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A Hybrid Simulated Annealing Algorithm for the Prize Collecting Travelling Salesman Problem

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Abstract - A salesman who travels between pairs of cities and collects a prize for each visited city and pays a penalty for each non visited city is considered as one of the most complex problems called The prize collecting Travelling salesman problem (PCTSP). The problem is NP-Hard and large-scale operations cannot be solved with exact algorithms at acceptable computational times. The purpose of this study is to provide hybrid simulation annealing (HSA) that is a hybrid method using the tabu search and simulated annealing technique to solve PCTSP. This HSA proposal not only prevents revisiting the solution but also preserves the stochastic nature. Finally the proposed HSA heuristic has been tested on eight sets of well-known benchmark, and the preliminary results indicate that the HSA is capable of solving real-world problems.

Keywords - The prize collecting travelling salesman problem, Simulated annealing, Tabu search, Meta-heuristics methods

I. INTRODUCTION

The PCTSP has several practical applications such as the daily scheduling formulated of the hot strip mill production [3]. Minimum total distance route for the helicopter tour planning for off shore oil fields; for tourist routes design [2] The PCTSP was presented by Egon Balas [1] as a scheduling model of the daily operations of a steel rolling mill. The author made some structural characteristics of the problem and two mathematical formulations for the PCTSP and cutting plans and lagrangean algorithms.

Several authors suggested different approaches for solving the PCTSP. Fischetti and Toth [4] expanded several bounding procedures for yield sequences of increasing lower bounds on the optimum value of the problem, based on different relaxations. A branch and bound algorithm was also developed that was used for small size problems.

Bienstock et al. [5] introduce an approximation algorithm for the TSP with fixed bounds. Goemans and Williamson [6] provided a 2-approximation procedure with no

minimum prize constraint to one version of the PCTSP. Dell'Amico et al. [8] developed a Lagrangean heuristic, which use a Lagrangean relaxation for generating starting solutions for the heuristic procedure. A procedure Adding-Nodes was used to obtain feasible solutions for PCTSP through the lower bound and a procedure Extension and Collapse seeks improving feasible solutions. Gomes et al. [8] utilized a GRASP heuristic with a variable neighborhood descent scheme to obtain upper bounds. Chaves et al.[9] extended the algorithm devised by [9]and proposed a GRASP procedure with a variable neighborhood search (VNS) descent combined with a variable neighborhood search to get good upper bounds. Feillet et al.[12]proposed a survey on TSP with profits that include the PCTSP ,identifying and comparing the complexity of different classes of applications, modelling approaches and exact or heuristic solution techniques. Chaves and Lorena proposed [10] two procedures based on clustering search to solve the PCTSP. The main difference between the two procedures is on how the initial solutions are generated: While the first procedure produces solutions by means of an evolutionary algorithm; the second creates starting solutions via a GRASP method associated with VNS/VND. In addition, Chaves and Lorena [9,11] developed a similar approach by devising hybrid algorithms based on clustering search associated with GRASP and VNS/VND. Pedro et al. [13] a tabu search (TS) approach is devised to solve an asymmetric version of the PCTSP.

The aim of this study is to presents a hybrid method using tabu search and simulated annealing technique to solve PCTSP called hybrid simulation annealing. This algorithm uses the features of two techniques and uses the strengths of each one. This proposed hybrid simulation annealing not only prevents revisiting the solution but also maintains the stochastic nature. The proposed algorithm have been illustrated to be efficient in finding optimal solutions in most cases and its performance is better than two other techniques.

II. Problem Definition

The Prize Collecting Travelling Salesman Problem (PCTSP) is a generalization of the Traveling Salesman Problem (TSP), a well-know NP-hard combinatorial optimization problem. A traveling salesman has to visit some cities, with each one having a prize and a penalty associated with it. The prize is collected whenever a city is visited; and the penalty is applied whenever the city is not visited. Further, there is a cost for traveling between two cities. The objective is to minimize the sum of the travel costs and penalties paid, but ensuring the collection of a minimum prize.

$w = \{1, 2, \dots, W\}$ Set of all nodes

Parameters:

N = Number of nodes

γ_i = Penalty for each node i ($i \in w, \gamma_i = \infty$)

p_i = Prize for each node i ($i \in w, p_i = 0$)

c_{ij} = Distance between node i and j , ($i, j \in w$)

p_{\min} = The minimum amount of prize to be collected

Decision variables:

X_{ij} = A binary variable that determines if arc (i, j) is part of the tour ($i, j \in w$)

Y_i = A binary variable that determines if node i is visited ($i \in w$)

f_{ij} = Auxiliary variable for sub-tour elimination constraints ($i, j \in w$)

Mathematical model:

$$(1) \quad \min z = \sum_{i=1}^w \sum_{j=1}^w c_{ij} X_{ij} + \sum_{i=1}^w \gamma_i (1 - Y_i)$$

s.t:

$$(2) \quad \sum_{i=1}^w \sum_{j=1}^w X_{ij} = Y_i \quad \forall i \in w$$

$$(3) \quad \sum_{i=1}^w \sum_{j=1}^w X_{ij} = Y_j \quad \forall j \in w$$

$$(4) \quad \sum_{i=1}^w p_i Y_i \geq p_{\min}$$

$$(5) \quad \sum_{j=1}^w f_{ji} - \sum_{j=1}^w f_{ij} = Y_i \quad \forall i \in w$$

$$(6) \quad f_{ij} \leq (N - 1) X_{ij} \quad \forall (i, j) \in W$$

$$(7) \quad Y_i, X_{ij} \in \{0, 1\} \quad f_{ij} \geq 0 \quad \forall (i, j) \in W$$

The objective function (1) represents the minimization of the sum of transportation cost and the incurred penalties cost. Constraints (2) and constraints (3) are Flow conservation constraints that ensure then only one arc arrives at it and one leaves this node. Constraints (4) ensure the minimum prize amount to be collected. Constraints (5) and constraints (6) are sub-tour elimination constraints. Finally, Constraint (7) defines the variable types.

III. METHODOLOGY

HYBRID SIMULATION ANNEALING ALGORITHM

The proposed hybrid simulation annealing (HAS) meta-heuristic is introduced for travelling salesman problem based on the hybridization of two algorithms, namely SA and TS. In the algorithm by the combination of simulated annealing (SA) and tabu search (TS), the number of solution revisits has been remarkably decreased. By using the proposed HAS, a number of solution revisits can be decreased by providing a short-term memory while keeping the stochastic nature of the SA algorithm also the proposed HAS, not only accepts better solutions but also the worse solutions but with a decreasing probability.

The proposed HAS have a number of advantages, including stochastic feature avoiding cycling and Short-term memory to escape from local optima. These characteristics limit the search from a previously visited solution and improve the performance of the conventional SA remarkably. In **Figure1** algorithmic steps of HSA is summarized

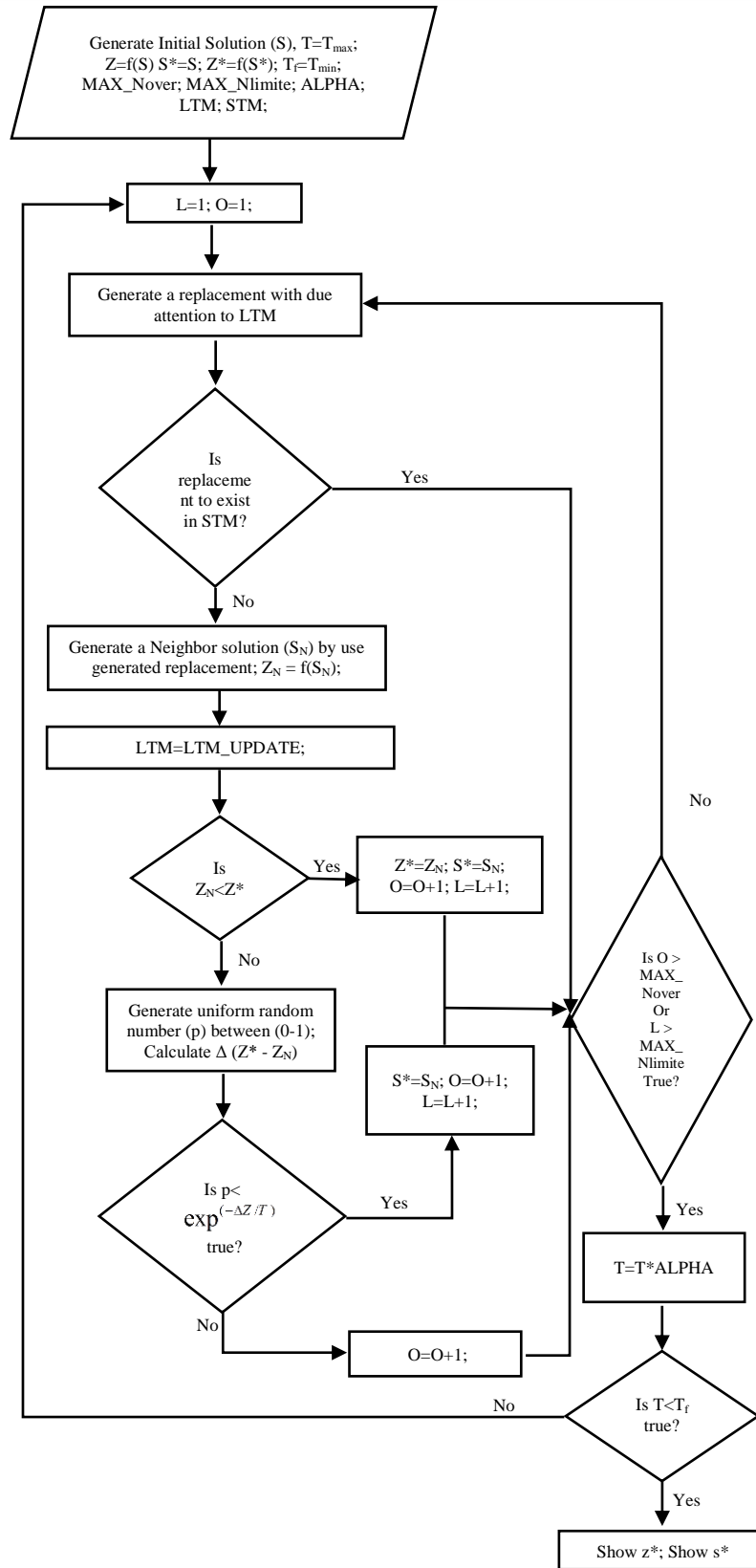


Figure 1. Algorithm of Hybrid Simulated Annealing

V. RESULTS

In this section, we present details of the implementation of the proposed HSA. For the testing of this technique, 8 example problems from the literature were used. The proposed HSA were written in MATLAB programming language and was run 40 times for every problem. For the comparison of the technique, GAP of the average results with due attention to the optimal solution was used.

$$GAP(\%) = \frac{(averageZ) - (optimalZ)}{(optimalZ)} \times 100$$

As we can observe from the results of Table 1, HSA meta-heuristic algorithm for solving the prize collecting travelling salesman problem is capable of finding optimal solutions. **Table 1** shows details of our results.

V. CONCLUSION

In this study, first we have presented The prize collecting travelling salesman problem then We have presented a hybrid simulation annealing (HAS) meta-heuristic based on combination of two well known techniques, tabu search and simulated annealing to solve the prize collecting travelling salesman problem. To validate and verify the proposed HSA, the technique was tested on 8 well known benchmark problem sets from literature. The proposed HAS have been illustrated to be efficient in finding optimal solutions in most cases.

Name of technique	Data of instances	Problem number	Name of instances	Number of city	Average Z	Optimal Z	GAP (%)
HAS	$n \in (20, 40, 60, 80)$ $Prize(p_i) \in [1, 100]$ $Penalty(\gamma_i) \in [1, 100]$ $Distance(c_{ij}) \in [1, 1000]$ $p_{min} = 0.2 \times \sum_{i=1}^n p_i$	1	20a	20	911.85	903	0.98
		2	40a	40	1018.3	996	2.23
		3	60a	60	1378.6	1314	4.91
		4	80a	80	1493.2	1384	7.89
	$n \in (20, 40, 60, 80)$ $Prize(p_i) \in [1, 100]$ $Penalty(\gamma_i) \in [1, 1000]$ $Distance(c_{ij}) \in [1, 10000]$ $p_{min} = 0.2 \times \sum_{i=1}^n p_i$	5	20b	20	11802	11677	1.07
		6	40b	40	11150.6	10776	3.47
		7	60b	60	15120.1	14236	6.21
		8	80b	80	15890	14484	9.7

Table 1. The summary of results

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