

Consideration of estimation error in multiple coherence approaches

**application to the inside/ outside coherence
analysis of aircraft flight tests**

ISMA 2018

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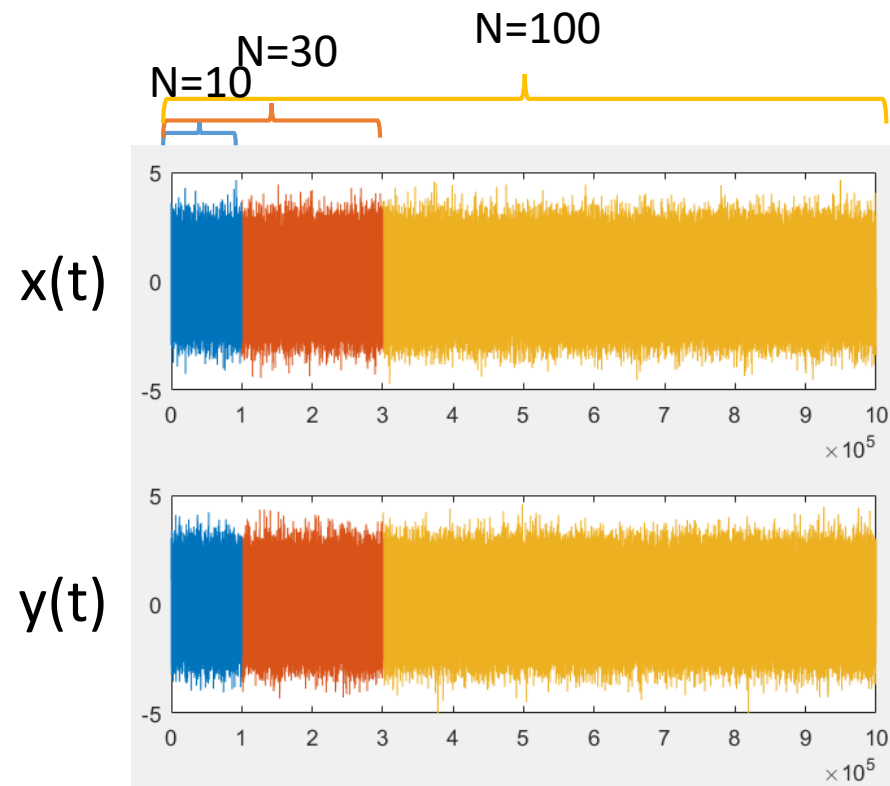
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- 2) Statistical test : the thresholded coherence
- 3) Small complication : windowing & overlap
- 4) MIMO case : the multiple thresholded coherence
- 5) Experimental application : aircraft flight tests

Estimated coherence of independant signals

Let's consider finite length realizations of two independant Gaussian signals $x(t)$ and $y(t)$, processed using Welch's periodogram.

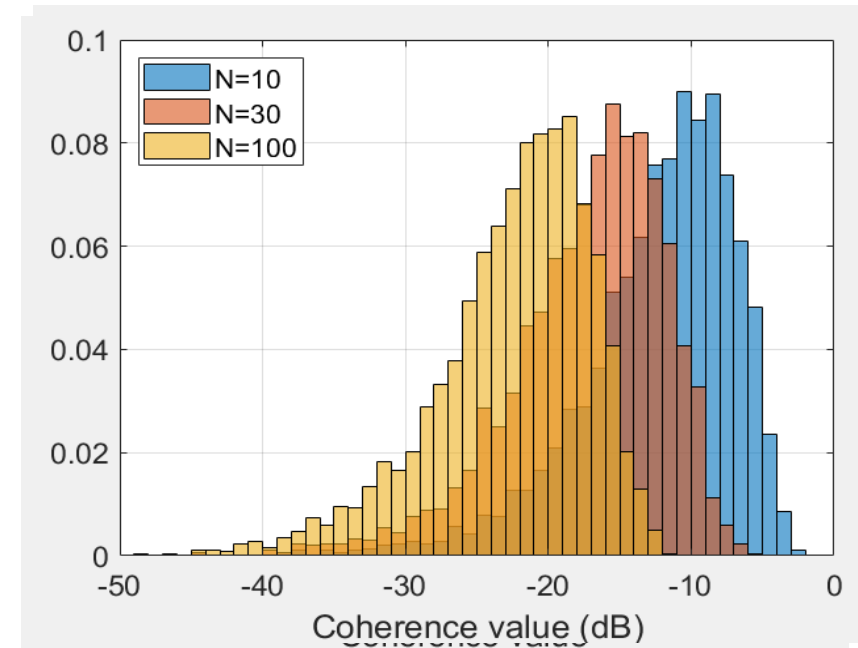
[N snapshots, 0% overlap, Uniform window]



P-Welch



$$\gamma^2 = \frac{|S_{xy}|^2}{S_{xx}S_{yy}}$$



Estimated coherence of independant signals

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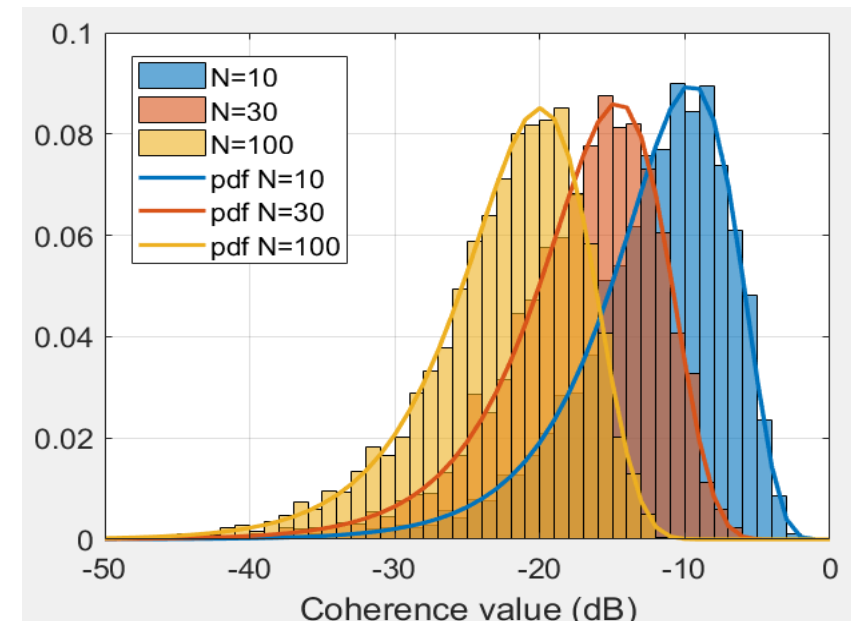
[N snapshots, 0% overlap, Uniform window]

Theoretical pdf [Carter, 1972] :

$$f_{\gamma^2}(x) = (N - 1)(1 - x)^{N-2} \quad (0 \leq x \leq 1)$$

With expected value and variance

$$E[\gamma_{xy}^2] = \frac{1}{N} \quad V[\gamma_{xy}^2] = \frac{1}{N^2} \frac{N - 1}{N + 1}$$



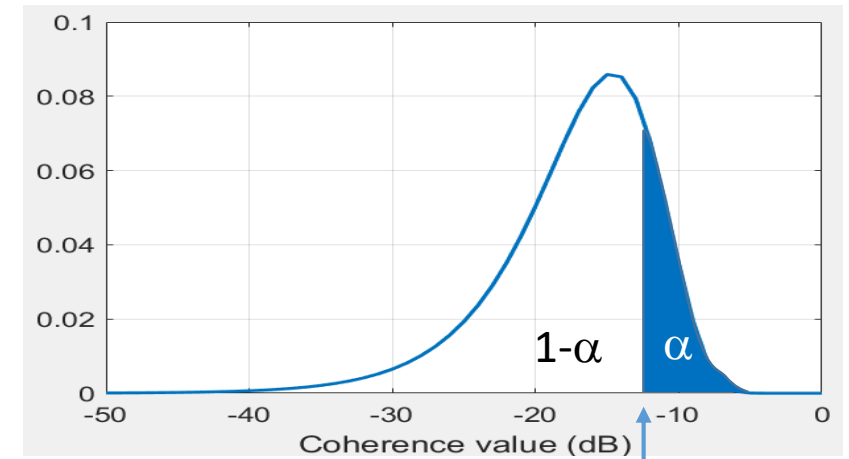
Statistical test : the thresholded coherence

Under an H_0 hypothesis that signals are uncorrelated, there is a small risk α that the coherence is greater than a threshold T :

$$p(\gamma_{xy}^2 < T) = \int_0^T (N-1)(1-x)^{N-2} dx = 1 - \alpha$$

which leads to $T(\alpha, N) = 1 - \alpha^{1/(N-1)}$

If the estimated γ^2 is lower than T , the H_0 hypothesis is accepted, which means that γ^2 is considered to be zero.

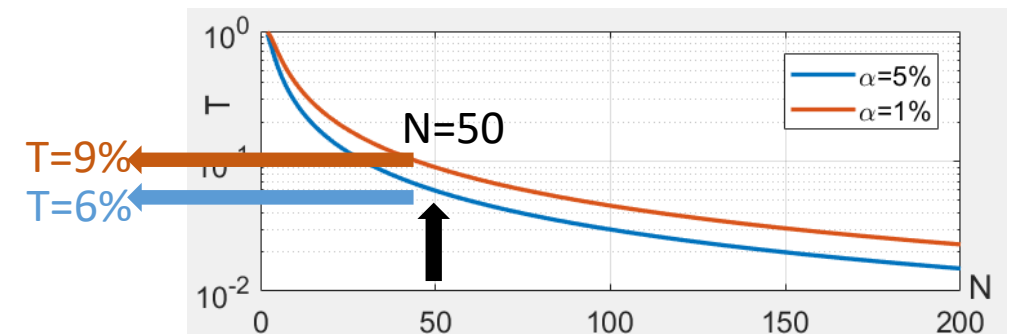


$10 \log(T)$

→ definition of the **Thresholded Coherence** :

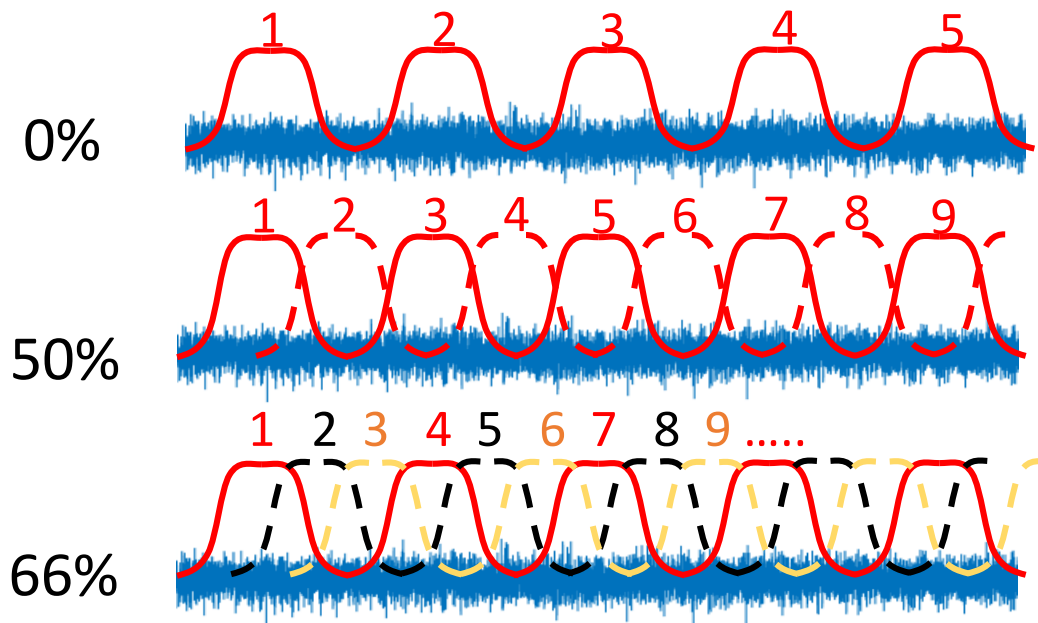
$$\gamma_{xy,\alpha}^2 = \mathbf{1}_{[\gamma_{xy}^2 > T(\alpha, N)]} \cdot \gamma_{xy}^2$$

Assuming H_0 , α represents the risk to be wrong deciding that the coherence is not nul. Typical values are set to 5 or 1 %



Small complication : windowing & overlap

- Windowing is used to reduce leakage
- Overlap is used to recover information dropped out by windowing.



For a given total record length, increasing overlap increases the number of snapshots.

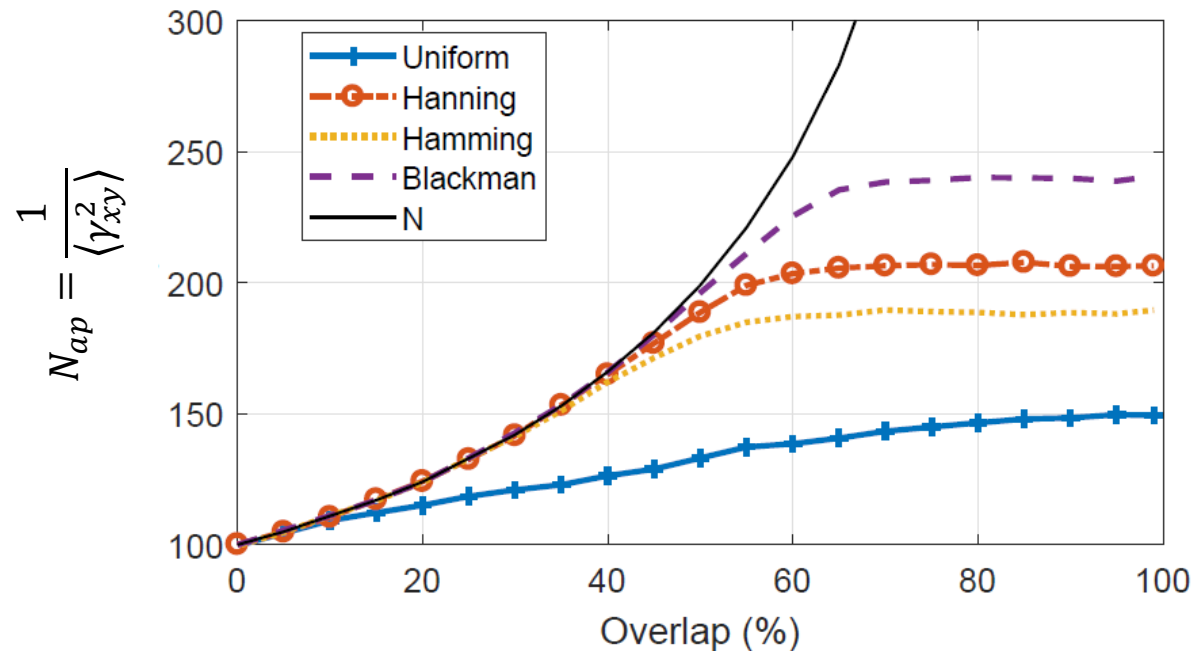
However, the « **apparent number of snapshots** » reaches a maximum value when the overlap tends to 100%

$$N_{ap} = \frac{1}{\langle \gamma_{xy}^2 \rangle} \quad \text{For } x, y \text{ independent}$$

Small complication : windowing & overlap

- Apparent value of N as a function of overlap
(value of N with no overlap $N_0 = 100$)

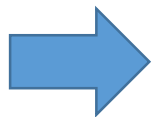
$$N_{ap} = \frac{1}{\langle \gamma_{xy}^2 \rangle}$$



When the overlap is high enough
(>66% for Hanning), $N_{ap} \approx \kappa N_0$

κ depends on N_0 , the window type and length [Welch, 67 ; Gallet 2011]

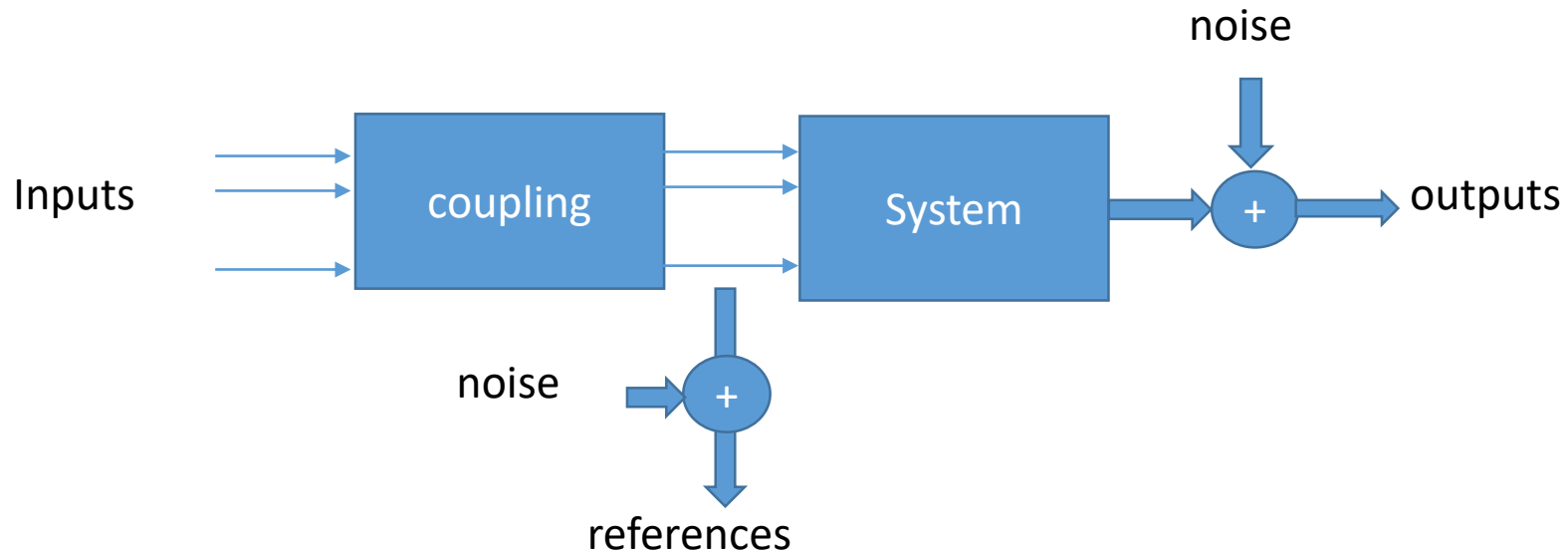
**For a Hanning window ,
when $N_0 > 100$, $\kappa \approx 2.07$**



*For the Thresholded Coherence with overlap,
the value of N is set to $N = \kappa N_0$*

MIMO case : the multiple coherence

- Need of multiple references :
 - source statistically complex (multiple incoherent components)
 - references are contaminated by noise

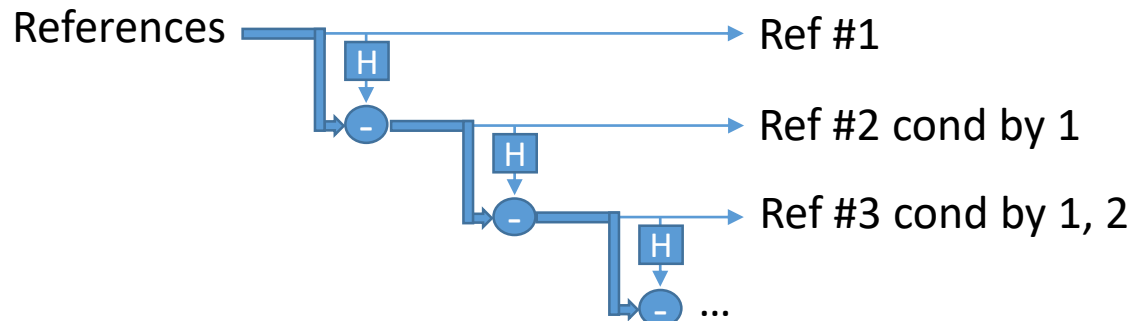


Difficulty : references are potentially strongly correlated

MIMO case : the multiple coherence

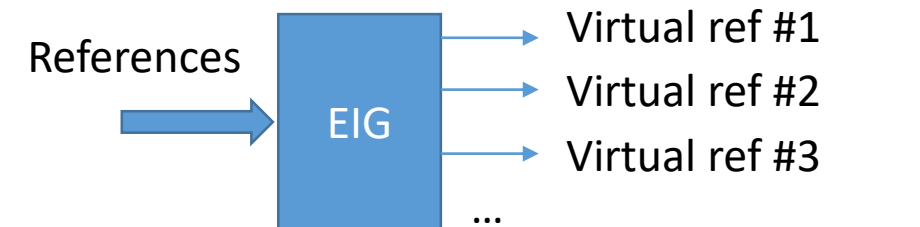
- Solution : build uncorrelated references

Conditioned Spectral Analysis [Bendat, 1976]



Partial coherences : coherence between conditioned references and outputs

Virtual Source Analysis [Price&Bernhard, 1986]



Virtual coherences : coherence between virtual references and outputs

└─ **Multiple coherence** : coherence between several references and outputs ─┘

MIMO case : the multiple coherence

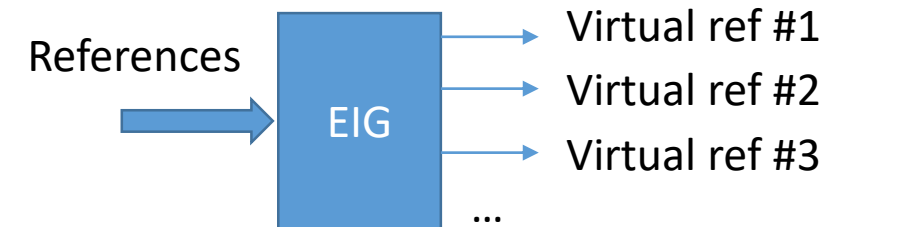
- Solution : build uncorrelated references

Easy with VSA : the multiple coherence is simply the sum of virtual coherences :

Multiple coherence $\rightarrow \Gamma_y^2 = \sum_{v=1}^n \gamma_{vy}^2 \leftarrow$ Virtual coherences

Under the H_0 hypothesis that references are incoherent with outputs, the virtual coherence follows the same probability law as the single coherence

Virtual Source Analysis [Price&Bernhard, 1986]



Virtual coherences : coherence between virtual references and outputs

MIMO case : the multiple coherence

- Solution : build uncorrelated references

Easy with VSA : the multiple coherence is simply the sum of virtual coherences :

Multiple coherence $\rightarrow \Gamma_y^2 = \sum_{v=1}^n \gamma_{vy}^2 \leftarrow$ Virtual coherences

Under the H_0 hypothesis that references are incoherent with outputs, the virtual coherence follows the same probability law as the single coherence

\rightarrow definition of the **Thresholded Multiple Coherence (TMC)**:

$$\Gamma_{y,\alpha}^2 = \sum_{v=1}^n \mathbf{1}_{[\gamma_{vy}^2 > T(\alpha, N)]} \cdot \gamma_{vy}^2$$

MIMO case : the multiple coherence

- Statistical behavior of the classical MC under the H0 Hypothesis

Virtual coherences are not independent random variables

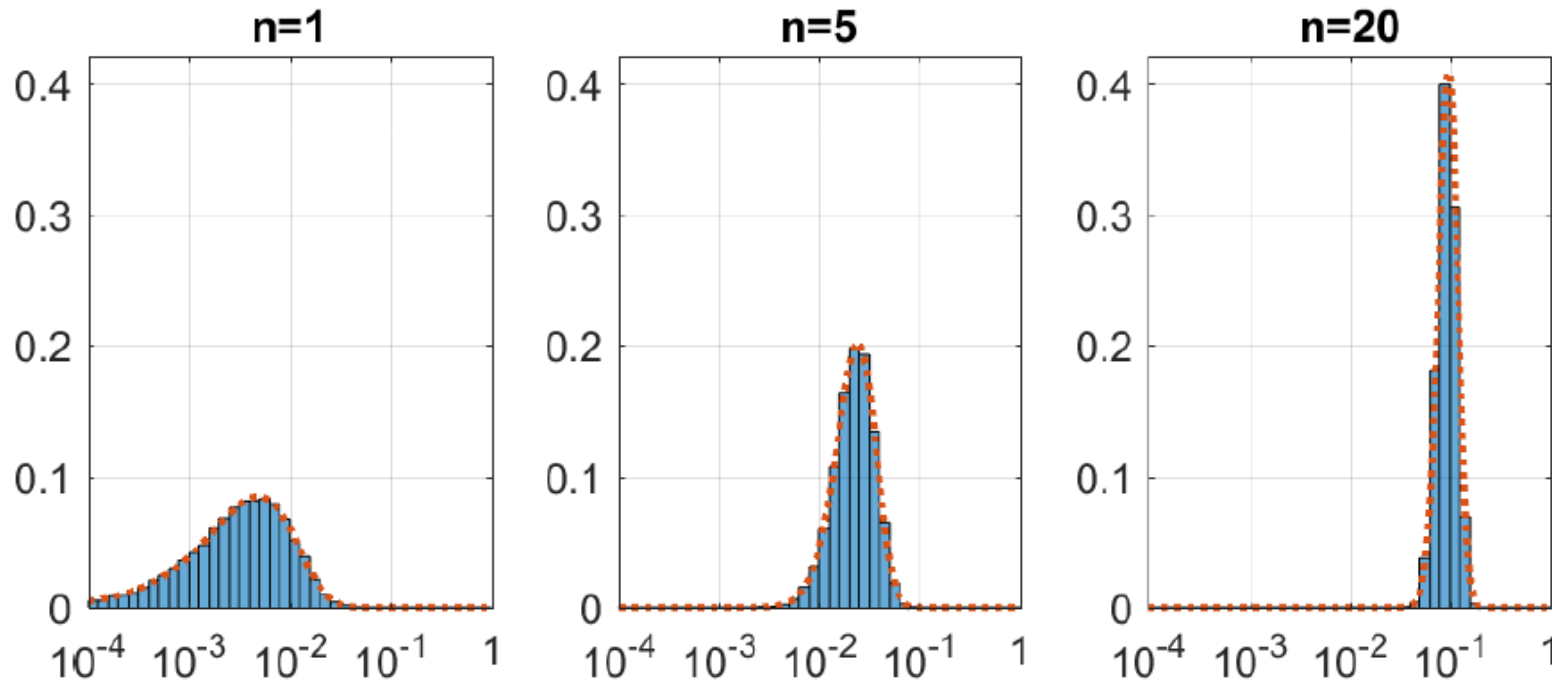
However, when N gets large, virtual coherences gets almost independent

$$\left. \begin{aligned} \Gamma_y^2 &= \sum_{v=1}^n \gamma_{vy}^2 \\ E[\gamma_{xy}^2] &= \frac{1}{N} \quad V[\gamma_{xy}^2] = \frac{1}{N^2} \frac{N-1}{N+1} \end{aligned} \right\} \Rightarrow \begin{aligned} &\text{Expected value and variance} \\ E(\Gamma_y^2) &\approx \frac{n}{N} \quad V(\Gamma_y^2) \approx \frac{n}{N^2} \end{aligned}$$

n= 1 references	; N=200 snapshots	$E(\Gamma^2) \approx 0.5\%$, std(Γ^2)=0.5%
n= 5 references	; N=200 snapshots	$E(\Gamma^2) \approx 2.5\%$, std(Γ^2)=1%
n=20 references	; N=200 snapshots	$E(\Gamma^2) \approx 10\%$, std(Γ^2)=2%

MIMO case : the multiple coherence

- Statistical behavior of the classical MC under the H0 Hypothesis



$n=1$ references	; N=200 snapshots	$E(\Gamma^2) \approx 0.5\%$, $\text{std}(\Gamma^2)=0.5\%$
$n=5$ references	; N=200 snapshots	$E(\Gamma^2) \approx 2.5\%$, $\text{std}(\Gamma^2)=1\%$
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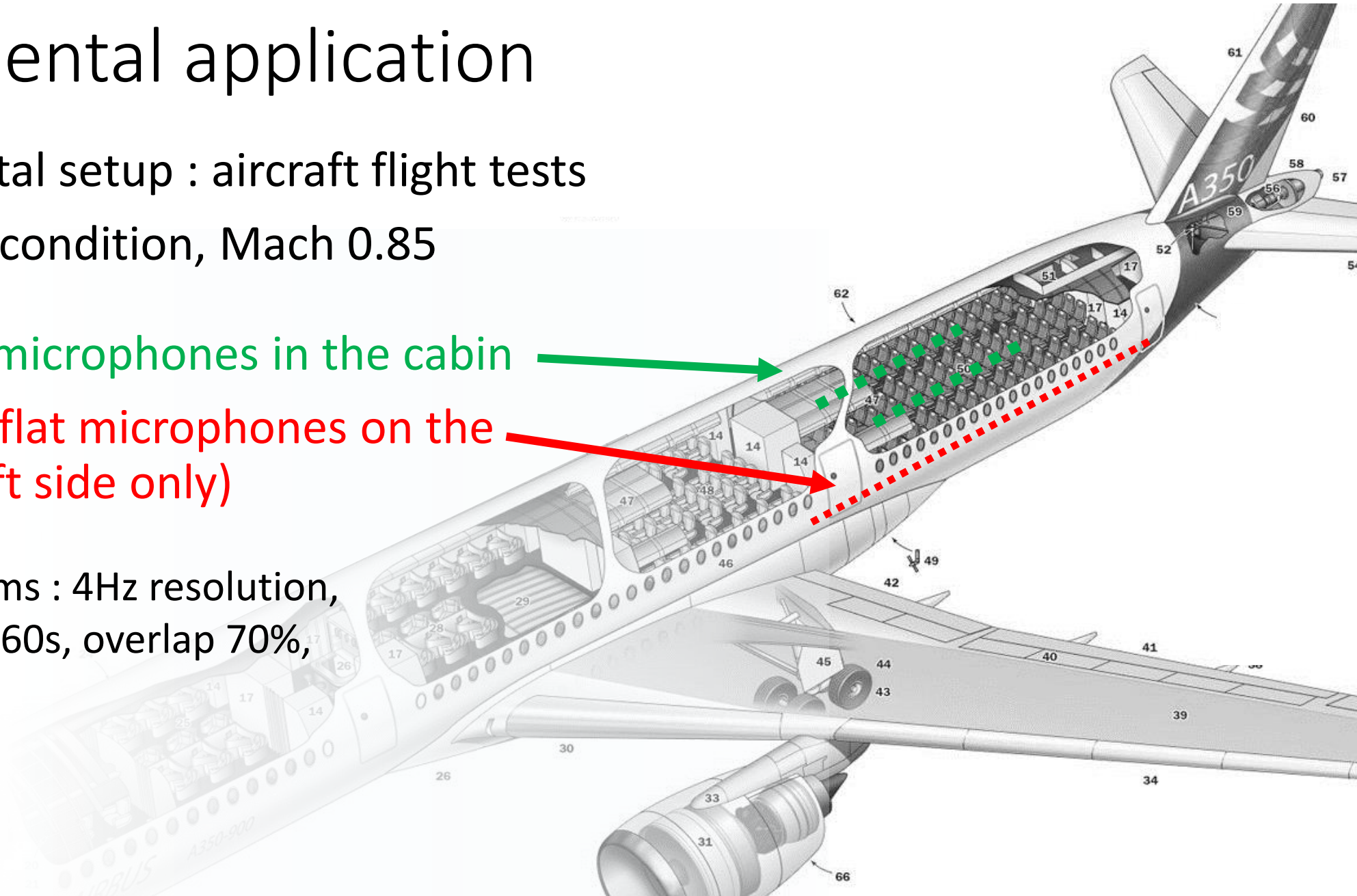
Experimental application

- Experimental setup : aircraft flight tests
Cruise flight condition, Mach 0.85

14 internal microphones in the cabin

35 external flat microphones on the fuselage (left side only)

P-Welch params : 4Hz resolution,
record length 60s, overlap 70%,
 $N_0=240$



Experimental application

Results are presented « averaged » on outputs :

→ Averaged Coherent Output Power

$$COP_{avg} = \frac{1}{m} \sum_{j=1}^m \Gamma_j^2 \cdot S_j$$

→ Global Multiple coherence

$$\Gamma_{glob}^2 = \frac{\sum_{j=1}^m \Gamma_j^2 \cdot S_j}{\sum_{j=1}^m S_j}$$

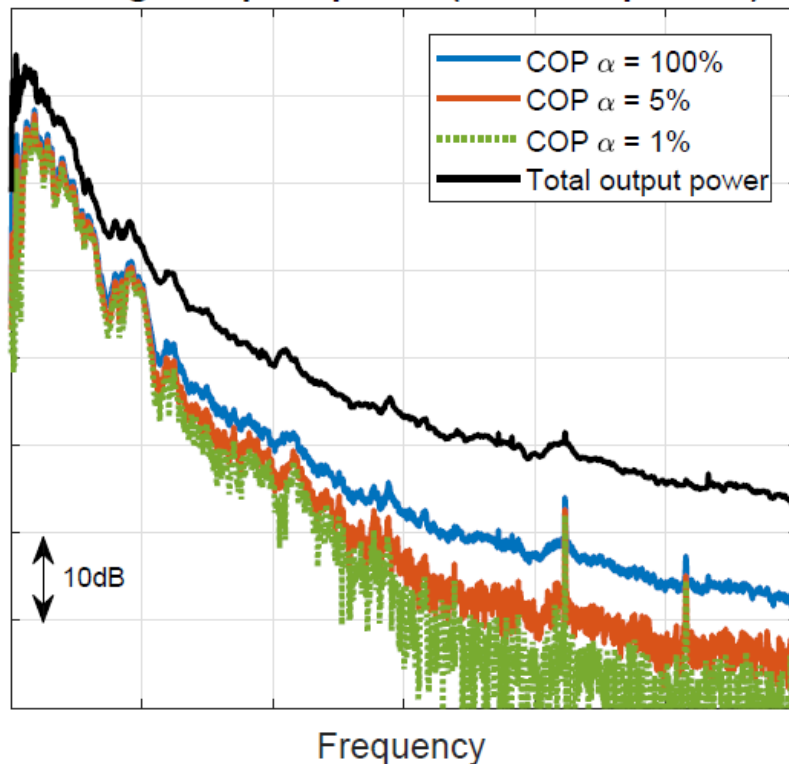
NB : Coherent Output Power (output j) : $COP_j = \Gamma_j^2 \cdot S_j$

Experimental application

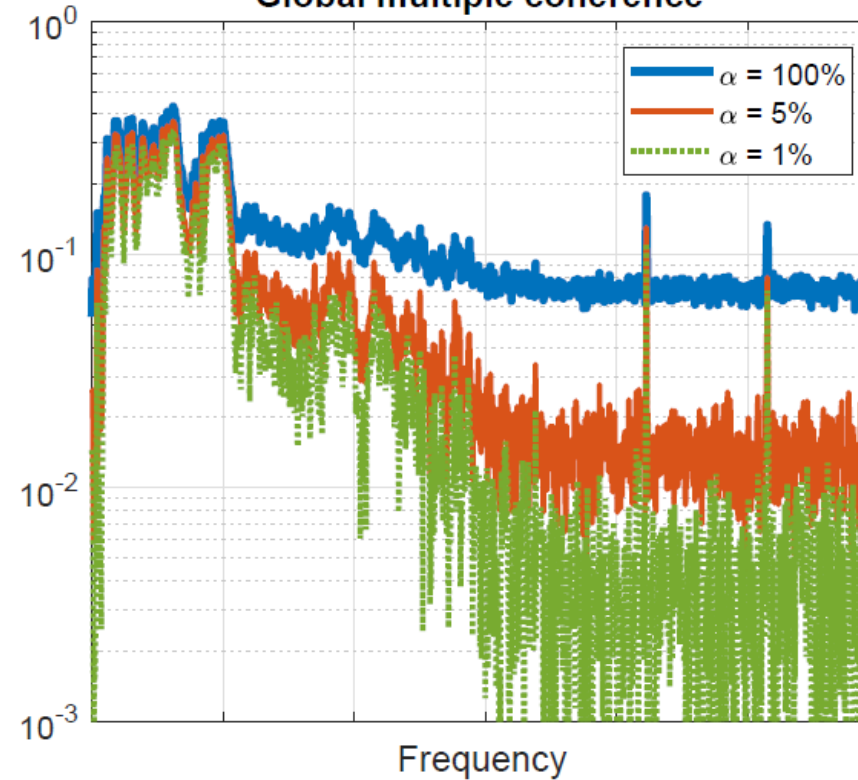
- References : **external microphones**

Contribution of external noise sources to the interior noise

Average Output Spectra (int. microphones)



Global multiple coherence



$\alpha=100\%$ correspond to the classical multiple coherence

$$\frac{n}{N} = \frac{35}{240 \times 2.07} = 7\%$$

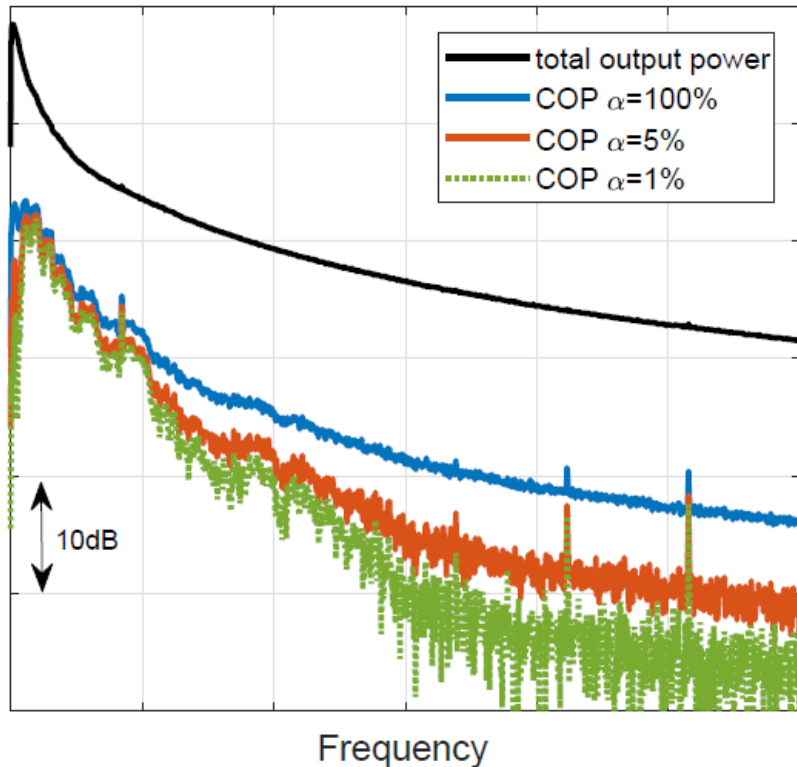
$\alpha=5, 1\%$ increases the dynamic of the Thresholded Multiple Coherence

Experimental application

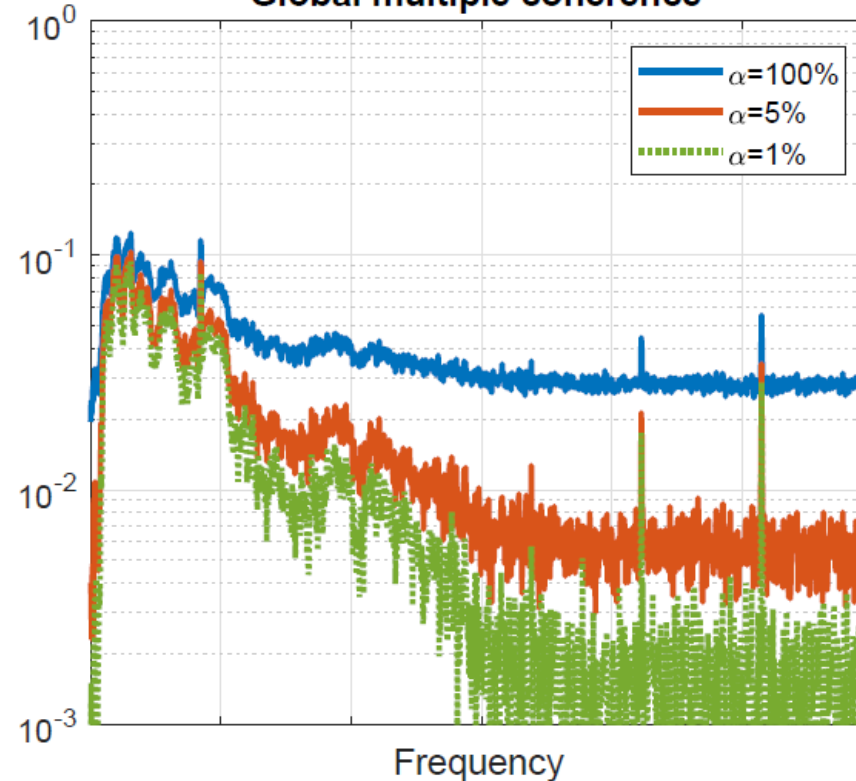
- References : **internal microphones**

Removing flow noise from external fuselage microphones

Average Output Spectra (ext. microphones)



Global multiple coherence



$\alpha=100\%$ correspond to the classical multiple coherence

$$\frac{n}{N} = \frac{14}{240 \times 2.07} = 3\%$$

Conclusion

The Thresholded coherence allows to determine the significance of the linear relationship between two signals

The concept has been extended in this work to the MIMO case with the Thresholded Multiple Coherence (TMC) with the following benefits:

- possibility of considering numerous references without the analysis limits of the standard multiple coherence
- Strong improvement of the dynamic

The risk α (false positive) remains a parameter to be selected. It could be adjusted as a function of a β risk (false negative)

This work was performed in the framework of Clean Sky 2 Joint Undertaking, European Union (EU), Horizon 2020, CS2-RIA, ADAPT project, Grant agreement no 754881.