} \approx 1+\alpha$):

$$\nu_1 = \nu_0\left(1+\frac{Gm_{abs0}}{c^2}\frac{\rho_1}{\rho_0(\rho_0+\rho_1)}\right).$$

Finally, we can easily express this relation as a function of the Schwarzschild radius of the considered planet ($R_S = \frac{2Gm_{abs0}}{c^2}$) and thus find a well-known formula of the ***theory of general relativity***:

$$\nu_1 = \nu_0\left(1+\frac{Gm_{abs0}}{c^2}\frac{\rho_1}{\rho_0(\rho_0+\rho_1)}\right) \Leftrightarrow \nu_1 = \nu_0\left(1+\frac{Gm_{abs0}}{c^2\rho_0}-\frac{Gm_{abs0}}{c^2(\rho_0+\rho_1)}\right),$$

i.e. $$\nu_1 = \nu_0\left(1+\frac{R_S}{2\rho_0}-\frac{R_S}{2(\rho_0+\rho_1)}\right).$$

5.3. Advance of the perihelion of the planets

The trajectory of an isolated planet around the Sun, according to classical mechanics, is an invariable ellipse. However, the observation shows that the perihelion of a planet moves slowly: its orbit is not fixed but rotates slowly in its plane.

In this paragraph, we will examine the impact of the variation of the Platonic speed of a planet near the Sun during its period of revolution.

We will thus find the well-known result, ***obtained by the theory of general relativity, concerning the advance of the perihelion of the planets***.

We have established, in article Hal-01756323, the relation:

$\frac{d\theta}{dT} = \frac{\cos\beta_i}{\rho_i}\sin\psi$; where $\cos\beta_i$ corresponds to the absolute speed of the studied object.

We will compare the results obtained by the quantification of the Keplerian approach adopted in the article hal-01756323 to the results obtained taking into account of the variation of the Platonic speed proposed here.

To clarify the situation, we will consider the following notations:

$d\theta_k$ and ρ_k will denote the quantities corresponding to the Keplerian approach;

$d\theta$ and ρ the corresponding quantities, modified by the current approach;

dT_P will denote an absolute duration measured by the planet considered and dT the corresponding absolute duration in the absence of a gravitational field;

v_{abs} denotes the modulus of the absolute velocity of the planet considered.

By designating by M_S the absolute mass of the Sun ($M_S = m_{abs1} = \frac{h}{c\Delta w_{01}\sin\beta_1}$), we have:

$$dT_P = dT\left(1-\frac{G}{c^2}\frac{M_S}{\rho}\right)$$

$$v_{abs} = \left(1-\frac{G}{c^2}\frac{M_S}{\rho}\right)\cos\beta$$

$$\rho = \frac{\rho_k}{1-\frac{G}{c^2}\frac{M_S}{\rho}}$$ (modulation due to the conservation of absolute angular momentum *)

$\frac{d\theta_k}{dT} = \frac{\cos\beta}{\rho_k}\sin\psi$, i.e. $d\theta_k = \frac{\cos\beta}{\rho_k}\sin\psi dT$.

From these preliminary data and the formula $d\theta = \frac{v_{abs}}{\rho}\sin\psi dT_P$, we get:

$$d\theta = \frac{v_{abs}}{\rho}\sin\psi dT_P \Rightarrow d\theta = \frac{\cos\beta}{\rho_k}\sin\psi dT\left(1-\frac{G}{c^2}\frac{M_S}{\rho}\right)^3.$$

With : $$\left(1-\frac{G}{c^2}\frac{M_S}{\rho}\right)^3 \approx 1-3\frac{G}{c^2}\frac{M_S}{\rho},$$

this implies : $$d\theta = \frac{\cos\beta}{\rho_k}\sin\psi dT - 3\frac{G}{c^2}\frac{M_S}{\rho}\frac{\cos\beta}{\rho_k}\sin\psi dT$$

i.e. $$d\theta = d\theta_k - \delta\theta \text{ , avec } \delta\theta = 3\frac{G}{c^2}\frac{M_S}{\rho}\frac{\cos\beta}{\rho_k}\sin\psi dT.$$

So we obtain : $$\delta\theta = 3\frac{G}{c^2}\frac{M_S}{\rho}d\theta_k.$$

Now, according to the Keplerian mechanics, $\rho = \frac{a\left(1-e^2\right)}{1+e\cos\theta_k}$.

Which leads us to: $$\delta\theta = 3\frac{G}{c^2}\frac{M_S}{a\left(1-e^2\right)}\left(1+e\cos\theta_k\right)d\theta_k .$$

During the complete revolution of a planet around the Sun, the advance of the perihelion is thus expressed as: $\Delta\theta = 3\frac{G}{c^2}\frac{M_S}{a\left(1-e^2\right)}\int_0^{2\pi}\left(1+e\cos\theta_k\right)d\theta_k$,

i.e. $$\boxed{\Delta\theta = \frac{6\pi GM_S}{c^2 a\left(1-e^2\right)}.}$$

*** Note on the modulation of the distance ρ :**

as we saw in the Keplerian approach proposed in articles hal-01584918 and hal-01756323, ***the conservation of the absolute angular momentum of the system*** leads to:

$$\rho_{k1}\frac{h\cos\beta_1}{c\Delta w_{01}\sin\beta_1}\sin\psi + \rho_{k2}\frac{h\cos\beta_2}{c\Delta w_{02}\sin\beta_2}\sin\psi = c_2,$$ where c_2 is constant. **[m1]**

(the distances here have been designated ρ_{k1} and ρ_{k2} to emphasize the Keplerian nature of this formula).

In this formula, the terms $\cos\beta_i$ correspond to the absolute velocities of the interacting bodies.

In the present study, it is therefore appropriate to substitute for them the terms v_{absi}, with :

$$v_{absi} = \left(1 - \frac{Gh}{c^3 \rho \Delta w_{0j} \sin \beta_j}\right) \cos \beta_i .$$

So the conservation of the absolute angular momentum of the system is expressed by:

$$\rho_1 \frac{h v_{abs1}}{c \Delta w_{01} \sin \beta_1} \sin \psi + \rho_2 \frac{h v_{abs2}}{c \Delta w_{02} \sin \beta_2} \sin \psi = c_2 , \quad \text{i.e.}$$

$$\rho_1 \frac{h \cos \beta_1}{c \Delta w_{01} \sin \beta_1} \left(1 - \frac{Gh}{c^3 \rho \Delta w_{02} \sin \beta_2}\right) \sin \psi + \rho_2 \frac{h \cos \beta_2}{c \Delta w_{02} \sin \beta_2} \left(1 - \frac{Gh}{c^3 \rho \Delta w_{01} \sin \beta_1}\right) \sin \psi = c_2$$

which induces, by comparison with **[m1]**, the relations: $\rho_{ki} = \left(1 - \frac{Gh}{c^3 \rho \Delta w_{0j} \sin \beta_j}\right) \rho_i$,

i.e. : $$\rho_i = \frac{\rho_{ki}}{1 - \frac{Gh}{c^3 \rho \Delta w_{0j} \sin \beta_j}} .$$

5.4. Stoppage of the absolute time

A ***singular result*** appears very quickly when one observes the Platonic velocity of a body C_2 (*possibly a photon*) in the gravitational field generated by a body C_1, located at a distance equal to its ***half-radius of Schwarzschild***.

Indeed, we have:

$$\overrightarrow{v_2} = \frac{\overrightarrow{dC_2}}{dT} = \left(1 - \frac{Gh}{c^3 \rho \Delta w_{01} \sin \beta_1}\right) \begin{pmatrix} \cos \beta_2 \cos \gamma \cos \varphi \\ \cos \beta_2 \cos \gamma \sin \varphi \\ \cos \beta_2 \sin \gamma \\ \sin \beta_2 \end{pmatrix},$$

with : $$\rho \to \rho_S = \frac{G}{c^2} m_{abs1} = \frac{Gh}{c^3 \Delta w_{01} \sin \beta_1} .$$

This leads to : $$\overrightarrow{v_2} \to \vec{0} .$$

In other words, the body C_2 ***tends to stop in the Platonic space***.

This surprising result, obtained quickly at a first approach, lets us glimpse at other results which a more refined study can offer within a quantum framework.

5.5. Some links with Planck units

If we repeat the interpretation proposed in paragraph 3:

« in the gravitational field generated by the body C_1 *, the* ***distance travelled***

by any moving object C_2 ***is reduced by the quantity*** $\delta T = \frac{Gh}{c^3 \rho}$

at each ***perception of an occurrence of the mass waves*** *generated by* C_1 *»,*

we notice the presence of the quantity $\frac{Gh}{c^3}$ which evokes the universe of Planck.

So, from $l_P = \sqrt{\frac{Gh}{2\pi c^3}}$ and $t_P = \sqrt{\frac{Gh}{2\pi c^5}} = \frac{l_P}{c}$ which correspond respectively to the length and time of Planck, we have, by posing $\rho = n l_P , n \in \mathbb{N}^*$:

$$\delta T = \frac{2\pi}{n} l_P \ , n \in \mathbb{N}^* .$$

And, thus, with the temporal relation $c\Delta t = \Delta T \sin \beta_2$ (cf the article hal-01378215):

each perception of an ***occurrence of the mass waves*** *generated by* C_1 *generates*

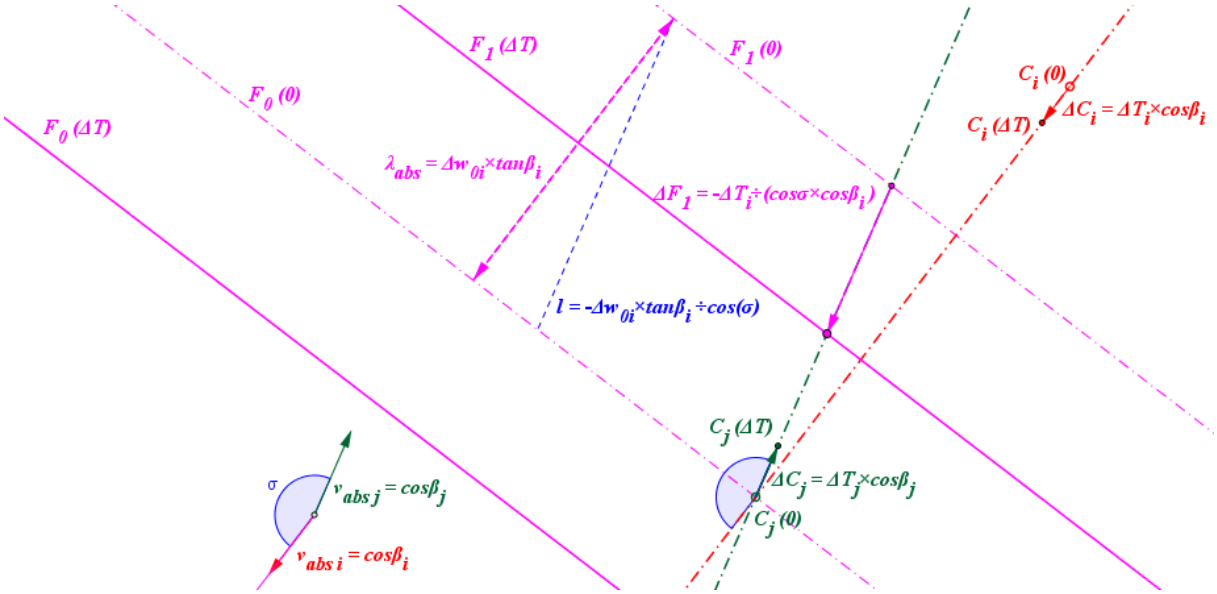
for the punctual body C_2 *a* ***proper temporal delay*** *equal to:*

$$\delta t = \frac{2\pi}{n} t_P \sin \beta_2 \ , n \in \mathbb{N}^* .$$

Note that, in this mathematical model, quanta related to the perception of an occurrence of De Broglie waves and inducing changes in absolute time and proper time are ***smaller than the corresponding Planck units***.

The mass waves, of which the celerity is greater than or equal to the speed of light (see the model proposed in the article hal-01207447), can thus be seen, mathematically, to attribute original and successful properties.

6. Frequency of the mass waves generated by the body C_i and perceived by C_j



We will repeat in this paragraph the calculations presented in the articles hal-01584918 and hal-01756323, ***taking into account the variations of absolute time*** *generated by the gravitational interaction bodies.*

Referring to the diagram above, consider the mass waves generated by the body C_i and perceived by the body C_j.

Let us denote by σ a measure of the angle $\left(\overrightarrow{v_{abs\,i}}, \overrightarrow{v_{abs\,j}}\right)$.

A first wavefront $F_0(0)$ is perceived by C_j when it is at $C_j(0)$.

The next wavefront F_1 is then located at $F_1(0)$, separated by the absolute wavelength $\lambda_{abs} = \Delta w_{0i} \tan \beta_i$ (in addition, see the diagram in the article hal-01584918, § 8).

After an absolute time ΔT (measured outside of any gravitational field), the bodies C_i and C_j are at $C_i(\Delta T)$ and $C_j(\Delta T)$ and the wavefront F_1 is at $F_1(\Delta T)$, with the relations:

$$\Delta F_1 = \frac{-\Delta T_i}{\cos\beta_i \cos\sigma} \quad \text{and} \quad \Delta C_j = \Delta T_j \cos\beta_j .$$

Thus, the body C_j perceives the front F_1 when the absolute duration ΔT satisfies the equation:

$$l = \Delta C_j + \Delta F_1 , \quad \text{i.e.} \quad -\frac{\Delta w_{0i} \tan\beta_i}{\cos\sigma} = \frac{-\Delta T_i}{\cos\beta_i \cos\sigma} + \Delta T_j \cos\beta_j ,$$

$$\text{i.e. :} \quad \Delta T = \frac{\Delta w_{0i} \sin\beta_i}{\left(1 - \frac{Gh}{c^3\rho}\frac{1}{\Delta w_{0j}\sin\beta_j}\right) - \cos\beta_i \cos\beta_j \cos\sigma \left(1 - \frac{Gh}{c^3\rho}\frac{1}{\Delta w_{0i}\sin\beta_i}\right)}$$

(***absolute period*** ΔT of the mass waves perceived by C_j).

It follows that the ***absolute frequency*** $f_{i,j}$ of this mass wave is given by:

$$f_{i,j} = \frac{1}{\Delta T} = \frac{\left(1 - \frac{Gh}{c^3 \rho} \frac{1}{\Delta w_{0j} \sin \beta_j}\right) - \cos \beta_i \cos \beta_j \cos \sigma \left(1 - \frac{Gh}{c^3 \rho} \frac{1}{\Delta w_{0i} \sin \beta_i}\right)}{\Delta w_{0i} \sin \beta_i}.$$

With the equality $\sigma = \pi$ in the particular case that we study in these articles, the absolute frequency $f_{i,j}$ of this mass wave perceived by the mobile C_j becomes:

$$f_{i,j} = \frac{1 + \cos \beta_i \cos \beta_j - \frac{Gh}{c^3 \rho} \left(\frac{1}{\Delta w_{0j} \sin \beta_j} + \frac{\cos \beta_i \cos \beta_j}{\Delta w_{0i} \sin \beta_i} \right)}{\Delta w_{0i} \sin \beta_i}. \qquad \textbf{[5]}$$

Remarks :

The ***absolute proper period*** ΔT_j of the mass waves perceived by C_j is:

$$\Delta T_j = \left(1 - \frac{Gh}{c^3 \rho} \frac{1}{\Delta w_{0i} \sin \beta_i}\right) \Delta T$$

$$\text{i.e.} \quad \Delta T_j = \frac{\Delta w_{0i} \sin \beta_i - \frac{Gh}{c^3 \rho}}{\left(1 - \frac{Gh}{c^3 \rho} \frac{1}{\Delta w_{0j} \sin \beta_j}\right) - \cos \beta_i \cos \beta_j \cos \sigma \left(1 - \frac{Gh}{c^3 \rho} \frac{1}{\Delta w_{0i} \sin \beta_i}\right)}.$$

It follows that the ***absolute proper frequency*** $f_{0i,j}$ of these mass waves is given by:

$$f_{0i,j} = \frac{1}{\Delta T_j} = \frac{\left(1 - \frac{Gh}{c^3 \rho} \frac{1}{\Delta w_{0j} \sin \beta_j}\right) - \cos \beta_i \cos \beta_j \cos \sigma \left(1 - \frac{Gh}{c^3 \rho} \frac{1}{\Delta w_{0i} \sin \beta_i}\right)}{\Delta w_{0i} \sin \beta_i - \frac{Gh}{c^3 \rho}}.$$

With the equality $\sigma = \pi$ in the particular case that we study in these articles, the ***absolute proper frequency*** $f_{0i,j}$ of these mass waves becomes:

$$f_{0i,j} = \frac{1 + \cos \beta_i \cos \beta_j - \frac{Gh}{c^3 \rho} \left(\frac{1}{\Delta w_{0j} \sin \beta_j} + \frac{\cos \beta_i \cos \beta_j}{\Delta w_{0i} \sin \beta_i} \right)}{\Delta w_{0i} \sin \beta_i - \frac{Gh}{c^3 \rho}}.$$

As in section 5.4, a ***singular result*** appears very quickly when one observes these results for a body C_j in the gravitational field generated by a body C_i, located at a distance equal to its ***half-radius of Schwarzschild***.

Indeed we have, with $\rho \to \dfrac{Gh}{c^3 \Delta w_{0i} \sin \beta_i}$, the following results:

$$\Delta T_j \to 0 \quad \text{and} \quad f_{0i,j} \to \infty .$$

7. Conclusion

Articles hal-01584918 and hal-01756323 offer a first quantified Platonic approach to Keplerian mechanics.

In particular, they create links between these laws and the De Broglie mass waves generated by the interacting bodies, through quantized changes in direction of their Platonic velocities.

By introducing a second quantified declination postulate, the article hal-01756323 also allows a quantified interpretation of the conservation of the absolute angular momentum of the considered system.

In addition, a comparative examination of these two articles, centered on the conservation of the absolute energy of the system, leads to envisage ***a quantified correction of the modulus of Platonic velocity*** of the moving objects in a gravitational field.

It thus appears that De Broglie mass waves modify, in a quantifiable way, not only the direction of Platonic velocities of the objects they affect, but also their modulus.

In the relativistic Platonic space, ***these quanta have an absolute character*** (they do not depend on the reference frames of observation) and are ***independent of the rest masses*** of the considered bodies.

Moreover, it should be emphasized that they are intimately related to the ***Planck units*** (by the presence of the quantity $\frac{Gh}{c^3}$ in their expression).

Thus, these postulates offer, ***in a relativistic framework***, a surprisingly correct ***quantified approach*** of the classical laws of gravitation applied to ***elliptic Keplerian orbits*** and allow, moreover, to quickly find several results offered by ***the theory of general relativity*** (advance of the perihelion of the planets, variations in the measurement of time in a satellite orbiting the Earth, gravitational redshift, ...).

As for the articles hal-01584918 and hal-01756323, these first results and their examination can certainly be quickly refined and completed (study of the deviation of light rays in a gravitational field, research on gravitational waves, ...).

And the principles retained deserve, of course, to be further enriched and deepened to propose, in a more general framework, a more complete approach of a quantum theory of gravitation.

In any case, the rapid obtaining of the few results set out in these introductory articles, in an original and quantified way, seems, at the very least, to favor the emergence of new and numerous questions, promising, in still unexplored directions .

Notably about the deep links that it suggests between the quantum effects of De Broglie mass waves and the laws of gravitation.

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