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WARRANTY COST ANALYSIS OF A TWO-COMPONENT SYSTEM WITH STOCHASTIC DEPENDENCE

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This paper analyses the expected warranty costs from the perspectives of the manufacturer and the consumer respectively. Both the non-renewing free replacement policy and the renewing replacement policy are examined regarding a two-component in series system with failure interaction between components. Our primary objective is to provide explicit expressions of the warranty costs allocations between the manufacturer and the consumer by taking into account the product service time. Numerical examples are given to demonstrate the applicability of the methodology. It is shown that, independent of the type of the warranty policy, the failure interaction between components has impact on the manufacturer profits and the consumer costs.

Keywords: free replacement warranty, two-component systems, stochastic dependence, imperfect repair

1. Introduction

A warranty is a contract between the manufacturer and the buyer which requests the manufacturer to repair, replace the product or to compensate the buyer if the product fails before a pre-determined time period, which

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is referred to as warranty period. Due to the development of technology and the intense competition between enterprises, warranty is more and more important and a lot of attention have been paid to it. Basically, there are two common types of warranty policies in the literature: the free replacement/repair warranty (FRW) ([1,2,3,4]) and the pro rata warranty (PRW) ([5,6,7]). The former indicates that the manufacturer covers the total cost of replacement/repair of the product before the expiration of the warranty and the latter means that within the warranty period, a failed product is repaired at a cost which is proportional to the product age at failure.

It is noticed in the literature that most of the the warranty cost analysis dealt with the single-item system or the multi-component system with independent items. However, this assumption is seldom valid in practice. Liu et al. [8] studied the warranty cost of multi-component systems in which the natural failure of one component may induce the instantaneous failure of other components under the renewing free replacement policy. In our study, we consider a two-component system containing different failure mechanisms with failure dependence between them. The warranty costs allocations between the manufacturer and the consumer under different warranty policies are calculated.

The paper is organised as follows. The system descriptions and the hypotheses are introduced in section 2. The warranty costs and profits from the perspectives of the manufacturer and the consumer are formulated in section 3. Section 4 is devoted to exact numerical calculations and system sensitivity analysis. Finally, we make conclusions in section 5.

2. Model description

The system descriptions and the warranty policies are given as follows.

- Component 1 and component 2 are in series.
- Component 1 is repairable with lifetime distribution $F(\cdot)$, where $F(0) = 0$. It is imperfectly repaired when failure occurs. Let X_i and a denote the inter-maintenance time between the $(i-1)$ th and the i th repair and the imperfect maintenance degree respectively. Then the virtual age of component 1 after the i th repair is $B_i = B_{i-1} + aX_i$, $i = 1, 2, \dots$ where $B_0 = 0$ and $0 \leq a \leq 1$.
- Whenever component 1 failure occurs, it causes a random amount of damage to component 2. Let Z_j ($j = 1, 2, \dots$) be the damages which are independently and identically distributed random vari-

ables with distribution function $H(\cdot)$, $H(0) = 0$. Damages are additive and the total damage induced by component 1 by time t is the sum of Z_j occurred by time t .

- Component 2 is non-repairable and it fails when its damage level exceeds a pre-determined threshold L , $L > 0$. The failure of component 2 induces the failure of component 1. The system is replaced at the failure of component 2.
- Denoted by $\{Y(t), t \geq 0\}$ the component 2 intrinsic damage level at time t and σ_L the first hitting time of the damage level L . Then without considering the damage induced by component 1 failure, the distribution function of σ_L is $G_{\sigma_L}(t) = \mathbb{P}\{\sigma_L \leq t\} = \mathbb{P}\{Y(t) \geq L\}$, $t \geq 0$.
- Both the component repair times and the system renewal duration time are negligible.
- Two types of warranty policies are provided to the consumer which are the non-renewing FRW and the FRW respectively:
 - under the non-renewing FRW
 - * component 1 is imperfectly repaired at its failure at time T_{f1} and the warranty of the system remains valid during the remaining $W - T_{f1}$ period.
 - * the system is replaced at the failure of component 2 at time T_{f2} and the warranty of the system remains valid only during the remaining $W - T_{f2}$ period.
 - under the RFRW,
 - * component 1 is imperfectly repaired at its failure at time T_{f1} and the warranty of the system remains valid during the remaining $W - T_{f1}$ period.
 - * the system is replaced at the failure of component 2 at time T_{f2} and a full system warranty period of length W is provided.
- The component 1 repair cost and the system replacement cost for the manufacture (resp. consumer) are denoted c_1 (resp. c_{11}) and c_2 (resp. c_{22}).

3. Warranty analysis

Let us denote $N(t)$ the component 1 failure number by time t , $V_n(\cdot)$ the distribution function of B_n , and $p_n(t) = \mathbb{P}(N(t) = n)$. According to [9],

the following equation can be easily derived.

$$p_n(t) = \int_0^{at} \frac{\bar{F}(y + \frac{at-y}{a})}{\bar{F}(y)} v_n(y) dy \quad (1)$$

where $\bar{F}(\cdot) = 1 - F(\cdot)$, $v_n(x) = \frac{d}{dx} V_n(x)$, $v_1(x) = \frac{1}{a} f(\frac{x}{a})$ and $v_{n+1}(x) = \frac{1}{a} \int_0^x \frac{f(y + \frac{x-y}{a})}{\bar{F}(y)} v_n(y) dy$ for $n \geq 1$ [9].

Besides, the component 2 lifetime distribution $F_s(t)$ can be given by

$$F_s(t) = p_0(t)G_{\sigma_L}(t) + \sum_{k=1}^{\infty} p_k(t) \int_0^{\infty} G_{\sigma_L-z}(t) dH^{*(k)}(z) \quad (2)$$

where $p_k(t)$ is the probability mass function of the component 1 failure number given in equation (1). $H(\cdot)$ is the damage distribution function and $H^{*(k)}(t)$ is the k -fold convolution of $H(t)$ with itself.

In the following, firstly the manufacture's expected costs under different warranty policies are presented.

3.1. The manufacturer's expected warranty costs

For simplicity, we call the system failure as minor failure if it contains only component 1 failure and major failure if both the two components in the system fail. Denote $\mathbb{E}(C(W))$ be the manufacturer's expected warranty cost under non-renewing FRW within warranty period W . Then we have the following theorem.

Theorem 3.1. *Under the non-renewing FRW, the expected manufacturer cost $\mathbb{E}(C(W))$ within the warranty period W is given by:*

$$\mathbb{E}(C(W)) = \int_0^W [1 + M(W-t)]k(t)dt$$

where $F_s(t)(f_s(t))$, $F(t)(f(t))$ are the lifetime distribution (density) functions of component 2 and component 1 respectively, $M(t)$ is the renewal function related with $F_s(t)$, $\bar{F}_s(t) = 1 - F_s(t)$, a is the imperfect maintenance degree of component 1, $v_n(t)$ is given in equation 1. $k(t) = c_1 \bar{F}_s(t) \sum_{n=0}^{\infty} n[av_n(at) - \int_0^{at} \frac{f(y + \frac{at-y}{a})}{\bar{F}(y)} v_n(y) dy] + c_2 f_s(t)$.

Under the RFRW, let $\mathbb{E}(C^R(W))$ be the expected manufacturer warranty cost under RFRW. We have the following theorem.

Theorem 3.2. *Under the RFRW, the expected system cost is*

$$\mathbb{E}(C^R(W)) = \sum_{n=0}^{\infty} \bar{F}_s(W) F_s^n(W) \left(nc_2 + c_1 \mathbb{E}(N(W)) + nc_1 \frac{\int_0^W \mathbb{E}(N(t)) dF_s(t)}{F_s(W)} \right)$$

where $F_s(t)$ is the component 2 lifetime distribution function, $\bar{F}_s(t) = 1 - F_s(t)$, $\mathbb{E}(N(t)) = \sum_{n=1}^{\infty} np_n(t)$ which is the expected failure times of component 1 before system failure.

3.2. The expected warranty profit and cost in $(0, T]$

In the following, by assuming that the consumer intends to put the product in service until time T , $W < T < \infty$. After T , the system is not repaired when failure occurs. Besides, after the product warranty period W , the consumer chose coming back to the original manufacturer for the maintenance.

In this paragraph, the warranty cost and profit to the consumer and the manufacturer respectively are examined. Hereafter we assume that component 1 is minimally repaired when failure occurs. Let $\mathbb{E}(C^s(T))$ and $\mathbb{E}(TP^s(T))$ be the total warranty cost of the consumer and the profit of the manufacturer respectively.

Theorem 3.3. *Under the RFRW, the expected maintenance cost of the consumer and the expected total profit of the manufacturer in $[0, T]$ are as follows:*

$$\begin{aligned}\mathbb{E}(C^s(T)) &= \int_W^T (1 + M(T - u))dh(u) \\ \mathbb{E}(TP^s(T)) &= \mathbb{E}(C^s(T)) - \mathbb{E}(C^1(T)) - c_2M(T)\end{aligned}$$

where

$$\begin{aligned}h(T) &= c_{22}(F_s(T) - F_s(W)) + c_{11} \int_W^T \bar{F}_s(t)d\mathbb{E}N(t) \text{ for } T \geq W \\ \mathbb{E}(C^1(T)) &= h_1(T) + \int_0^T h_1(T - t)dM(t) \\ h_1(T) &= c_1\mathbb{E}(N(T)) - c_1 \int_0^T F_s(t)d\mathbb{E}(N(t))\end{aligned}$$

$M(t)$ is the system renewal function related with $F_n(t)$ and $\mathbb{E}N(t)$ is the expected failure number of component 1 in $[0, t]$ before system replacement. $F_s(t)(f_s(t)), F(t)(f(t))$ are the lifetime distribution (density) functions of component 2 and component 1 respectively, $\bar{F}_s(t) = 1 - F_s(t)$.

Under the non-renewing FRW, the manufacturer pays the full warranty cost if the system fails within the warranty period and the warranty is not renewed. It is reasonable to assume that the consumer is provided with a

new system with a full warranty if the major failure occurs in $(W, T]$ and he/she covers himself/herself the full purchase cost.

Let $\mathbb{E}(C^{sn}(T))$ and $\mathbb{E}(TP^{sn}(T))$ be the expectations of the total warranty cost of the consumer and the profit of the manufacturer respectively.

Theorem 3.4. *Under the non-renewing FRW, when component 1 failures occur according to a Poisson process, the expected warranty cost of the consumer and the expected total profit of the manufacturer in $[0, T]$ are derived as follows:*

$$\begin{aligned}\mathbb{E}(C^{sn}(T)) &= h_n(T) + \int_0^T h_n(T-u) dM_U(u) \\ \mathbb{E}(TP^{sn}(T)) &= \mathbb{E}(C^{sn}(T)) - c_1 \mathbb{E}N(T) - c_2 M(T)\end{aligned}$$

where

$$F_U(t) = F_s(t) - \int_0^W \overline{F}(t-x) dM(x) \quad \text{for } t \geq W \text{ and } 0 \text{ otherwise}$$

$$h_n(t) = c_{22} F_U(t) + c_{11} \int_W^t \overline{F}_U(\theta) d\mathbb{E}N(\theta) \quad \text{for } t \geq W \text{ and } 0 \text{ otherwise}$$

$M_U(t)(M(t))$ is the system renewal function related with $F_U(t)(F_n(t))$ and $\mathbb{E}N(t)$ is the expected failure number of component 1 in $[0, t]$ before system replacement. $F_s(t)(f_s(t)), F(t)(f(t))$ are the lifetime distribution (density) functions of component 2 and component 1 respectively, $\overline{F}_s(t) = 1 - F_s(t)$.

4. Numerical examples

Here we suppose that component 1 is Weibull distributed with lifetime distribution function $F(t) = 1 - e^{(-\lambda t^b)}, t > 0$. The damage induced to component 2 by component 1 failure has exponential distribution with expectation μ . The natural deterioration of component 2 follows a homogeneous Gamma process [10,11] with density function

$$g_{\alpha t, \beta}(u) = \frac{\beta^{\alpha t} u^{\alpha t - 1} e^{-\beta u}}{\Gamma(\alpha t)}$$

where $\Gamma(\alpha) = \int_0^\infty u^{\alpha-1} e^{-u} du$.

Figure 1 shows the consumer's warranty costs and the manufacturer's profits under the RFRW with various values of W and μ . The expected warranty cost of the consumer (resp. the expected profit of the manufacturer) is a decreasing function of the warranty period. It is easy to understand that as the warranty period increases, the manufacturer has to cover

more warranty costs. In presence of interaction between components, for short warranty period, the manufacturer profit increases. As the warranty period gets longer, the average total manufacturer benefit (resp. the consumer total average cost) decreases. Which means in presence of interaction between components, a long warranty period is more cost efficient for the consumer.

The manufacture profits and consumer costs under both RFRW and non renewing FRW with different maintenance costs and initial warranty period are presented in Table 1. Both the manufacturer profit and the consumer cost decrease with the value of W . As W gets larger, more maintenance costs are covered by the manufacturer which induces the manufacture profit and the consumer cost. Both the consumer cost and the manufacturer profit increase with the consumers maintenance cost units c_1 and c_2 .

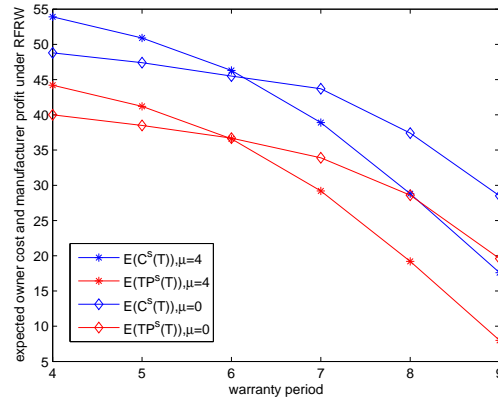


Figure 1. The expected consumer cost and manufacturer profit under RFRW with various values of W and μ when $\lambda = 1/64, b = 2, a = 1, \alpha = 4, \beta = 2, L = 20, c_1 = 5, c_2 = 25, c_{11} = 1, c_{22} = 4, T = 20$

5. Conclusions

In this study, a warranty cost and profit model for a two-component in series system is developed. The warranty cost and profit from both point of views, the manufacturer and the consumer, are derived. The numerical results show that the failure interaction between components can affect the system lifetime and therefore the warranty cost and the profit. The initial warranty length has impacts on the manufacturer's and the consumer's product quality preferences. It is recommended to the manufacturer consid-

Table 1. The warranty costs and profits under the RFRW and the non-renewing FRW with different values of c_1 , c_2 and W when $\lambda = 1/8, b = a = 1, \mu = 5, \alpha = 4, \beta = 2, L = 20, c_{11} = 1, c_{22} = 4, T = 20$

parameters cost/profit	$c_1 = 5, c_2 = 25$			$c_1 = 5, c_2 = 15$		
	$W = 4$	$W = 6$	$W = 9$	$W = 4$	$W = 6$	$W = 9$
$\mathbb{E}(C^s(T))$	50.0	40.9	17.3	31.5	25.0	10.5
$\mathbb{E}(TP^s(T))$	40.0	30.8	7.3	21.5	15.0	0.45
$\mathbb{E}(C^{sn}(T))$	50.5	42.9	28.1	29.6	25.6	17.0
$\mathbb{E}(TP^{sn}(T))$	40.4	32.8	18.0	20.6	16.7	8.2

ering the warranty cost/profit for better system reliability evaluation and profit maximization.

References

- [1] M. Park and H. Pham. Cost models for age replacement policies and block replacement policies under warranty. *Applied Mathematical Modelling* 40(9), 5689-5702, 2016. 43, 498-508, 2016.
- [2] R. H. Yeh, G. C. Chen, and M. Y. Chen. Optimal age-replacement policy for nonrepairable products under renewing free-replacement warranty. *IEEE transactions on reliability* 54(1), 92-97, 2015.
- [3] Y. H. Chien. A general age-replacement model with minimal repair under renewing free-replacement warranty. *European Journal of Operational Research* 186(3), 1046-1058, 2008.
- [4] M. N. Darghouth, A. Chelbi, and D. Ait-Kadi. A profit assessment model for equipment inspection and replacement under renewing free replacement warranty policy. *International Journal of Production Economics* 135(2), 899-906, 2012.
- [5] C. C. Wu, C. Y. Chou, and C. Huang. Optimal burn-in time and warranty length under fully renewing combination free replacement and pro-rata warranty. *Reliability Engineering & System Safety* 92(7), 914-920, 2007.
- [6] Y. H. Chien. The effect of a pro-rata rebate warranty on the age replacement policy with salvage value consideration. *IEEE Transactions on Reliability* 59(2), 383-392, 2010.
- [7] M. Jain and S. Maheshwari. Discounted costs for repairable units under hybrid warranty. *Applied Mathematics and Computation* 173(2), 887-901, 2006.
- [8] B. Liu, I. Wu and M. Xie. Cost analysis for multi-component system with failure interaction under renewing free-replacement warranty. *European Journal of Operational Research* 243(3), 874-882, 2015.
- [9] M. Kijima. Some results for repairable systems with general repair. *Journal of Applied probability* 89-102, 1989.
- [10] M. Fouladirad, A. Grall, and L. Dieulle. On the use of on-line detection for maintenance of gradually deteriorating systems. *Reliability Engineering & System Safety* 93(12), 1814-1820, 2008.
- [11] J. M. Van Noortwijk. A survey of the application of gamma processes in maintenance. *Reliability Engineering & System Safety* 94(1), 2-21, 2009.