Estimation-based algorithm for a stochastic one-commodity pick-up & delivery travelling salesman problem

S. Hadjadj, H. Kheddouci

Estimation-based algorithm for a stochastic one-commodity pick-up & delivery travelling salesman problem

S. Hadjadj and H. Kheddouci

Laboratoire d'informatique en image et systèmes d'information, Bâtiment Nautibus 43, bd du 11 Novembre 1918 69622 VILLEURBANNE CEDEX
mohamed-seddik.hadjadj@liris.cnrs.fr
hamamache.kheddouci@univ-lyon1.fr

1 Introduction

This work is carried out in collaboration with a company which specializes in the sale of ready-mix concrete.
Ready-mix concrete is normally delivered in *mixer trucks*. This type of truck is heavy, cumbersome, expensive, and can be disproportionate in some cases, especially when delivering small quantities of concrete.
Therefore, the company wants to propose a new delivery method using small containers (500 litre bins) to reduce delivery costs and deal more effectively with orders of small quantities.
This new method is a two-step process :

1. A vehicle delivers a number of bins of concrete to the customer ;
2. The next day, the vehicle returns to the customer to pick up empty bins.

To ensure the profitability of this method, the company needs a decision support system that can generate efficient **pick-up & delivery tours** taking into account vehicle capacity constraint and **recycling constraint**.
Indeed, if a bin is totally empty and clean when it is picked up from a customer, it could be directly supplied to another customer. Otherwise, it must be immediately routed to a recycling centre before it can be delivered again (the unconsumed concrete is then recycled and the bin cleaned). Knowing that the state of a bin is uncertain before the vehicle arrives at customer's location, the a priori planned vehicle route may changes during time to include recycling centre(s) whenever necessary. This uncertainty involves dealing with **stochastic vehicle routing**.
This paper aims to provide an efficient approach to build pick-up & delivery tours minimizing the loss of quality caused by potential detours to recycling centres.

2 Literature Review

We consider a stochastic One-Commodity pick-up & delivery travelling salesman problem.

2.1 Pick-up & delivery problems

There are three main classes of pick-up & delivery problem in the literature :

One-to-one problems One or more vehicle have to carry n commodities, where each commodity has a specific origin and destination. One of the best known examples of this class is the *Dial-a-Ride problem* which consists in transporting people from an origin to a destination. The problem has been studied for both single [4] and multiple [5] vehicle cases, with various types of constraints related to ride times, time windows [6, 7]...

One-to-many-to-one problems Commodities are divided into "delivery commodities" and "pick-up commodities". One or more vehicle have to carry the delivery commodities from the depot to the customers and the pick-up commodities from the customers to the depot. Assuming that n_p is a set of pick-up customers, and n_d a set of delivery customers, two cases have been distinguished for these problems : single demands, where $n_p \cap n_d = \emptyset$, and combined demands, where $n_p \cap n_d \neq \emptyset$. For the latter case, [8] consider various possible path types such as *Hamiltonian* path, where each customer is visited once such that pick-up and delivery are performed simultaneously, as well as *Double-path* where each customer that has a combined demand (pick-up and delivery) is visited twice, the first time for a pick-up, the second for a delivery. Several heuristics have been proposed for both path types for the single and the multi-vehicle cases [9, 10]...

Many-to-many problems One or more vehicle have to transport goods between customers knowing that each customer can be a source or a destination of any type of good. Among the problems of this class, the *One-Commodity pick-up and delivery travelling salesman problem* was introduced in [11]. A single vehicle with a known and finite capacity has to carry a single commodity between pick-up customers and delivery customers, a picked up commodity can be supplied to a delivery customer. This problem is known to be *NP-Hard*. Moreover, checking the existance of a feasible solution is an *NP-Complete* problem [13]. Studies on such problems are relatively scarce. A branch and cut algorithm has been proposed in [11] for small instances, and two heuristics have been developed in [12] to tackle larger instances, in particular by defining "the infeasibility of a path", and adapting the nearest neighbourhood heuristic to increase the chance of obtaining a feasible solution. Furthermore, [14] have proposed a hybrid method combining GRASP (greedy randomized adaptive search procedure) and VND (variable neighbourhood descent) metaheuristics. This method gave better results than the previously proposed ones.

For a detailed survey on pick-up and delivery problems, we refer the reader to [15].

2.2 Stochastic/Dynamic vehicle routing problems

Vehicle routing problems can be classified according to the information quality and evolution. Thus, an input information can be deterministic or stochastic, and it can be known in advance or revealed during the tour.
A taxonomy of vehicle routing problems based on these two dimensions is proposed by [16]. Four types of vehicle routing problems are then distinguished :

Static and deterministic problems Input is known in advance and doesn't change over time. This is the most studied type of problem, but it generally doesn't fit with real-world applications, where some information cannot be known beforehand.

Static and stochastic problems Here, some information is a stochastic variable which is revealed gradually during the execution of the tour. However, the a priori planned routes cannot change during the execution of the tour except in some special cases. For example, if the considered stochastic variable is the customers request, or in other words, if customers may request a visit with a certain probability, the a priori planned route may change only to skip customers that do not require a visit. Several types of stochastic variables have been studied in the literature : stochastic travel times [17], where travel times between customers is a random variable, stochastic customers, where customers may request a visit with a certain probability [3]...

Dynamic and deterministic problems Some information is totally unknown beforehand and is revealed only during the execution of the tour. Vehicle tours are then changed in real time, during the execution of the tour according to revealed information.

Dynamic and stochastic problems This type of problem is a combination of the latter two types described above. Some information is a stochastic variable that can be used to build a priori tours taking into account possible future events, and routes are adapted in real time according to revealed information.

For more details on stochastic and dynamic vehicle routing problems, we refer the reader to the surveys of [16] and [1].

2.3 Stochastic/Dynamic pick-up & delivery problems

Most studies tackling pick-up & delivery problems consider the static case in which all information is known beforehand and does not change during time. However, some papers deal with the dynamic case where some information is only revealed during the tour and the a priori tour is adapted progressively in real time. A few of these works exploit stochastic information to anticipate future events, [18] present some of these papers. However, to the best of our knowledge, there is no work dealing with the One-commodity travelling salesman problem in a stochastic case.

The problem considered in this paper can be classified as a static and stochastic One-commodity travelling salesman problem. It is static because changes are not allowed during the execution of tour except for detours to recycling centres. Stochastic because we have probabilistic information through historical data about potential future detours. It is a One-commodity travelling salesman problem because a single-vehicle has to carry one commodity from a set of pick-up customers to a set of delivery customers.

3 Problem Formulation

The pick-up & delivery travelling salesman problem(1-PDTSP) can be defined on a complete graph $G = (V, E)$ as follows :

- $V = \{0, 1, ..., n\}$ is a set of $n+1$ nodes representing the n customers ($n = n_d + n_p$, where n_d is the number of delivery customers and n_p the number of pick-up customers). Node 0 represents the depot ;
- $E = \{(i, j), i, j \in V, i \neq j\}$ is a set of edges representing connections between customers ;
- $C = \{c_{i,j}, (i, j) \in E\}$ represents the travel distance between customers i and j ($c_{i,j} = c_{j,i}, \forall (i, j) \in E$) ;
- $D = \{d_i, i \in V\}$ is a set of customers demands ($|d_i|$ is the number of bins to deliver to / pick up from customer i, $d_i < 0$ for delivery customers and > 0 for pick-up customers) ;

Given a vehicle with a known and finite maximum capacity Q, and assuming that :

- $x_{i,j}$ is a boolean variable such that:

$$\begin{array}{ll} x_{i,j} = 1 & \text{if customer } j \text{ is visited immediately after customer } i; \\ x_{i,j} = 0 & \text{otherwise.} \end{array}$$

- q_i the number of bins in the vehicle after his visit to customer i.

Our objective is to find a Hamiltonian cycle that minimizes the total travel distance, ie :

$$min \sum_{i=0}^{n} \sum_{j=0}^{n} x_{i,j} c_{i,j} \tag{1}$$

Subject to :

$$\sum_{j=0}^{n} x_{i,j} = 1 \qquad \forall i \in \{0, 1, ..., n\} \tag{2}$$

$$\sum_{i=0}^{n} x_{i,j} = 1 \qquad \forall j \in \{0, 1, ..., n\} \tag{3}$$

$$q_i + x_{i,j} d_j \leq Q \qquad \forall i, j \in \{0, 1, ..., n\} \tag{4}$$

$$q_i + x_{i,j} d_j \geq 0 \qquad \forall i, j \in \{0, 1, ..., n\} \tag{5}$$

Constraints (2) et (3) ensure that each customer is visited exactly once, while constraints (4) and (5) relate to vehicle capacity.
A picked up bin can be supplied to a delivery customer if necessary. However, if a bin is not totally clean and empty when it is picked up from a customer, it must be firstly routed to one of the R available recycling centres around the customer's location before it can be supplied again. We can consider the R available recycling centres as a "priority customer" that may requires a visit after each of the n_p pick-up customers. Then, we define p_i as the probability that the "priority customer" requires a visit immediately after customer i ($p_i = 0$ for all delivery customers). The problem can then be seen as a travelling salesman problem with stochastic customer requests.

4 Estimation-based algorithm

To tackle the 1-SPDTSP described above, we propose an estimation-based heuristic adapted from the approach presented in [2] for the probabilistic travelling salesman problem.
This approach is based on a local search method which starts from an initial feasible solution S, and tries to improve it by moving to S', a feasible neighbouring solution of S, such that $f(S') < f(S)$. The process is repeated until no improvement can be found.

$ImprovedSolution \leftarrow True$;
$S \leftarrow IntialSolution$;
while $ImprovedSolution$ **do**
  $N \leftarrow Neighborhood\ (S)$;
  $N \leftarrow RomoveUnfeasibleSolutions\ (N)$;
  for $S' \in N$ **do**
    if $f(S') < f(S)$ **then**
      $S \leftarrow S'$;
    end
  end
  $ImprovedSolution \leftarrow False$;
end

Algorithm 1: Local search principle

4.1 Neighbourhood structure

We use the 1-shift algorithm introduced in [3] to generate the neighbourhood of a given solution S. This method consists in changing the position of a customer in a tour from i to j. Customers which are at positions $i+1, i+2, ..., j$ of the tour are then shifted backwards (see figure 1).

4.2 Feasibility checking

For each generated solution, we unsure that capacity constraints described in section 3 are respected. A feasible solution is a tour in which the total number of bins loaded on the vehicle never exceeds the maximum capacity Q of the vehicle, and is never negative. Assuming that q_i is the number of bins in the vehicle after visiting customer i, figure 1 presents an example of feasible and infeasible solution.

Given a feasible solution S and an 1-shift neighbouring solution S' of S obtained by shifting a

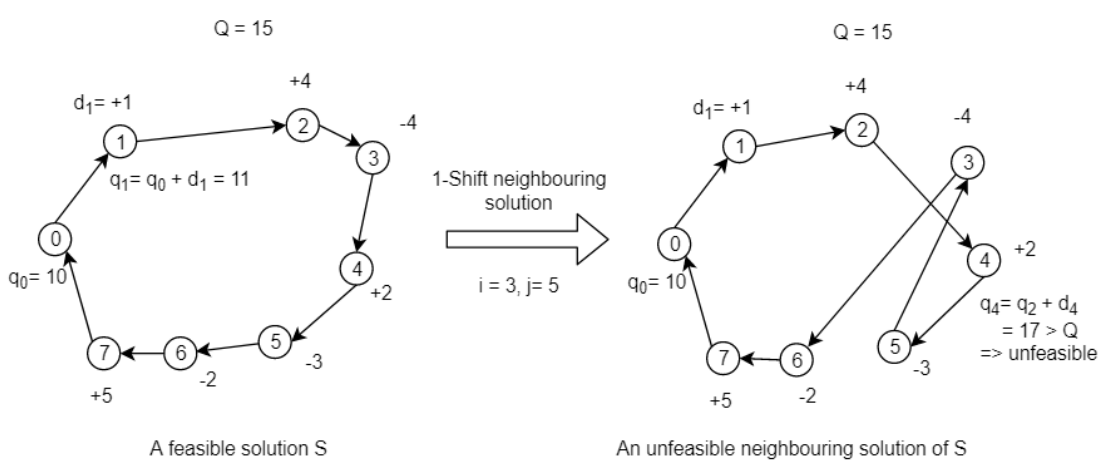

Fig. 1: 1-Shift algorithm

customer from position i to j. It can easily be shown that S' is feasible if and only if the partial tour from customer i to customer j is feasible. Indeed, to check to feasibility of a neighbouring solution, we only check the feasibility of the tour between positions i and j.

4.3 Objective function

In our case, the objective function f to minimize is the total travel distance of the vehicle. However, since we cannot know in advance the travel distance of an a priori solution S due to potential detours to recycling centres (see figure 2), we use the following unbiased estimator of $f(S)$ as a criterion to move from a solution to another one :

$$\hat{f}_M(S) = \frac{1}{M}\sum_{r=1}^{M} f(S, \omega_r)$$

This estimator was proposed by [2] for the probabilistic travelling salesman problem. The idea is to estimate the quality of an a priori solution S from a set of M simulations of possible a posteriori solutions. An a posteriori solution is obtained by associating a binary vector ω with the a priori solution such that $\omega[i] = 1$ if a detour to a recycling centre is required immediately after visiting customer i, 0 otherwise (see vector ω in figure 2).
Thus, given an a priori solution S (that does not include recycling detours) :

1. M possible a posteriori solutions (including potential detours) are generated by associating M vectors ω with the a priori solution S;
2. For each generated a posteriori solution, $f(S, \omega_i)$, the travel distance of the a posteriori solution given by ω_i is calculated. $f(S, \omega_i) = f(S) + TDL - \sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i x_{i,j} c_{i,j}$, where :
   - $f(S)$ is the travel distance of the a priori solution S (without detours) ;
   - TDL is the Total Detour Length of the a posteriori solution (see example in figure 2).
3. $\hat{f}_M(S) = \frac{1}{M}\sum_{r=1}^{M} f(S, \omega_r)$ is calculated and considered as an estimator of $f(S)$.

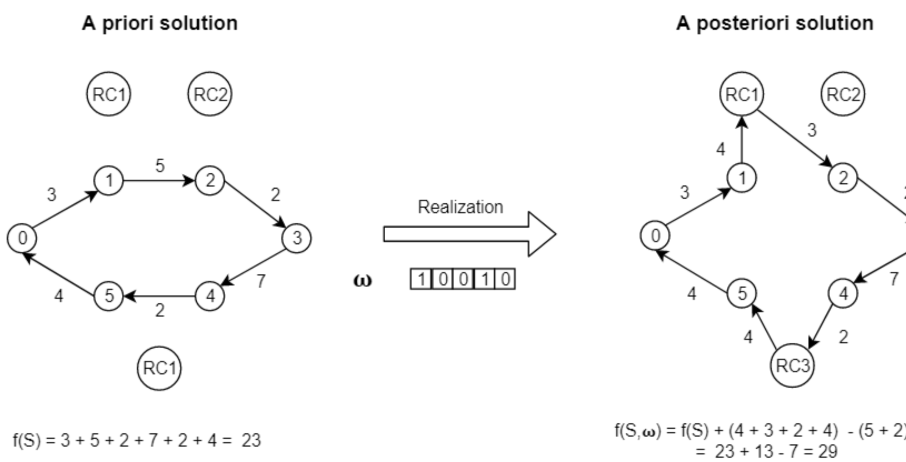

Fig. 2: A priori solution VS a posteriori solution

Note that ω is generated according to the set $P = \{p_i, i \in V\}$ of probabilities that recycling detour is required after visiting customer i. Therefore, $\omega[i] = 0$ for all delivery customers because recycling detour may occur only when picking up bins.

4.4 Recycling centre choice

Since we consider R available recycling centres in our problem, each time a detour to recycling centre is required, we must choose among the R possibilities we have. Therefore, we calculate the travel distance caused by the detour to each of the R available recycling centres to choose the one that minimizes the detour length (see figure 3).

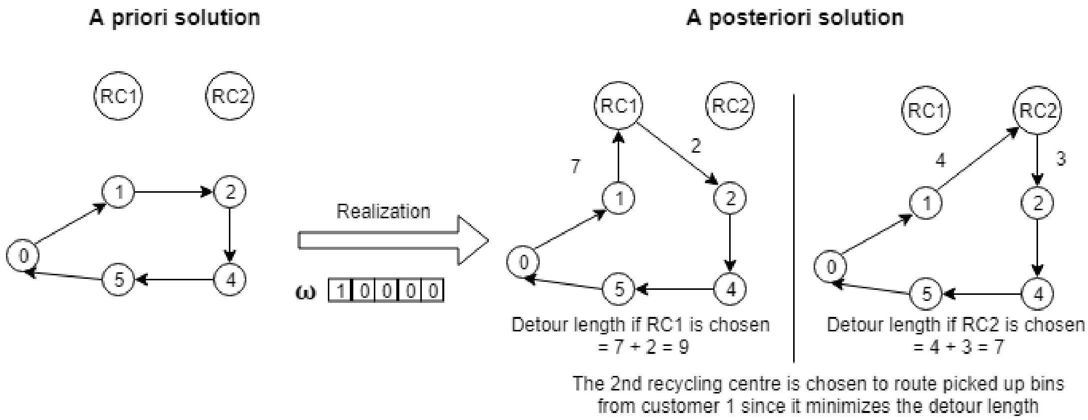

Fig. 3: Recycling centre choice

5 Computational results

The algorithm was implemented in Java, and executed on AMD A10-7700K Radeon R7, 3.40 GHz With 8 GB RAM.

We tested the performance of our algorithm on the Euclidian PDTSP instances generated by [19]. The number of customers in these instances vary between 25 and 200. The first four customers of each instance have been chosen to be the recycling centres, the remaining nodes are assumed to be the customers. For each customer i, we determined whether a recycling detour is required after visiting i or not (we determined an "effective scenario" for each instance). The boolean variables were generated according to a fixed probability P. We generated scenarios for $P \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$. Then, the a priori solutions found by our algorithm were evaluated according to the fixed scenarios, with different values for the parameter M, the number of simulated a posteriori solutions (see section 4.3). For $M = 0$, our algorithm doesn't simulate a posteriori solutions. It is then equivalent to a classic local search which doesn't take into account stochastic information.

Table 1 shows the average solution cost obtained by our Estimation-Based Local Search for the instances described above (we fixed the neighbourhood size to 200).

First, we observe that, for each class of instances, solution costs increase as the parameter p increases. This is due to the fact that a higher probability p involves a greater risk of requiring detours to recycling centres and thus, a greater risk of increasing the solution cost. However, we can see that this increase is smaller as the number of generated a posteriori solutions (M) is greater.

Figure 4 shows the percentage of travel distance due to detours when $p = \{0.1, 0.3, 0.5, 0.7, 0.9\}$, and for $M = \{0, 25, 50, 100\}$. The results show the effectiveness of our approach in minimizing the detours impact on the solution cost, especially when $p \geq 0,5$. Indeed, the objective of our estimation-based local search is to anticipate possible detours during the tour and take them into account when generating a priori solutions. Therefore, the more detours may occur during a tour, the more interesting our approach is. In other words, and as we can observe in table 1, our estimation-based heuristic always obtains the best solutions in comparison with the classic local search (the one with $M = 0$) when $p > 0,1$. Moreover, the results are generally better when $M = 100$. Thus, a greater number of a posteriori simulations gives generally a more accurate evaluation of an a priori solution.

6 Conclusion and perspectives

We presented in this a paper an Estimation-based local search to tackle a stochastic one-commodity pick-up & delivery travelling salesman problem. The objective of our approach is to build efficient vehicle routes that minimize loss of quality due to potential changes during the tour. We tested our algorithm on the Euclidian PDTSP instances proposed in [19]. We adapted the instances to fit our constraints and collected the results with different parameter values. The experiments show the effectiveness of our algorithm, especially when dealing with large instances, and when detours

Table 1: Estimation-based local search solutions for the Euclidian PDTSP instances

		Number of sample solutions M			
Probability	Number of customers	0	25	50	100
$p = 0.1$	25	**544,26**	555.9	551,5	565,4
	50	**843,33**	895	893.1	860.6
	75	1121,1	1119	1138,66	**1111,5**
	100	**1414,56**	1432,5	1494	1457,5
	150	2007,07	2017	2013	**1993,3**
	200	2603,73	2601.1	2496	**2489,16**
$p = 0.3$	25	618,5	596	**574,26**	577,3
	50	1020,03	1018,2	1051,3	**1001,1**
	75	1446,9	1474,4	1398,12	**1392,8**
	100	1874,26	1862,2	**1813.7**	1841,1
	150	2673,83	2641,14	2594	**2569,4**
	200	3592,26	3617,14	3504,2	**3495,6**
$p = 0.5$	25	706,66	688,3	681.13	**680,6**
	50	1281,33	1210,15	1187	**1161,7**
	75	1848,86	1817,21	1832,2	**1804,4**
	100	2233,96	2214,1	2157	**2149,4**
	150	3416,86	3411,14	**3378,9**	3386.6
	200	4445,53	4431,37	4376,1	**4348,4**
$p = 0.7$	25	782,06	771	**734,3**	746,2
	50	1480,83	1535,2	1457,5	**1447,3**
	75	2172	2267,3	2169	**2125,2**
	100	2666,5	2517,3	2500,1	**2491,7**
	150	4047,16	4006,7	4011,6	**3992,8**
	200	5490,2	5397,3	**5325,8**	5332,3
$p = 0.9$	25	867,8	804,4	**786,4**	799,1
	50	1682,06	1633,9	**1526,1**	1589,1
	75	2472	2480,4	2366,5	**2306,2**
	100	3108,8	3116,5	2915,2	**2903**
	150	4741,8	4886,3	4605,36	**4552,52**
	200	6457,8	6384,8	6376,6	**6301,1**

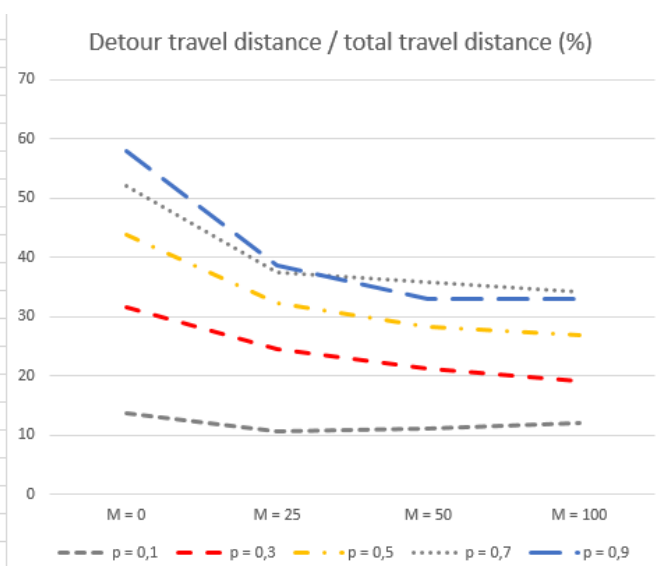

Fig. 4: Recycling detour's impact on total travel distance

are more likely to occur.
In this paper, we proposed a static and stochastic approach to tackle our vehicle routing problem. To improve further the obtained results, future works will be devoted to the development of a dynamic and stochastic approach which can exploit stochastic information to build efficient routes that can dynamically change to fit potential unexpected events during the vehicle tour.

References

1. RITZINGER, Ulrike, PUCHINGER, Jakob, et HARTL, Richard F. A survey on dynamic and stochastic vehicle routing problems. International Journal of Production Research, 2016, vol. 54, no 1, p. 215-231.
2. BIRATTARI, Mauro, BALAPRAKASH, Prasanna, STTZLE, Thomas, et al. Estimation-based local search for stochastic combinatorial optimization using delta evaluations: a case study on the probabilistic traveling salesman problem. INFORMS Journal on Computing, 2008, vol. 20, no 4, p. 644-658.
3. BERTSIMAS, Dimitris. Probabilistic combinatorial optimization problems. 1988. Thse de doctorat. Massachusetts Institute of Technology.
4. PSARAFTIS, Harilaos N. A dynamic programming solution to the single vehicle many-to-many immediate request dial-a-ride problem. Transportation Science, 1980, vol. 14, no 2, p. 130-154.
5. CORDEAU, Jean-Franois. A branch-and-cut algorithm for the dial-a-ride problem. Operations Research, 2006, vol. 54, no 3, p. 573-586.
6. TOTH, Paolo et VIGO, Daniele. Fast local search algorithms for the handicapped persons transportation problem. In : Meta-Heuristics. Springer, Boston, MA, 1996. p. 677-690.
7. CORDEAU, Jean-Franois et LAPORTE, Gilbert. A tabu search heuristic for the static multi-vehicle dial-a-ride problem. Transportation Research Part B: Methodological, 2003, vol. 37, no 6, p. 579-594.
8. GRIBKOVSKAIA, Irina, HALSKAU SR, yvind, LAPORTE, Gilbert, et al. General solutions to the single vehicle routing problem with pickups and deliveries. European Journal of Operational Research, 2007, vol. 180, no 2, p. 568-584.
9. HOFF, Arild et LKKETANGEN, Arne. Creating lasso-solutions for the traveling salesman problem with pickup and delivery by tabu search. Central European Journal of Operations Research, 2006, vol. 14, no 2, p. 125-140.
10. CHEN, Jeng-Fung et WU, Tai-Hsi. Vehicle routing problem with simultaneous deliveries and pickups. Journal of the Operational Research Society, 2006, vol. 57, no 5, p. 579-587. and pickups
11. HERNNDEZ-PREZ, Hiplito et SALAZAR-GONZLEZ, Juan-Jos. A branch-and-cut algorithm for a traveling salesman problem with pickup and delivery. Discrete Applied Mathematics, 2004, vol. 145, no 1, p. 126-139.
12. HERNNDEZ-PREZ, Hiplito et SALAZAR-GONZLEZ, Juan-Jos. Heuristics for the one-commodity pickup-and-delivery traveling salesman problem. Transportation Science, 2004, vol. 38, no 2, p. 245-255.
13. HERNNDEZ-PREZ, H. Traveling salesman problems with pickups and deliveries. Disertation, University of La Laguna, Spain, 2004.
14. HERNNDEZ-PREZ, Hiplito, RODRGUEZ-MARTN, Inmaculada, et SALAZAR-GONZLEZ, Juan Jos. A hybrid GRASP/VND heuristic for the one-commodity pickup-and-delivery traveling salesman problem. Computers & Operations Research, 2009, vol. 36, no 5, p. 1639-1645.
15. BERBEGLIA, Gerardo, CORDEAU, Jean-Franois, GRIBKOVSKAIA, Irina, et al. Static pickup and delivery problems: a classification scheme and survey. Top, 2007, vol. 15, no 1, p. 1-31.
16. PILLAC, Victor, GENDREAU, Michel, GURET, Christelle, et al. A review of dynamic vehicle routing problems. European Journal of Operational Research, 2013, vol. 225, no 1, p. 1-11.
17. KENYON, Astrid S. et MORTON, David P. Stochastic vehicle routing with random travel times. Transportation Science, 2003, vol. 37, no 1, p. 69-82.
18. BERBEGLIA, Gerardo, CORDEAU, Jean-Franois, et LAPORTE, Gilbert. Dynamic pickup and delivery problems. European journal of operational research, 2010, vol. 202, no 1, p. 8-15.
19. GENDREAU, Michel, LAPORTE, Gilbert, et VIGO, Daniele. Heuristics for the traveling salesman problem with pickup and delivery. Computers & Operations Research, 1999, vol. 26, no 7, p. 699-714.