

# Tight interval inclusions with compensated algorithms

Stef Graillat<sup>1</sup>, and Fabienne Jézéquel<sup>1,2</sup>

<sup>1</sup> Sorbonne Université, CNRS, Laboratoire d'informatique de Paris 6, LIP6,  
F-75005 Paris, France

<sup>2</sup> Université Panthéon-Assas, 12 place du Panthéon, F-75231 Paris  
Cedex 05, France

stef.graillat@lip6.fr, fabienne.jezequel@lip6.fr

**Keywords:** interval arithmetic, directed rounding, compensated algorithms, error-free transformations, floating-point arithmetic, numerical validation, rounding errors, summation algorithms, dot product, Horner scheme

Compensated algorithms consist in computing the rounding errors of individual operations and then adding them later to the computed result. This makes it possible to increase the accuracy of the computed result efficiently. Computing the rounding error of an individual operation is possible through the use of a so-called *error-free transformation*. In this talk, we show that it is possible to use compensated algorithms for having tight interval inclusions. We study compensated algorithms for summation, dot product and polynomial evaluation. We prove that the use of directed rounding makes it possible to get narrow inclusions with compensated algorithms. This is due to the fact that error-free transformations are no more exact but still sufficiently accurate to improve the numerical quality of results.

## References

- [1] S. Boldo, S. Graillat, and J.-M. Muller. On the robustness of the 2Sum and Fast2Sum algorithms. *ACM Trans. Math. Softw.*, 44(1):4:1–4:14, July 2017.
- [2] S. Graillat, F. Jézéquel, and R. Picot. Numerical validation of compensated summation algorithms with stochastic arithmetic. *Electronic Notes in Theoretical Computer Science*, 317:55–69, 2015.
- [3] S. Graillat, F. Jézéquel, and R. Picot. Numerical validation of compensated algorithms with stochastic arithmetic. *Applied Mathematics and Computation*, 329:339–363, 2018.

- [4] S. Graillat, Ph. Langlois, and N. Louvet. Algorithms for accurate, validated and fast polynomial evaluation. *Japan J. Indust. Appl. Math.*, 2-3(26):191–214, 2009. Special issue on State of the Art in Self-Validating Numerical Computations.
- [5] T. Ogita, S. M. Rump, and S. Oishi. Accurate sum and dot product. *SIAM Journal on Scientific Computing*, 26(6):1955–1988, 2005.