A Nominal Model for Vehicle Dynamics and Estimation of Input Forces and Tire Friction

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Abstract

In this paper a 16 DoF vehicle model is developped and discussed. Then some parial models are considered and justified for the design of robust estiamtors using sliding mode approach in order to identify the tire-road friction or input variables.

Index Terms

Vehicle dynamics, Sliding Modes observer, Robust nonlinear observers, tire forces estimation.

I. INTRODUCTION

In recent years, increase of safety demand in vehicles motivated research and developement in the field of active safety. More and more new safety systems are installed on vehicle for real-time monitoring and controlling the dynamic stability (EBS, ABS, ESP). Car accidents may occur for several reasons which involve either the driver or vehicle components or environment. One of the important factors determining vehicle dynamics including safety is road friction and evolution of the contact forces. Thus for vehicles and road safety analysis, it is necessary to take into account the contact force characteristics. However, tire forces and road friction are difficult to measure directly. In literature, their values are often deduced by some experimentally approximated models[2][9][1]. Tire forces are represented by nonlinear functions of wheel slip.Generally the partial and approximated models used are not fully justified and their validity is often limited.This makes the forces and parameters difficult to estimate on line for vehicle control applications and detection and diagnosis for driving monitoring and surveillance. In this work we try to highlight approximations made and give details allowing to evaluate what is really neglected. Robust observers looking forward are based on the physics of interacting systems (the vehicle, the driver and the road).

Recently, many analytical and experimental studies have been performed on estimation of the frictions and contact forces between tires and road [3][4]. In [7][4], application of sliding mode control is proposed. Observers based on the sliding mode approach have been also used in [8]. In [5] an estimation based using the least squares method and Kalman filtering is applied for estimation of contact forces. Gustafsson in [1] presented a tire/road friction estimation methode based on Kalman filter to give a relevant estimates of the slope of μ versus slip (λ), that is, the relative differnence in wheel velocity. Carlson in [9] presented an estimator for longitudinal stiffness and wheel effective raduis using stok vehicle sensors and GPS for low values of slip. Robust observers with unknown inputs are efficient for estimation of road profile and for estimation of the contact forces [8]. Robut observers with unknown inputs have been shown to be efficient for estimation of road profile [12][**?**] and for estimation of the contact forces. Tracking and braking control reduce wheel slip. This can be done also by means of its regulation while using sliding mode approach for observation and control.[3]. This enhances the road safety leading better vehicle adherence and maneuvers ability. The vehicle controllability in its environment along the road admissible trajectories remain an important open research problem [6].

In this paper, modelling of the contact forces and interactions between a vehicle and road is revisited in the objective of on line force estimation by means of robust observers coupled with a robust and adaptive estimation for contact forces. We propose an observer to estimate the vehicle state and an adaptive estimator for tire forces identification. The designed observer is based on the sliding mode approach. The main contribution is the on-line estimation of the tire force needed for control.

We focus our work, as presented in this paper, first on modeling and second on on-line estimation of the tires forces. We estimate the vehicle state and identify tire forces. The main contribution is the emphisize of the rational behind partial approximated models and the on-line estimation of the tire force needed for control. Tire forces can be represented by the nonlinear (stochastic) functions of wheel slip. The deterministic tire models encountered are complex and depend on several factors (as load, tire pressure, environmental characteristics, etc.). The proposed estimation procedure has to be robust enough to avoid model complexity. It can then be used to detect some critical driving situations in order to improve the security. This approach can be used also in several vehicle control systems such as Anti-look Brake Systems (ABS), traction control system (TCS), diagnosis systems, etc...

The estimations are produced using only the angular wheel position as measurement by the specially designed robust observer based on the super-twisting second-order sliding mode. The proposed method of estimation is verified through one- wheel simulation model with a "Magic formula" tire model and then application results (on a Peugeot 406) show an excellent reconstruction of the velocities, tire forces and radius estimation.

II. VEHICLE MODELING

1) Mechanical Models: 4 DOF system: In order to give an idea for the system modeling, we start with a simple example. In this part we develop a 4 Degrees Of Freedom (DOF) model. This means that we consider only the vertical motion (along Oz axis) and the 3 rotations. Let us consider for simplicity a table moving in 4 DOF, made of a rigid body (see figure **??**) with a mass M, as length L and wide l. The table thikness is h. We first consider the reference frame R_1 attached to the table and the absolute reference R_0.

The table is only excited by 4 forces F_1, F_2, F_3 and F_4 du to the springs attached at the points:P_{FT1} /R_1.; P_{FT2} /R_1 for the rear right and left and P_{FT3} /R_1. P_{FT4} /R_1 for the front right and left respectively. Each point is refered by its cartesian coordinates (x, y, z) (distances to the centre of gravity (CDG) / relative to R_1). There are no lateral nor longitudinal forces $F_5 = F_6 = 0$. The inertia tensor is $I = diag\,(Ixx, Iyy, Izz)$. The table model is obtained applying the Lagrange formalism, considering the generalized coordinate vector $q = [z, \psi, \varphi, \theta]^T \in \Re^4$ and Γ_a the external forces vector.

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + V(q, \dot{q}) = \Gamma_a \tag{1}$$

z is the vertical displacement of the gravity center, ψ the roll angle, φ the pitch angle, θ the yaw angle. $\dot{q}$ is the velocity vector and $\ddot{q}$ the accelerations one. The inertia matrix is $M(q)$ and $C(q, \dot{q})$ represent the coriolis and centrifugal effects. $G(q) = [gM, 0, 0, 0]^T$ is the gravitation effect. Frictions can be considered and are noted $V(q, \dot{q})$.The system is passive, then we have the following property: $\dot{M}(q) - 2C(q, \dot{q})$ is an antisymétric matrix.

The external forces applied to the table are expressed in the cartesian space (F_{Ti} : longitudinal force, F_{Li} : lateral force, F_{Ni} : normal force): $F = [F_{T1}, F_{L1}, F_{N1}, F_{T2}, F_{L2}, F_{N2}, F_{T3}, F_{L3}, F_{N3}, F_{T4}, F_{L4}, F_{N4}]^T$. The transversal and lateral exerted forces are assumed nul.

$$\Gamma_a = J^T(q)F \tag{2}$$

with $J(q)$ the (12×4) jacobian matrix defined at the 4 contact points. This describes the fact that contact points are the connections betwen environment (in the absolute reference frame) and the system's body (in its refernce frame).

The applied forces vector is then reduced to only normal force, for simplicity of presentation (this will not be the case of the vehicle, here we have then $F = [F_{N1}, F_{N2}, F_{N3}, F_{N4}]^T$ and then

$$J^T(q) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ J_{3,3} & J_{3,6} & J_{3,9} & J_{3,12} \\ J_{4,3} & J_{4,6} & J_{4,9} & J_{4,12} \end{bmatrix} \tag{3}$$

$$M(q) = \begin{bmatrix} M & 0 & 0 & 0 \\ 0 & M_{2,2} & M_{2,3} & M_{2,4} \\ 0 & M_{3,2} & M_{3,3} & 0 \\ 0 & M_{4,2} & 0 & M_{4,4} \end{bmatrix} \tag{4}$$

Please note that the inertia matrix is not diagonal despite considering only the chassis alone and that different frames must be considered with geometric and kinematic models relating cartesian and operational spaces.

a) The Coriolis and centrifugal effects matrix $C(q,\dot{q})$::

$$C(q,\dot{q}) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{2,2} & C_{2,3} & C_{2,4} \\ 0 & C_{3,2} & C_{3,3} & C_{3,4} \\ 0 & C_{4,2} & C_{4,3} & 0 \end{bmatrix}$$

With as exemple we give only diagonal coefficients:

$$\begin{aligned} C_{2,2} = {} & 1/8\,\dot{\varphi}\,Iyy\,\sin(2\,\varphi{+}2\,\theta) + 1/8\,\dot{\varphi}\,Iyy\,\sin(2\,\varphi{-}2\,\theta) - 1/4\,\dot{\varphi}\,Iyy\,\sin(2\,\varphi) \\ & -1/8\,\dot{\varphi}\,Izz\,\sin(2\,\varphi{+}2\,\theta) - 1/8\,\dot{\varphi}\,Izz\,\sin(2\,\varphi{-}2\,\theta) - 1/4\,\dot{\varphi}\,Izz\,\sin(2\,\varphi) \\ & +1/2\,\dot{\varphi}\,Ixx\,\sin(2\,\varphi) + 1/8\,\dot{\theta}\,Iyy\,\sin(2\,\varphi{+}2\,\theta) - 1/8\,\dot{\theta}\,Iyy\,\sin(2\,\varphi{-}2\,\theta) \\ & +1/4\,\dot{\theta}\,Iyy\,\sin(2\,\theta) - 1/8\dot{\theta}\,Izz\,\sin(2\,\varphi{+}2\,\theta) + 1/8\,\dot{\theta}\,Izz\,\sin(2\,\varphi{-}2\,\theta) \\ & -1/4\,\dot{\theta}\,Izz\,\sin(2\,\theta) \end{aligned}$$

$$C_{3,3} = -1/2\,\sin(2\,\theta)\,(Iyy - Izz)\dot{\theta}$$

b) The Transpose Jacobian matrix:: The applied forces vector is then reduced to $F = [F_{N1}, F_{N2}, F_{N3}, F_{N4}]^T$

$$J = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ J_{2,1} & J_{2,2} & 0 & J_{2,4} & J_{2,5} & 0 & J_{2,7} & J_{2,8} & 0 & J_{2,10} & J_{2,11} & 0 \\ J_{3,1} & J_{3,2} & J_{3,3} & J_{3,4} & J_{3,5} & J_{3,6} & J_{3,7} & J_{3,8} & J_{3,9} & J_{3,10} & J_{3,11} & J_{3,12} \\ J_{4,1} & J_{4,2} & J_{4,3} & J_{4,4} & J_{4,5} & J_{4,6} & J_{4,7} & J_{4,8} & J_{4,9} & J_{4,10} & J_{4,11} & J_{4,12} \end{bmatrix}$$

This 4 DOF model gives only an idea of how much complex is the nominal model of vehicle dynamics. In what follows we restart the procédure to compute the vehicle model assuming it is composed with a body (chassis) with 6 DOF and four wheels attached to this body by four suspensions.

A. Complete 16 DoF model

In litterature, many studies deal with vehicle modelling [10][11]. This kind of systems are complex and nonlinear composed with many coupled subsystems: wheels, motor and system of braking, suspensions, steering, more and more in borad and inbedded electronics. Let us represent the vehicle (like eg a Peugeot 406) by the scheme of figure 1 and define the following notations.

ftbpFU3.2113in1.8157in0ptVehicle dynamics and reference framesVehiiclvehicl.jpg

The vehicle body receives as excitations external forces and moments following the three axes: - Longitudinal, - Lateral, - Vertical. These come from interaction of the wheels and road, from perturbations (wind for example), gravity and vehicle drive line. Let us consider the basic reference fixed frame R. Whe can consider the vehicle as made of 5 sub-systems: chassis whit 6 DOF and then 4 wheels with their suspensions. Each of the rear wheels has 2 DoF. The front ones are driven wheels with 3 DoF each. Then we have 16 DoF. Let the generalized variables be in the vector $q \in R^{16}$, defined as

$$q^T = [x, y, z, \theta_z, \theta_y, \theta_x, z_1, z_2, z_3, z_4, \delta_3, \delta_4, \varphi_1, \varphi_2, \varphi_3, \varphi_4]$$

where x, y, et z represent displacements in longitudinal, laterl and vertical direction. angles of roll, pitch and yaw are θ_x, θ_y et θ_z respectively. The suspensions elongations are noted z_i: ($i = 1..4$). δ_i: stands for the steering angles (for wheels numbered as $i = 3, 4$), finally φ_i: are angles wheels rotations ($i = 1..4.$). Vectors $\dot{q}, \ddot{q} \in R^{16}$ are respectively velocities and corresponding accelerations. $M(q)$ is the inertia matrix and $C(q,\dot{q})\dot{q}$ are coriolis and centrifugal forces. The gravity term is G . Suspensions forces are $V(q,\dot{q}) = K_v\dot{q} + K_p q$ with respevtively damping and stifness matrices K_v , K_p. We can define as dynamic equations of the vehicle by applying the principles fundamental of the dynamics (see [13]):

$$\Gamma + J^T F = M\ddot{q} + C(q,\dot{q})\dot{q} + Kq + G \tag{5}$$

with as parameters only to give an idea

$$M = \begin{bmatrix} \bar{M}_{1,1} & \bar{M}_{1,2} & \bar{M}_{1,3} & 0 & 0 \\ \bar{M}_{2,1} & \bar{M}_{2,2} & \bar{M}_{2,3} & \bar{M}_{2,4} & \bar{M}_{2,5} \\ \bar{M}_{3,1} & \bar{M}_{3,2} & \bar{M}_{3,3} & 0 & 0 \\ 0 & \bar{M}_{4,2} & 0 & \bar{M}_{4,4} & 0 \\ 0 & \bar{M}_{5,2} & 0 & 0 & \bar{M}_{5,5} \end{bmatrix}$$

$$C=\begin{bmatrix} 0 & \bar{C}_{12} & \bar{C}_{13} & 0 & 0 \\ 0 & \bar{C}_{22} & \bar{C}_{23} & \bar{C}_{24} & \bar{C}_{25} \\ 0 & \bar{C}_{32} & \bar{C}_{33} & 0 & 0 \\ 0 & \bar{C}_{42} & 0 & 0 & 0 \\ 0 & \bar{C}_{52} & 0 & 0 & \bar{C}_{55} \end{bmatrix}$$

and $J^T = \begin{bmatrix} J_{1,1} & J_{1,2} & J_{1,3} & J_{1,4} \\ J_{2,1} & J_{2,2} & J_{2,3} & J_{2,4} \\ J_{3,1} & J_{3,2} & J_{3,3} & 0 \\ 0 & 0 & 0 & J_{4,4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

This is just to show that we can decompose our system as coupled subsystems. Let us say five coupled subsystems, that we have considered in our previous works. This has been computed using a symbolic computation software considering 16 generalized variables: 6 for position and orientation of body, 4 as suspensions ones, 2 for front wheels steering and 4 as wheels rotations. The matrices M, C and K are of dimensions 16×16. F is input forces vector acting on wheels, it has 12 components(3 forces (longitudinal, lateral and normal) $\times$ 4 wheels), Γ represent extra inputs for perturbations.. In the following application this model has been reduced and simplified assuming as nominal behaviour a normal driving situation [6].

B. Coupled sub models

We can split the previous model, without approximations, in five parts as follows. This leads us to Rotations and orientation motions of the body:

$$\begin{bmatrix} J_{4F} \\ J_{5F} \\ J_{6F} \end{bmatrix} = \bar{M}_{12}^T \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \bar{M}_{22} \begin{bmatrix} \ddot{q}_4 \\ \ddot{q}_5 \\ \ddot{q}_6 \end{bmatrix} +$$

$$+\bar{M}_{23} \begin{bmatrix} \ddot{q}_{11} \\ \ddot{q}_{21} \\ \ddot{q}_{31} \\ \ddot{q}_{41} \end{bmatrix} + \bar{M}_{24} \begin{bmatrix} \ddot{q}_{12} \\ \ddot{q}_{22} \\ \ddot{q}_{32} \end{bmatrix}$$

$$+\bar{M}_{25} \begin{bmatrix} \ddot{q}_{42} \\ \ddot{q}_{33} \\ \ddot{q}_{43} \end{bmatrix} + \bar{C}_{22} \begin{bmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} + \bar{C}_{23} \begin{bmatrix} \dot{q}_{11} \\ \dot{q}_{21} \\ \dot{q}_{31} \\ \dot{q}_{41} \end{bmatrix} +$$

$$+\bar{C}_{24} \begin{bmatrix} \dot{q}_{12} \\ \dot{q}_{22} \\ \dot{q}_{32} \end{bmatrix} + \bar{C}_{25} \begin{bmatrix} \dot{q}_{42} \\ \dot{q}_{33} \\ \dot{q}_{43} \end{bmatrix}$$

The body's translations dynamics is given by:

$$\begin{bmatrix} F_{LT} \\ F_{TT} \\ F_{NT} \end{bmatrix} = \bar{M}_{11} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} + \bar{M}_{12} \begin{bmatrix} \ddot{q}_4 \\ \ddot{q}_5 \\ \ddot{q}_6 \end{bmatrix} + \tag{6}$$

$$+\bar{M}_{13} \begin{bmatrix} \ddot{q}_{11} \\ \ddot{q}_{21} \\ \ddot{q}_{31} \\ \ddot{q}_{41} \end{bmatrix} + \bar{C}_{12} \begin{bmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} \tag{7}$$

Suspensions dynamics is written:

$$\begin{bmatrix} J_{7F} \\ J_{8F} \\ J_{9F} \\ J_{10F} \end{bmatrix} = \bar{M}_{1,3} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \bar{M}_{2,3} \begin{bmatrix} \ddot{q}_4 \\ \ddot{q}_5 \\ \ddot{q}_6 \end{bmatrix} +$$

$$+\bar{M}_{3,3}\begin{bmatrix}\ddot{q}_{11}\\ \ddot{q}_{21}\\ \ddot{q}_{31}\\ \ddot{q}_{41}\end{bmatrix}+\bar{C}_{32}\begin{bmatrix}\dot{q}_{4}\\ \dot{q}_{5}\\ \dot{q}_{6}\end{bmatrix}+\bar{C}_{33}\begin{bmatrix}\dot{q}_{11}\\ \dot{q}_{21}\\ \dot{q}_{31}\\ \dot{q}_{41}\end{bmatrix}+$$
$$+\begin{bmatrix}R_{11} & & & \\ & R_{21} & & \\ & & R_{31} & \\ & & & R_{41}\end{bmatrix}\begin{bmatrix}q_{11}\\ q_{21}\\ q_{31}\\ q_{41}\end{bmatrix}+\begin{bmatrix}G_7\\ G_8\\ G_9\\ G_{10}\end{bmatrix}$$

The previous 16 DoF model is then equivalent to the following one:

$$\begin{bmatrix}F_T\\ F_2\\ F_3\\ 0\\ U_5\end{bmatrix}=M\begin{bmatrix}\ddot{\xi}_1\\ \ddot{\xi}_2\\ \ddot{\xi}_3\\ \ddot{\xi}_4\\ \ddot{\xi}_5\end{bmatrix}+C\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_2\\ \dot{\xi}_3\\ \dot{\xi}_4\\ \dot{\xi}_5\end{bmatrix}+\begin{bmatrix}0\\ 0\\ R_{33}\xi_3\\ 0\\ 0\end{bmatrix}+\begin{bmatrix}G_1\\ G_2\\ G_3\\ 0\\ 0\end{bmatrix}$$

In the last expression, we can remark that splitting the model can be realized and this model is helpful, when using reduced models, to identify what is neglected regard to our proposed nominal model with 16 DoF. The dynamic equations can be reduced, in case where we assume that motion is normal driving in a normal straith road, to translations and rotations of the body, and wheels plus supension motions. For translations we find often in literature:

$$\begin{array}{l} m\dot{v}_x=\sum F_x\\ m\dot{v}_y=\sum F_y\\ m\dot{v}_z=\sum F_z\end{array}\tag{8}$$

where m is the total mass of the vehicle and $v=[v_x,v_y,v_z]^T$ describe the vehicle velocities along x, y, z. In the left hand side of this approximate model are the forces $\sum F_x$, $\sum F_y$ and $\sum F_z$ applied in directions of x, y and z and the balance of the moments ($\sum M_x, \sum M_y, \sum M_z$), give rotations following the three directions x, y and z, is given by:

$$J\begin{bmatrix}\ddot{\theta}\\ \ddot{\phi}\\ \ddot{\psi}\end{bmatrix}=\begin{bmatrix}\sum M_x\\ \sum M_y\\ \sum M_z\end{bmatrix}\tag{9}$$

The wheel angular motions can be written:

$$\begin{aligned}\dot{\omega}_{fl} &= \frac{1}{I_\omega}(C_{fl}-R_\omega F_{xf1})\\ \dot{\omega}_{fr} &= \frac{1}{I_\omega}(C_{fr}-R_\omega F_{xf2})\\ \dot{\omega}_{rl} &= \frac{1}{I_\omega}(C_{rl}-R_\omega F_{xr1})\\ \dot{\omega}_{rr} &= \frac{1}{I_\omega}(C_{rr}-R_\omega F_{xr2})\end{aligned}\tag{10}$$

with ω_f and ω_r are the rotation velocities of the front and rear wheel, C_{mi} is the motor couple applied at wheel i and T_i is the braking couple applied at wheel i. Let r_1 be the distance between the center of gravity and the front axis and r_2 the distance between the center of gravity and the rear axis.

C. Partial models

The complete model is difficult to use in control applications. It involves several variables which are not available for measurement or not observable. The most part of applications deal with simplified and partial models. Let us consider, for our robust observer, the simplified motion dynamics of a quarter-vehicle model, capturing only nominal behavior [3] [6]. This model retains the main characteristics useful for the longitudinal dynamic. For a global application, this method can be easily extended to the complete vehicle and involve the four coupled wheels. The amount of neglected parts in the modeling can be considered to evaluate robustness of proposed estimators.

Applying Newton's law to one isolated wheel gives:.

$$\begin{aligned} m\dot{v}_x &= F_x \\ J_r\dot{\omega} &= T - rF_x \end{aligned}$$

where m is the vehicle mass and J_r, r are the inertia and effective radius of the tire respectively. v_s is the linear velocity of the vehicle, ω is the angular velocity of the considered wheel. T is the accelerating (or braking) torque, and F_x is the tire/road friction force. The tractive (respectively braking) force, produced at the tire/road interface when a driving (braking) torque is applied to pneumatic tire, has opposite direction of relative motion between the tire and road surface. This relative motion exhibits the tire slip properties. The wheel-slip is due to deflection in the contact patch. The longitudinal wheel slip is generally called the slip ratio and is described by a kinematic relation as [9].

$$\lambda = \frac{|v_r - v_x|}{\max(v_r, v_x)} \tag{11}$$

where v_r is the wheel velocity. Representing the adhesion coefficient as a function of the wheel slip yields the adhesion characteristic $\mu(\lambda)$, which depends on the road surfaces as shown in the following figure 2.ftbphFU3.5959in2.0773in0ptW slipWheel slipimfqzl00.wmf

The figure 2 shows the relations between coefficient of road adhesion μ and longitudinal slip λ for different road surface conditions. It can be observed that all curves $\mu(\lambda)$ start at $\mu = 0$ for zero slip, which corresponds to the non-braking and non accelerating, free rolling wheel. With a linear increasing slip ratio from 3% to 20%. Beyond this maximum value the slope of the adhesion characteristic is maximum and then slope becomes negative. At a slip ratio of 100% the wheel is completely skidding, which corresponds to the locking of the wheel. The adhesion characteristic plays an essential role for both the design and the validation of ABS. Overall, to improve the performance of an ABS it is desirable to have some real-time information about the adhesion characteristic.

By assuming that the longitudinal forces are proportional to the transversal ones, we can expressed theses forces as follows, where F_z is the vertical force of the wheel.

$$F_x = \mu F_z \tag{12}$$

The verticla forces that we use in our model are function of the longitudinal acceleration and the height of the center of gravity. The vertical force can be represented as:

$$Fz = \frac{m}{2(l_f + l_r)}(gl_r - h\dot{v}_x) \tag{13}$$

where h is the height of the center of gravity, l_f is the distance between the center of gravity and the front axis center of gravityand l_r is the distance between the center of gravity and the rear axis center of gravity.

III. OBSERVER DESIGN

The sliding mode technique is an attractive approach [14]. The primary characterestic of SMC is that the feedback signal is discontinuous, switching on one or several manifolds in the state-space. In what follows, we develop a second order differentiator in order to obtain estimates of the tire road friction.

A. *High Order Sliding Mode Observer (HOSM)*

In this part we will use a Robust Differentiation Estimator (RDE) to deduce our estimations. Consider a smooth dynamics function, $s(x) \in \mathbb{R}$. The system containing this variable may be closed by some possibly-dynamical discontinuous feedback where the control task may be to keep the output $s(x(t)) = 0$. Then provided that successive total time derivatives $s, \dot{s}, \ddot{s}...s^{(r-1)}$ are continuous functions of the closed system state space variables, and the t-sliding point set is non-empty and consist locally of Filippov trajectories.

$$s = \dot{s} = \ddot{s} = ... = s^{(r-1)} = 0 \tag{14}$$

is non-empty and consists locally of Filippov trajectories. The motion on set [16][17] is called r-sliding mode (rth-order sliding mode) [18][19].

The HOSM dynamics converge toward the origin of surface coordinates in finite time always that the order of the sliding controller is equal or bigger than the sum of a relative degree of the plant and the actuator. To estimate the derivatives s_1 and s_2 without its direct calculations of derivatives, we will use the 2^{nd}-order exact robust differentiator of the form [19]

$$\begin{aligned} \dot{z}_0 &= v_0 = z_1 - \lambda_0 \left|z_0 - s_\omega\right|^{\frac{2}{3}} sign(z_0 - s_\omega) \\ \dot{z}_1 &= v_1 = -\lambda_1 sign(z_1 - v_0)^{\frac{1}{2}} sign(z_1 - v_0) + z_2 \\ \dot{z}_2 &= -\lambda_2 sign(z_2 - v_1) \end{aligned}$$

where z_0, z_1 and z_2 are the estimate of s_ω, s_1 and s_2, respectively, $\lambda_i > 0$, $i = 0, 1, 2$. Under condition $\lambda_0 > \lambda_1 > \lambda_2$ the third order sliding mode motion will be established in a finite time. The obtained estimates are $z_1 = s_1 = \dot{s}_\omega$ and $z_2 = s_2 = \ddot{s}_\omega$ then they can be used in the estimation of the state variables and also in the control.

B. Cascaded Observers - Estimators

In this section we use the previous approach to build an estimation scheme allowing to identify the tire road friction. The estimations will be produced in three steps, as cascaded observers and estimator, reconstruction of informations and system states step by step. This approach allow us to avoid the observability problems dealing with inappropriate use of the complete modeling equations. For vehicle systems it is very hard to build up a complete and appropriate model for global observation of all the system states in one step. Thus in our work, we avoid this problem by means of use of simple and cascaded models suitable for robust observers design.

The first step produces estimations of velocities. The second one estimate the tire forces (vertical and longitudinal ones) and the last step reconstruct the friction coefficient.

The robust differentiation observer is used for estimation of the velocities and accelerations of the wheels. The wheels angular positions and the velocity of the vehicles body v_x, are assumed available for measurements. The previous Robust Estimator is useful for retrieval of the velocities and accelerations.

1^{st} Step:

$$\begin{aligned} \dot{\widehat{\theta}} &= v_0 = \widehat{\omega} - \lambda_0 \left|\theta - \widehat{\theta}\right|^{\frac{2}{3}} sign(\theta - \widehat{\theta}) \\ \dot{\widehat{\omega}} &= v_1 = \dot{\widehat{\omega}} - \lambda_1 sign(\widehat{\omega} - v_0)^{\frac{1}{2}} sign(\widehat{\omega} - v_0) \\ \ddot{\widehat{\omega}} &= -\lambda_2 sign(\dot{\widehat{\omega}} - v_1) \end{aligned}$$

The convergence of these estimates is garanteed in finite time t_0.

2^{nd} Step: In the second step we can estimate the forces F_x and F_z. Then to estimate F_x we use the following equation,

$$J\dot{\widehat{\omega}} = T - R_{ef}\widehat{F}_x \tag{15}$$

In the simplest way, assuming the input torques known, we can reconstract F_x as follows:

$$\widehat{F}_x = \frac{(T - J\dot{\widehat{\omega}})}{R_{ef}} \tag{16}$$

$\dot{\widehat{\omega}}$ is produced by the Robust Estimator (RE). Note that any estimator with output error can also be used to enhance robustness versus noise. In our work, in progress actually, the torque T wil be also estimated by means of use of additional equation from engine behaviour related to accelrating inputs.

After those estimations, their use in the same time with the system equations allow us to retrieve de vertical forces F_z as follows. To estimate F_z we use the following equation

$$\widehat{F}_z = \frac{m}{2(l_f + l_r)}(gl_r - h.\dot{\widehat{v}}_x) \tag{17}$$

$\widehat{v}_x$ is produced by the RE.

3^{rd} Step: At this step it only remains to estimate the adherence or friction coefficient. To this end we assume the vehicle rolling in a normal or steady state situation in order to be able to approximate this coefficient by the following formula

$$\widehat{\mu} = \frac{\widehat{F}_x}{\widehat{F}_z} \tag{18}$$

IV. SIMULATION AND EXPERIMENTAL RESULTS

In this section, we give somme realistic simulation results in order to test and validate our approach and the proposed observer. In simulation, the state and forces are generated by use of a car simulator called VEDYNA [20]. In this simulator the model involved is more complex than the one of 16 DoF presented in the first part of the paper. Comparind the simplified model to the 16 DoF one, let us evaluate the robustnes of estimation The VeDyna simulated braque torque is shown in figure 3.ftbphFU3.6296in1.9847in0ptBraking torquetorqueFigure

Figure 4 showes the measured and estimated wheel angular position. This signal is used to estimat velocities and accelerations.ftbphFU3.5198in1.8844in0ptAngular displacementsAngular displacementsFigure

Figure **??** shows the estimated wheel velocity.ftbphFU3.1868in1.5264in0ptangular velocityAngular velocity-imedgf08.wmfIn the figure **??**, we represente the estimation of vehicle velocity. The figure shows the good convergence to the actual vehicle velocity.ftbphFU3.4082in1.6276in0ptVehicle velocityVelocityFigure

Figure **??** shows the obtained vehicle acceleration. The observer allows a good estimation of angular velocity and acceleration.ftbphFU3.3676in1.5463in0ptEstimated and measured acceleration AccelerationFigure

The last step gives us the estimated longitudinal forces F_x and normal forces F_z wich are presented in figure 8 and 9.ftbphFU3.2552in1.4529in0ptLongitudinal forceLongitudinal forceFigureftbphFU3.122in1.6094in0ptNormal force F_zNormal forceimedi30f.wmf

Finally road friction coefficient is deduced and presented in 10.ftbphFU3.1946in1.8481in0ptRoad frictionRoad frictionimedi30g.wmf

V. CONCLUSIONS

In this work we have tryed to highlight all approximations made in general when using simplified models and this paper gives some details allowing to evaluate what is really neglected. In second part od this paper, we have proposed an effecient and robust estimator baser on the second order sliding mode differentiator. This is used to build an estimation sheme allowing to identify the tire road frictions and input forces which are non observable when using the complete model and standard sensors. The estimations produced finite time converging measurements of model inputs, in three steps by cascaded observers and estimators. This method shows very good performances in simulations conducted using a more complex model (than the 16 DoF one) involved in VeDyna car simulator. Tire forces (vertical and longitudinal ones) are also estimated correctly. Simulation results are presented to illustrate the ability of this approach to give estimation of both vehicle states and tire forces. The robustness versus uncertainties on model parameters and neglected dynamics has also been emphasized in simulations. Application of this approach with inclusion of torque estimation using a simplified model for the engine behaviour, is in progress.

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VI. Appendix A

Definition of the matrices involved.in the model.

$$\bar{M}_{11}= \begin{bmatrix} M_{1,1} & 0 & 0 \\ 0 & M_{2,2} & 0 \\ 0 & 0 & M_{3,3} \end{bmatrix};$$

$$\bar{M}_{12} = \bar{M}_{21}^T= \begin{bmatrix} M_{1,4} & M_{1,5} & M_{1,6} \\ M_{2,4} & M_{3,5} & M_{2,6} \\ 0 & M_{3,5} & M_{3,6} \end{bmatrix};$$

$$\bar{M}_{13} = \bar{M}_{31}^T = \begin{bmatrix} M_{1,7} & M_{1,8} & M_{1,9} & M_{1,10} \\ M_{2,7} & M_{2,8} & M_{2,9} & M_{2,10} \\ M_{3,7} & M_{3,8} & M_{3,9} & M_{3,10} \end{bmatrix};$$

$$\bar{M}_{23} = \bar{M}_{32}^t = \begin{bmatrix} M_{4,7} & M_{4,8} & M_{4,9} & M_{4,10} \\ M_{5,7} & M_{5,8} & M_{5,9} & M_{5,10} \\ M_{6,7} & M_{6,8} & M_{6,9} & M_{6,10} \end{bmatrix}$$

$$\bar{M}_{24} = \bar{M}_{42}^T = \begin{bmatrix} M_{4,11} & M_{4,12} & M_{4,13} \\ M_{5,11} & M_{5,12} & M_{5,13} \\ 0 & 0 & 0 \end{bmatrix};$$

$$\bar{M}_{2,5} = \bar{M}_{52}^T \begin{bmatrix} M_{4,14} & M_{4,15} & M_{4,16} \\ M_{5,14} & M_{5,15} & M_{5,16} \\ 0 & M_{6,15} & M_{6,16} \end{bmatrix}$$

$$\bar{M}_{2,2} = \begin{bmatrix} M_{4,4} & M_{4,5} & M_{4,6} \\ M_{5,4} & M_{5,5} & M_{5,6} \\ M_{6,4} & M_{6,5} & M_{6,6} \end{bmatrix};$$

$$\bar{M}_{3,3} = \begin{bmatrix} M_{7,7} & 0 & 0 & 0 \\ 0 & M_{8,8} & 0 & 0 \\ 0 & 0 & M_{9,9} & 0 \\ 0 & 0 & 0 & M_{10,10} \end{bmatrix};$$

$$\bar{M}_{4,4} = \begin{bmatrix} M_{11,11} & 0 & 0 \\ 0 & M_{12,12} & 0 \\ 0 & 0 & M_{13,13} \end{bmatrix};$$

$$\bar{M}_{5,5} = \begin{bmatrix} M_{14,14} & 0 & 0 \\ 0 & M_{15,15} & 0 \\ 0 & 0 & M_{16,16} \end{bmatrix}$$