A three-phase matheuristic for the Packaging and Shipping Problem

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Abstract

E-commerce has been continuously growing in the last years to a primary retail market. Recently in France, the threshold of 1 billion of online transactions was overcome. Due to a high demand fluctuation of e-commerce, the workforce sizing for the logistic chain is a challenging problem. Companies have to develop good strategies to have a sustainable workforce size while guaranteeing a high-level service.

In this paper, we consider the management of workforce for a warehouse of an e-commerce company. Specifically, we address issues as i) How the workforce at the warehouse can be determined; ii) What is the daily operational production planning; iii) How the demand peaks can be smoothed, and the production maintained ideally constant over the time horizon.

To provide answers to the issues, we introduce the Packaging and Shipping Problem (PSP). The PSP looks for a solution approach that jointly determines the workforce over a multi-period horizon and daily operational plans while minimizing the total logistics cost. We considered two strategies that aim to enhance the flexibility of the process and the efficiency of resources use: *reassignment* and *postponement*. To tackle the Packaging and Shipping Problem we propose a model, and a three-phase matheuristic. This heuristic proves to be competitive with respect to the direct solution of the model with a commercial solver on real-life based instances.

1 Introduction

E-commerce has been continuously growing in the last years to a primary retail market. In 2016 in France, the threshold of 1 billion of online transactions was overcome for a total of 72 billion Euros [1]. At the same time, new challenges arise in the e-commerce sector. Due to high demand fluctuation, the workforce sizing for the logistic chain is a challenging problem: a high number of workers guarantees demand satisfaction to face all situations, but at a high cost. On the other side, reducing the personnel results in delays that lead to customer dissatisfaction and negative impacts on the company image. The company has to develop appropriate strategies leading a sustainable workforce size while guaranteeing a high-level service.

In this paper, we aim to model and to provide a solution method for a problem occurring in the management of workforce for a warehouse of an e-commerce company. This problem

[1]http://www.fevad.com/bilan-2016-e-commerce-france-cap-70-milliards-a-ete-franchi/

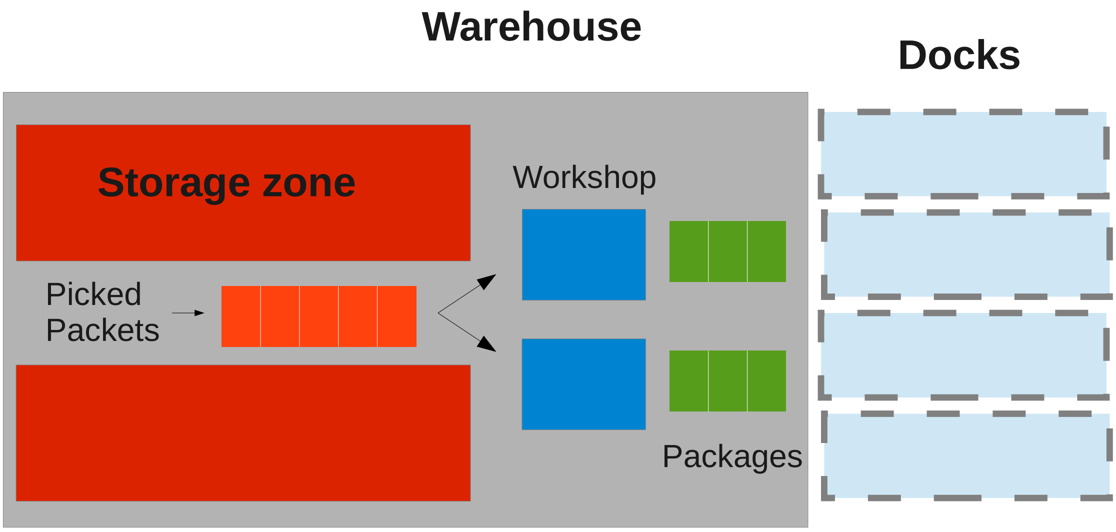

Figure 1: Warehouse sketch

was originally motivated by a real-case study introduced to us by one of the leading third-party logistics company. We address here three main issues: i) How the workforce at the warehouse can be determined; ii) What is the daily operational production planning; iii) How the demand peaks can be smoothed and the production maintained ideally constant over the horizon.

To address the last issue and face the high fluctuation of the demand, we propose two strategies to smooth the workload, i.e., *production postponement* and *demand reassignment*. Roughly speaking, the first strategy allows delaying production on successive days, while the second strategy allows assigning a given order to delivery service different than the one selected by the customer. We detail such policies below.

To provide answers to the first two issues, we address the Packaging and Shipping Problem (PSP) to optimize the preparation order process. In this problem, we deal with tactical and operational decisions simultaneously. In particular, we take into account the management of the workforce over a multi-day horizon and the determination of the number of workers required for all the shifts of each day. Moreover, we provide the operational planning required to prepare the total demand received during each day of the planning horizon.

From a managerial point of view, the model and the matheuristic we propose can help companies in two different ways. First, based on historical data it can help understanding the value of postponement and re-assignment strategies. Companies would have an insight on the gain that would achieve adopting such strategies and comparing this gain with the cost of implementing them. Second, it can help to determine the number employees needed to ensure production in different scenarios. Such decision support can be obtained in an offline fashion, based on historical data to hire employees, or in an online fashion based on real-time data to determine the number of temporary workers that should be hired to guarantee production. Finally, the model provides a detailed production planning taking into account postponement and re-assignment strategies.

The process of an order received at a warehouse consists of different sequential phases. First, the items that form the order need to be picked up in the storage zone, then they need to be put in standard packages and finally loaded in appropriate trucks. In the case considered here, the trucks transfer packages to carrier hubs where they are sorted according to their final destination. A a representation of the different areas of a warehouse is given in Figure 1.

For simplicity, a volume is associated with each order. As an order can include several items, the volume represents the number of packages required to contain these items. Also, during online ordering, the customer can choose among different delivery options, called *modes*, e.g., standard or express delivery service. Each mode is associated with different delivery delays (for example the standard mode delivers within 3-5 business days or the express mode within 24 hours) and prices to be paid by the customers. Notice that the word *mode* is used to distinguish between different transportation services. In other papers, as in Alptekinoğlu and Tang [3], the modes are the different modes of distribution, as in-store, mail-order or internet-based services.

The tackled problem consists in determining the daily operations within the warehouse. Each day is composed by *shifts*, and a shift by *periods*. During each shift, a certain number of employees is in charge of the demand preparation. Workers are associated with a *productivity*, i.e., the number of packages they can prepare during a period. The employees can be *permanent* or *temporary*. An employee works for the entire shift and guarantees a productivity level during each period of the shift. Temporary workers can be hired for one shift only, usually when the workload is estimated to be high. On the other side, they cost more than a permanent worker. It is assumed that the productivity of a permanent worker is higher than the productivity of a temporary worker.

The number of workers assigned to each shift is a decision variable of the problem. The preparation plan for each time period needs to be determined, namely, for each order, the number of packages prepared at each time period needs to be set. Note that an order with more than one package can be prepared during different, and not necessarily consecutive, periods.

Once an order has been processed, its packages need to be loaded in the trucks. Trucks need to be present at the warehouse docks to be loaded. Each truck is associated with one delivery mode, and it contains only packages of orders delivered with that mode. Then, if orders associated with different modes are processed at a given period, at least one truck for each mode must be docked.

Since packages related to an order can be prepared during different periods, they can be loaded in different trucks. Moreover, at each period, the number of packages prepared is limited by the number of workers and the number of docked trucks.

The warehouse, where operations take place, has a limited number of docks that, consequently, limits the number of trucks that are loaded simultaneously. The docks are a critical resource that impacts the overall fluidity of picking and shipping operations. We thus define a truck movement policy that manages spatially and temporally the use of docks. The proposed truck movement policy is detailed in Section 3.

After describing the general structure of the warehouse and the classical steps of order picking and shipping, we introduce two new strategies that aim to enhance the flexibility of the process and the efficiency of resources use: *reassignment* and *postponement*.

Reassignment allows delivering an order, i.e., all associated packages, with another mode than the one selected by the customer. When reassignment is operated, the company pays a penalty that represents either an additional delivery cost (when for example the change is from the standard mode to the express mode) or the dissatisfaction of the customer for a possible late delivery (when the change is from the express mode to the standard). Due to the penalty cost, reassignment takes place when it allows to hire fewer workers or to use fewer trucks.

Postponement comes into play due to the multi-period nature of the problem. Since we need to plan the operations for several successive days, we then introduce the possibility to *postpone* to next days the process of some orders. As a counterpart, a penalty is paid because a postponement leads to a delay. As for the reassignment case, when the preparation of a given order is postponed, the preparation of all its packages is postponed. Postponement can be combined with reassignment when, for instance, some orders associated with the standard delivery mode are delivered with the express mode of the day after.

The PSP looks for a solution that determines the workforce over a multi-period horizon and operational daily plans which minimize the total cost of shipping and picking operations. Specifically, the total cost is computed as the sum of the cost of hiring the workers (permanent and temporary), the cost of the trucks used, the penalty cost for order postponements and reassignments, and the dock utilization.

The contribution of this paper is threefold. First, we define the PSP and provide a mathematical programming formulation. Second, we consider two strategies, reassignment and postponement, to smooth the high demand peaks that characterize e-commerce demand profiles. Third, we present a three-phase matheuristic induced by a natural decomposition of the proposed model to obtain efficient solutions.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 formally introduces the problem and presents a mathematical model for the PSP. Section 4 describes the three-phase algorithm we developed to obtain good solutions for the PSP. Section 5 presents the computational results. Finally conclusions are drawn in Section 6.

2 Literature review

The Packaging and Shipping Problem described in Section 1 aims to simultaneously determine the number of employees, the reassignment and postponement of demands, and a complete and detailed production planning. In the following, we review some academic papers that share similarities with our work. From a general point of view, the PSP belongs to the category of *integrated* problems, where different level decisions are taken simultaneously, i.e., strategic and tactical decisions, or tactical and operational decisions. It is well known that integrated decisions lead to cost savings (Chandra and Fisher [8]) against the increase of the problem complexity. Integration of decisions increases the coordination of the distribution system and, consequently, its efficiency. It results in cost reductions (from 3% to 20%, Strack et al. [21]) or storage levels reduction and leading to better use of the resources. Moreover, taking decision sequentially and without taking into account interaction among decisions can lead to approaches that provide infeasible solutions. In the recent past, the scientific community devoted a growing interest to this class of problems.

The PSP integrates tactical decisions (determination of the number of employees) as long as operational decisions (production planning). An example of the tactical-operational integrated problems is the Production Routing Problem (PRP, Absi et al. [1]), where a lot-sizing problem is combined with a vehicle routing problem. Another example of an integrated problem where lot-sizing decision, as well as production and transportation decisions, are considered can be found in [18]. The reader interested in production routing problems is referred to the survey of Adulyasak et al. [2]. The work of Hiassat et al. [12] goes a step further. In they work they deal with the location-inventory-routing problem where perishable products are considered. Here strategic, tactical and operational decision are simultaneously taken.

PSP does not deal with routing issues but coordinates production and vehicle loading (shipment release). Two examples of works coordinating production and shipping are Baptiste et al. [5] and Lee et al. [13]. In Baptiste et al. [5] production lines are organized with batches that are subsequently loaded into trucks and shipped to their final destinations. The problem consists in determining the production plan in such a way that a set of full trucks is dispatched to different destinations. The plan needs to respect logistics constraints (number of production lines, number of docks) and to minimize the delivery costs.

Lee et al. [13] study the coordination of inbound and outbound flows of a product at a warehouse. The inbound flow guarantees inventory level satisfaction, and it is managed by coordinating production at the manufacturer level. From the warehouse, a set of distribution centers is served using full truckload. The problem is modeled as a two-echelon inventory lot-sizing problem with shipment scheduling and looks for a solution that minimizes the transportation costs and inventory cost required for the replenishment of the warehouse and the distribution centers.

The PSP considers different delivery services. With this respect, the work of Wang and Lee [22] addresses a variant of the PSP. As in the PSP, they consider the possibility to deliver the products manufactured by the company using different delivery services: standard and express shipping modes. Modes have different costs and travel times. In Wang and Lee [22] production has to be scheduled in such a way that products can be transported at customer locations before prespecified due dates. Another example of integrated optimization problem that considers different transportation modes is presented in Siddiqui et al. [19]. This work deals with an integrated inventory and transportation problem that occurs when petroleum needs to be moved to refineries. The authors consider the possibility of choosing between pipelines and maritime transportation taking into account environmental risks.

Employees management in the PSP relies only on *demand modeling* (following the terminology of Ernst et al. [10]). In the PSP, we have to determine the number of employees that are needed during each shift to process the total demand. More complex rostering problems require the determination of a detailed working plan for each employee. Applications can be found in freight handling at cargo terminals (Rong and Grunow [17]), in demand satisfaction of time-dependent requirements for check-in of individual flights (Stolletz and Zamorano [20]).

Postponement strategies have been widely studied. The results obtained highlight the benefit of such policies. We can say that a problem involves a postponement strategy when it allows delaying some operations. In Carbonara and Pellegrino [7] *postponement consists of delaying activities in the supply chain until customer order information becomes available*. Here we decide to postpone a delivery if this is beneficial for the company. The Inventory Routing Problem (IRP, Bertazzi et al. [6]) can be seen as a vehicle routing problem with a postponement policy. If a customer consumes q units of product per day, the delivery company is not obliged to deliver q units per day, but Tq units of products over a horizon of T days to avoid stockouts. This allows to postpone services and to deliver larger quantities in fewer visits. Significant cost savings can then be generated (Pang and Muyldermans [15]). Mahar and Wright [14] develop a dynamic strategy for online orders fulfillment for a multi-channel retail company. Online orders are accumulated before being assigned to a specific site for fulfillment. This policy results in cost reductions compared with an assign-as-order-arrive strategy.

The third main characteristic of our problem is the reassignment policy, i.e., assignment of an order to a delivery channel different than the one selected by the customer. The Lot-Sizing Problem (LSP) with one-way substitution shares similar characteristics. In this case, the demand can be satisfied by recovering used items. In case of running out of used items, new items can be supplied instead (see for example Piñeyro and Viera [16]). This generates a loss for the company that sells products at a lower price. Opposite substitution (supply recovered items instead of new ones) is not allowed. In our problem, the reassignment strategy can be seen as substituting delivery mode. However, the PSP differs from the LSP, since both substitutions are considered.

Last, problems integrating decisions of the same level have been tackled. For example, when order-to-order production (scheduling) and transportation decisions are coupled, decisions at the operational level are integrated. The reader is refereed to Chen [9] for a recent survey of such models.

$\mathcal{H}$	Set indexing periods in the planning horizon
$\bar{\mathcal{H}}$	Set indexing postponement periods
$\mathcal{D}_h$	Set indexing orders to be processed during period h
$\mathcal{V}$	Set indexing the delivery modes
H	Periods in the planning horizon
$\bar{H}$	Postponement periods in the planning horizon
D_h	Number of orders to be processed during period h
d	A specific order
V	Number of delivery modes
vol_{hd}	Number of packages that compose order d of period h
v_{hd}	Delivery mode of order d of period h
r_{hd}	Release date of order d of period h
$p^{\bar{h}}_{v_{hd}v}$	Penalty due to postponement to $\bar{h}$ and to re-assignment to v order d of period h
t_v	Departure slot of vehicles associated to mode v
Q	Truck capacity
c^{truck}	Truck cost
N_{max}	Number of docks
S	Number of shifts per period
T	Number of slots per period
$\bar{T}$	Number of slots per shift
$start_{hs}$	Starting slot for shift s at period h
end_{hs}	Ending slot for shift s at period h
c^{per}_{hs}	Cost of a permanent employee working at shift s of period h
c^{temp}_{hs}	Cost of a temporary employee hired at shift s of period h
$prod^{per}_{hs}$	Number of packages that can be prepared by a permanent employee during shift s of period h
$prod^{temp}_{hs}$	Number of packages that can be prepared by a temporary employee during shift s of period h
e^{max}_{hs}	Maximum number of permanent employees at shift s of period h

Table 1: Notation

3 Problem definition, notation and model

In this section, we formally define the PSP and we provide a mix-integer mathematical formulation for the problem. Table 1 contains the notation used in the paper.

We consider a planning horizon of H periods indexed in $\mathcal{H} = \{0, \ldots, H-1\}$. Typically, a period is as a day. At each period $h \in \mathcal{H}$, D_h orders have to be processed (indexed in $\mathcal{D}_h = \{0, \ldots, D_h - 1\}$). Orders revealed on period h need to be prepared in one of the following $\bar{H}$ periods, indexed in $\bar{\mathcal{H}} = \{0, \ldots, \bar{H} - 1\}$. When $\bar{h} = 0$, $\bar{h} \in \bar{\mathcal{H}}$, orders are not postponed. There are V available delivery modes, indexed in $\mathcal{V} = \{0, \ldots, V-1\}$.

Each order $d \in \mathcal{D}_h$ of period h is characterized by its

- volume vol_{hd}: the number of packages it is composed;
- delivery mode v_{hd};
- time slot at which the order becomes known (release date of the order) r_{hd};
- penalty $p^{\bar{h}}_{v_{hd}v}$ for processing the order at period $h + \bar{h}$ and assigning it to mode v.

As an example, $\bar{h} = 1$ corresponds to a postponement of one day, while $v_{hd} \neq v$ corresponds to a reassignment. Naturally $p^0_{v_{hd}v_{hd}} = 0$. We assume that the penalty for postponing a package from period h to period $h + \bar{h}$ or changing its mode is identical for all orders.

Each delivery mode v is characterized by its departure slot t_v. No truck associated with mode v will be available after t_v. All the trucks have the same capacity Q and the same cost c^{truck}. Moreover, $N_{\max}$ is the number of available docks at the warehouse, thus at most $N_{\max}$ trucks can be simultaneously loaded.

Each period h is divided into S shifts , and each shift into $\bar{T}$ slots. It follows that each period is divided into $T = S\bar{T}$ slots. Each shift s of period h is characterized by its

- starting slot $start_{hs}$;
- ending slot end_{hs};
- cost for a permanent employee c^{per}_{hs};
- cost for a temporary employee c^{temp}_{hs};
- number of packages a permanent employee can prepare $prod^{per}_{hs}$;
- number of packages a temporary employee can prepare $prod^{temp}_{hs}$;
- maximum number of permanent employees $e^{\max}_{hs}$.

Trucks are managed according to the following *truck movement policy.* Each truck is assigned to one and only one delivery mode, i.e., it will carry only packages assigned to that mode. Trucks can be made available at the docks at any slot. When a truck is fully loaded, it is undocked by the end of the slot. As a consequence, the dock, it has occupied, becomes free for use at the beginning of the next slot. If necessary, more than one truck per mode can be simultaneously docked. If the truck is not fully loaded at the end of a slot, it remains docked for the next slot. Non-full trucks for mode v are undocked in two cases: at slot t_v or when no package for mode v will be assigned to the corresponding mode during the following slots of the period.

Over the planning horizon, the PSP aims to determine the number of employees and trucks, an order process planning that consists in identifying the exact slot during which each package of each order is processed, and a truck management planning (i.e., when to dock and undock trucks) in order to minimize the sum of the employees and trucks costs, penalties, and the docks occupation.

Let us now present the mathematical formulation of the PSP. We first introduce the variables of the model. For each period $h \in \mathcal{H}$, for each postponement period $\bar{h} \in \bar{\mathcal{H}}$, for each $d \in \mathcal{D}_h$, for each mode $v \in \mathcal{V}$ and for each shift $s \in \mathcal{S}$ we have:

- tactical variables:
  - z^{per}_{hs} the number of permanent workers working on shift s of period h;
  - z^{temp}_{hs} the number of temporary workers working on shift s of period h;
- operational variables:
  - $x^{\bar{h}v}_{hd}$ equals 1 if the order d of period h is prepared in period $h + \bar{h}$ and assigned to mode v, 0 otherwise;
  - y^{vt}_h equals 1 if the number of empty trucks for mode v during period h at a slot $\bar{t} \geq t$ is not null, 0 otherwise;

- $f_{hd}^{\bar{h}vt}$ indicates the number of packages of order d prepared in slot t of period $h+\bar{h}$ assigned to mode v;
- w_h^{vt} is the number of docked trucks for mode v at period h in slot t;
- u_h^{vt} is the number of empty trucks for mode v that are docked at period h in slot t;
- k_h^{vt} is the residual capacity of trucks at period h in slot t for mode v.

The mathematical model for the PSP reads as follows:

$$
\begin{aligned}
(PSP) \quad & \min \sum_{h\in\mathcal{H}} \sum_{d\in\mathcal{D}_h} \sum_{\bar{h}\in\bar{\mathcal{H}}} \sum_{v\in\mathcal{V}} p^{\bar{h}}_{v_{hd}v}\, vol_{hd}\, x_{hd}^{\bar{h}v} + \sum_{h\in\mathcal{H}} \sum_{s\in\mathcal{S}} (c_{hs}^{per} z_{hs}^{per} + c_{hs}^{temp} z_{hs}^{temp}) + \\
& + c^{truck} \sum_{h\in\mathcal{H}} \sum_{v\in\mathcal{V}} \sum_{t\in\mathcal{T}} u_h^{vt} + \alpha \sum_{h\in\mathcal{H}} \sum_{v\in\mathcal{V}} \sum_{t\in\mathcal{T}} w_h^{vt} && (1)\\
\text{s.t.} \quad & \sum_{\bar{h}\in\bar{\mathcal{H}}} \sum_{v\in\mathcal{V}} \sum_{t\in\mathcal{T}} f_{hd}^{\bar{h}vt} = vol_{hd} \quad \forall h\in\mathcal{H},\ \forall d\in\mathcal{D}_h && (2)\\
& \sum_{t\in\mathcal{T}} f_{hd}^{\bar{h}vt} \le vol_{hd}\, x_{hd}^{\bar{h}v} \quad \forall h\in\mathcal{H},\ \forall d\in\mathcal{D}_h,\ \forall \bar{h}\in\bar{\mathcal{H}},\ \forall v\in\mathcal{V} && (3)\\
& \sum_{\bar{h}\in\bar{\mathcal{H}}} \sum_{v\in\mathcal{V}} x_{hd}^{\bar{h}v} = 1 \quad \forall h\in\mathcal{H},\ \forall d\in\mathcal{D}_h && (4)\\
& f_{hd}^{0vt} = 0 \quad \forall h\in\mathcal{H},\ \forall d\in\mathcal{D}_h,\ \forall v\in\mathcal{V},\ 0\le t< r_{hd} && (5)\\
& \sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\ge 0}} \sum_{d\in\mathcal{D}_{h-\bar{h}}} f_{(h-\bar{h})d}^{\bar{h}v0} + k_h^{v0} = Q u_h^{v0} \quad \forall h\in\mathcal{H},\ \forall v\in\mathcal{V} && (6)\\
& \sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\ge 0}} \sum_{d\in\mathcal{D}_{h-\bar{h}}} f_{(h-\bar{h})d}^{\bar{h}vt} + k_h^{vt} = k_h^{v(t-1)} + Q u_h^{vt} \quad \forall h\in\mathcal{H},\ \forall v\in\mathcal{V},\ 0<t\le t_v && (7)\\
& \sum_{t=\bar{t}}^{t_v} u_h^{vt} \le N_{\max}\, y_h^{v\bar{t}} \quad \forall h\in\mathcal{H},\ \forall v\in\mathcal{V},\ 0\le \bar{t}\le t_v && (8)\\
& y_h^{v\bar{t}} \le \sum_{t=\bar{t}}^{t_v} u_h^{vt} \quad \forall h\in\mathcal{H},\ \forall v\in\mathcal{V},\ 0\le \bar{t}\le t_v && (9)\\
& u_h^{v0} \le w_h^{v0} \quad \forall h\in\mathcal{H},\ \forall v\in\mathcal{V} && (10)\\
& Q u_h^{vt} + k_h^{v(t-1)} \le Q w_h^{vt} + Q(1-y_h^{vt}) \quad \forall h\in\mathcal{H},\ \forall v\in\mathcal{V},\ 0<t\le t_v && (11)\\
& Q u_h^{vt} + k_h^{v(t-1)} - k_h^{v(t_v-1)} \le Q w_h^{vt} \quad \forall h\in\mathcal{H},\ \forall v\in\mathcal{V},\ 0<t\le t_v && (12)\\
& \sum_{v\in\mathcal{V}} w_h^{vt} \le N_{\max} \quad \forall h\in\mathcal{H},\ \forall t\in\mathcal{T} && (13)\\
& \sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\ge 0}} \sum_{d\in\mathcal{D}_{h-\bar{h}}} \sum_{v\in\mathcal{V}} f_{(h-\bar{h})d}^{\bar{h}vt} \le prod_{hs}^{per} z_{hs}^{per} + prod_{hs}^{temp} z_{hs}^{temp} \\
& \qquad \forall h\in\mathcal{H},\ \forall s\in\mathcal{S}, start_{hs}\le t\le end_{hs} && (14)\\
& z_{hs}^{per} \le e_{hs}^{\max} \quad \forall h\in\mathcal{H},\ \forall s\in\mathcal{S} && (15)\\
& z_{hs}^{temp} \le z_{hs}^{per} \quad \forall h\in\mathcal{H},\ \forall s\in\mathcal{S} && (16)\\
& x_{hd}^{\bar{h}v} \in \{0,1\}\ \forall h\in\mathcal{H},\ \forall d\in\mathcal{D}_h,\ \forall \bar{h}\in\bar{\mathcal{H}},\ \forall v\in\mathcal{V} && (17)\\
& y_h^{vt} \in \{0,1\}\ \forall h\in\mathcal{H},\ \forall v\in\mathcal{V},\ \forall t\in\mathcal{T} && (18)
\end{aligned}
$$

$$z_{hs}^{per}, z_{hs}^{temp} \in \mathbb{N} \; \forall h \in \mathcal{H}, \; \forall s \in \mathcal{S} \tag{19}$$

$$f_{hd}^{\bar{h}vt} \in \mathbb{N} \; \forall h \in \mathcal{H}, \; \forall d \in \mathcal{D}_h, \; \forall \bar{h} \in \bar{\mathcal{H}}, \; \forall v \in \mathcal{V}, \; \forall t \in \mathcal{T} \tag{20}$$

$$w_h^{vt}, k_h^{vt}, u_h^{vt} \in \mathbb{N} \; \forall h \in \mathcal{H}, \; \forall v \in \mathcal{V}, \; \forall t \in \mathcal{T} \tag{21}$$

Before analyzing the model, we can underline from variable x and f definitions that orders are processed individually. The model offers a highly accurate tracking of order processing information, and it is possible to retrieve for each order the exact slot of its process. Such accurate tracking is required in e-commerce for the management of the whole delivery route of the order as well as for the management of customer relationship.

The objective function (1) aims to minimize the cost of processing all the orders. This cost is given by the sum of four terms computed over the planning horizon. The first term is the sum of all penalties due to postponements and reassignments. The second term in the objective function is the total labor cost computed as the sum of all workers costs, while the third is the cost of the used trucks. The fourth term is a measure of the docks occupation, and it is incremented each time a dock is occupied by a truck during one time slot. We express this term in dock-slot as it is the case when we measure an amount of work in man-hour or man-day units. The coefficient α converts this term from the dock-slot unit into a cost unit.

Constraints (2) ensure that all the packages that compose an order are prepared. Constraints (3) and Constraints (4) impose that each order is assigned to only one mode and processed entirely during the same period. Constraints (5) forbid to prepare orders before their release date. Constraints (6)–(7) is the packages flow conservation: processed packages are loaded in an already docked truck with some residual capacity or in an empty truck. These constraints are formulated differently for the first slot of each period. Note that the truck residual capacities are updated at every slot. Constraints (8) (resp. Constraints (9)) force variables y_h^{vt} to be one (resp. zero) if (resp. if no) additional trucks for the mode v will be used during the slot t or the slots after t of period h.

Together constraints (10)–(11) and (12) enable to apply the truck movement policy explained earlier in this section. They update variables w_h^{vt} that represent the exact number of docks occupied by the trucks associated to each mode at each slot. At the first slot of each period, constraints (11)–(12) have specific formulation (10) that corresponds to the first slot of each period. These formulations are different from the general form because they do not implicate truck residual capacities. In Constraints (10)–(11), a new truck docked at a given slot (variables u_h^{vt}) naturally implies that a dock is occupied. On the other side, a truck remains on dock only if other trucks associated with the same mode are expected to be used in the upcoming slots ($y_h^{vt} = 1$). Constraints (12) complete the truck movement policy by handling the particular case, not handled by Constraints (10)–(11), where a truck already docked should remain on dock because a number of packages, inferior to the current residual capacity, is expected in the upcoming slots. Note that it is possible to formulate Constraints (10)–(11) and (12) in a more compact way, by expressing variables y_h^{vt} in terms of expected upcoming packages instead of expected upcoming trucks. The proposed formulation was preferred since it presents a good separability, in the sense that Constraints (8) and (9) involve variables associated with one same period.

Constraints (13) impose a limit on the number of docks available. Constraints (14) impose that the number of packages to be prepared in each slot should not exceed the production capacity of the workers. Constraints (15) impose a limit on the number of permanent workers. Constraints (16) ensure that there are not more temporary workers than permanent workers. Otherwise, we assume that permanent workers should be on duty. Constraints (17)–(21) define the integrality or binary requirements.

To conclude the section, we give in the following complexity results for the PSP.

Proposition 1. *The Packaging and Shipping Problem (PSP) is $\mathcal{NP}$-hard.*

Proof. The proof is given in Appendix A. □

4 A three-phase matheuristic

To solve the PSP, we propose an algorithm based on a three-phase matheuristic. The choice for developing a matheuristic is motivated by the observation that the model introduced in Section 3 presents a natural decomposition into subproblems involving types of decisions ranging from tactical ones to operational ones. Our three-phase approach sequentially solves three sub-problems of the PSP, in a way that the solution of each sub-problem is the input for the next phase. The solution of the third sub-problem is, in turn, a solution for the PSP. Following the classification of matheuristics proposed by Ball [4], our procedure falls into the *decomposition approach* category: sub-problems are sequentially solved to identify a feasible solution for the original problem.

In our three-phase approach, the first phase solves a relaxation of the PSP model presented in Section 3. It determines the workers needed to process all the orders. In other words, for all $h \in \mathcal{H}$ and $s \in \mathcal{S}$ we fix the values of variables z_{hs}^{per} and z_{hs}^{temp}. This phase leads to take the tactical decisions under aggregated operational constraints. The second phase determines the complete orders process planning and sets the reassignments and the postponements. Specifically, it determines, for each $h \in \mathcal{H}$, $d \in \mathcal{D}_h$, $\bar{h} \in \bar{\mathcal{H}}$, $v \in \mathcal{V}$, the values of variables $x_{hd}^{\bar{h}v}$. This phase focuses at the operational level based on tactical decisions taken at the previous phase of the method. The solution provided by the second phase does not consider docks occupation minimization but provides a feasible solution for PSP. Therefore, the algorithm could be stopped after this phase.

If the algorithm is continued, the output of the second phase is used as input for the last phase which considers the truck movement policy and minimizes the dock occupation. This phase refines operational decisions to optimize the dock occupation. The different phases are detailed in the next sections. An outline of the three-phase procedure is given in Algorithm 1.

For each of the three phases, we propose a speed-up technique to decrease computation times. In particular, we compute two valid lower-bounds on the objective function for the models solved in phase I and phase II. Each of these lower bounds is used to define a stopping criterion. Before starting the third phase of the algorithm, we implement an order *aggregation* procedure which groups all the orders with the same characteristics. Indeed, the reassignments and the postponements are determined in phase II, and the aggregation procedure does not reduce the set of feasible solutions as it will be further explained in Section 4.4.3.

This decomposition approach is based on the distinction of the different decisions regarding their nature. In the first phase, the tactical decisions, namely, the workers needed for production, are determined. The second and the third phases focus on the operational decisions. First, we determine a complete and feasible planning, then we optimize the production planning again to minimize the quay occupancy. Sections 4.1–4.3 present the three phases of the algorithm. Section 4.4 presents the speed-up techniques.

4.1 Phase I - Production capacity

This phase determines the number of workers required during each shift of the planning horizon. To achieve this aim, we solve a relaxation of the model (PSP) presented in Section 3. The relaxation does not consider the truck management issues, i.e., Constraints (6)–(13) are not taken into account. The relaxation of the model (PSP) is based on the following proposition.

Algorithm 1 Three-phase algorithm

```
1: Phase I
2: Compute a lower-bound for model PSP I: LBI (Section 4.4.2)
3: while Time limit not reached do
4:    Solve model PSP I (Section 4.1)
5:    for all Feasible solutions found do
6:       if Solution optimality proved or solution cost equal to LBI then
7:          Go to Step 13
8:       end if
9:    end for
10:   end
11: end while
12: end
13: Fix the number of workers based on the solution of model PSP I
14: Phase II
15: Compute a lower-bound for model PSP II: LBII (Section 4.4.1)
16: while Time limit not reached do
17:    Solve model PSP II (Section 4.2)
18:    for all Feasible solutions found do
19:       if Solution optimality proved or solution cost equal to LBII then
20:          Go to Step 26
21:       end if
22:    end for
23:    end
24: end while
25: end
26: Fix reassignments and postponements based on solution of model PSP II
27: Phase III
28: Aggregate orders (Section 4.4.3)
29: while Time limit not reached do
30:    Solve model PSP III (Section 4.3)
31: end while
32: end
33: Disaggregate orders
```

Proposition 2. *The following model :*

$$(PSP\ I) \quad \min \sum_{h\in\mathcal{H}} \sum_{d\in\mathcal{D}_h} \sum_{\bar{h}\in\bar{\mathcal{H}}} \sum_{v\in\mathcal{V}} p^{\bar{h}}_{v_{hd}v}\, vol_{hd}\, x^{\bar{h}v}_{hd} + \sum_{h\in\mathcal{H}} \sum_{s\in\mathcal{S}} (c^{per}_{hs} z^{per}_{hs} + c^{temp}_{hs} z^{temp}_{hs}) \tag{22}$$

$$s.t.\quad (2)\text{–}(5)$$

$$\sum_{v\in\mathcal{V}} \Big(\sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}} \sum_{d\in\mathcal{D}_{h-\bar{h}}} f^{\bar{h}vt}_{(h-\bar{h})d} + k^{vt}_h \Big) \leq QN_{\max} + QV \quad \forall h\in\mathcal{H},\ 0\leq t\leq t_v \tag{23}$$

$$(14)\text{–}(20)$$

is a valid relaxation for model (PSP).

In this model, we change the objective function of (PSP I) compared to (1), and we substitute Constraints (6)–(13) by Constraints (23).

Proof. By summing Constraints (6) over all modes in $\mathcal{V}$, we obtain (using Constraints (10)):

$$\sum_{v\in\mathcal{V}} \Big(\sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}} \sum_{d\in\mathcal{D}_{h-\bar{h}}} f^{\bar{h}vt}_{(h-\bar{h})d} \Big) \leq Q \sum_{v\in\mathcal{V}} (w^{vt}_h + (1-y^{vt}_h)) - \sum_{v\in\mathcal{V}} k^{vt}_h$$

From Constraint (13) it follows:

$$\sum_{v\in\mathcal{V}} \Big(\sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}} \sum_{d\in\mathcal{D}_{h-\bar{h}}} f^{\bar{h}vt}_{(h-\bar{h})d} + k^{vt}_h \Big) \leq QN_{\max} + Q\Big(\sum_{v\in\mathcal{V}} (1-y^{vt}_h)\Big) - \sum_{v\in\mathcal{V}} k^{vt}_h$$

From Constraint (21) on the variables, it follows that the term $\sum_{v\in\mathcal{V}} k^{vt}_h$ is positive, then

$$\sum_{v\in\mathcal{V}} \Big(\sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}} \sum_{d\in\mathcal{D}_{h-\bar{h}}} f^{\bar{h}vt}_{(h-\bar{h})d} + k^{vt}_h \Big) \leq QN_{\max} + Q\Big(\sum_{v\in\mathcal{V}} (1-y^{vt}_h)\Big)$$

Finally, the term $\sum_{v\in\mathcal{V}}(1-y^{vt}_h) \in \{0,\ldots,V\}$ equals V when all the variables y^{vt}_h equal 0, i.e., when the process is ended. Then, we obtain:

$$\sum_{v\in\mathcal{V}} \Big(\sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}} \sum_{d\in\mathcal{D}_{h-\bar{h}}} f^{\bar{h}vt}_{(h-\bar{h})d} + k^{vt}_h \Big) \leq QN_{\max} + QV$$

□

(PSP I) is solved with a commercial solver, and the solution is used to determine the number of workers assigned to each shift over the planning horizon.

4.2 Phase II - Reassignment and postponement

Based on the decisions obtained in phase I, the second phase of the algorithm determines the assignment of each order to a period and to a delivery mode (variables $x^{\bar{h}v}_{hd}$). The productivity capacity during each shift is known from phase I, i.e., the values of variables z^{per}_{hs} and z^{temp}_{hs} are now fixed. Moreover, we do not minimize the platform occupation, i.e., the term $\sum_{h\in\mathcal{H}} \sum_{v\in\mathcal{V}} \sum_{t\in\mathcal{T}} w^{vt}_h$ is removed from the objective function. The model solved in this phase is the following (the variables z^{per}_{hs} and z^{temp}_{hs} are replaced by the parameters ζ^{per}_{hs} and ζ^{temp}_{hs}):

$$
\text{(PSP II)} \quad \min \sum_{h \in \mathcal{H}} \sum_{d \in \mathcal{D}_h} \sum_{\bar{h} \in \bar{\mathcal{H}}} \sum_{v \in \mathcal{V}} p^{\bar{h}}_{v_{hd} v} \, vol_{hd} \, x^{\bar{h}v}_{hd} + c^{truck} \sum_{h \in \mathcal{H}} \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} u^{vt}_h \tag{24}
$$

s.t. (2)–(13)

$$
\sum_{\substack{\bar{h} \in \bar{\mathcal{H}} \\ h-\bar{h} \geq 0}} \sum_{d \in \mathcal{D}_{h-\bar{h}}} \sum_{v \in \mathcal{V}} f^{\bar{h}vt}_{(h-\bar{h})d} \leq prod^{per}_{hs} \zeta^{per}_{hs} + prod^{temp}_{hs} \zeta^{temp}_{hs} \quad \forall h \in \mathcal{H}, \ \forall s \in \mathcal{S}, \forall t = start_{hs}, \dots, end_{hs} \tag{25}
$$

(17)–(18)

(20)–(21)

Note that the solution obtained after phase II is a feasible solution for the (PSP) model: by construction, it satisfies Constraints (2)–(21). This solution can be used as an initial feasible solution in the last phase.

4.3 Phase III - Dock management

In the last phase, the platform occupancy is optimized, i.e., we minimize the number of slots during which vehicles are present at the docks. The workers to hire, i.e., the values of variables z^{per}_{hs} and z^{temp}_{hs} and the possible reassignments or postponements of orders, i.e., the values of variables $x^{\bar{h}v}_{hd}$, are fixed and are parameters of the model (indicated with $\chi^{\bar{h}v}_{hd}$). The mathematical model solved in this phase is the following:

$$
\text{(PSP III)} \quad \min \sum_{h \in \mathcal{H}} \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} w^{vt}_h \tag{26}
$$

$$
\text{s.t.} \quad \sum_{t \in \mathcal{T}} f^{\bar{h}vt}_{hd} \leq vol_{hd} \, \chi^{\bar{h}v}_{hd} \quad \forall h \in \mathcal{H}, \ \forall d \in D_h, \ \forall \bar{h} \in \bar{\mathcal{H}}, \ \forall v \in \mathcal{V} \tag{27}
$$

$$
\sum_{\substack{\bar{h} \in \bar{\mathcal{H}} \\ h-\bar{h} \geq 0}} \sum_{d \in \mathcal{D}_{h-\bar{h}}} \sum_{v \in \mathcal{V}} f^{\bar{h}vt}_{(h-\bar{h})d} \leq prod^{per}_{hs} \zeta^{per}_{hs} + prod^{temp}_{hs} \zeta^{temp}_{hs} \quad \forall h \in \mathcal{H}, \ \forall s \in \mathcal{S}, \forall t = start_{hs}, \dots, end_{hs} \tag{28}
$$

(2), (5)–(13)

(18), (20)–(21)

The solution provided by phase III is the final solution obtained for the PSP.

4.4 Speed-up techniques

To speed-up the algorithm, we have developed two valid lower-bounds on the objective function values of models (PSP I) and (PSP II). The lower-bounds are given by the solution of two specific arc-flows problems. Since the computation of these lower-bounds follows the same lines, we detail only the computation of the lower bound for (PSP II).

4.4.1 Lower-bound for (PSP II)

We first recall that in phase II, the objective function is given by the sum of the penalties due to the reassignments and the postponements plus the cost of used trucks. The lower-bound is obtained by solving the following relaxation of (PSP II).

$$\text{(RPSP II)} \quad \min \sum_{h\in\mathcal{H}}\sum_{v\in\mathcal{V}}\sum_{\bar{h}\in\mathcal{H}}\sum_{\bar{v}\in\mathcal{V}} p_{v\bar{v}}^{\bar{h}} \xi_{hv}^{\bar{h}\bar{v}} + c^{truck} \sum_{h\in\mathcal{H}}\sum_{v\in\mathcal{V}} \zeta_h^v \tag{29}$$

$$D_{hv} + \sum_{\substack{\bar{h}\in\bar{\mathcal{H}} \\ h-\bar{h}\geq 0}} \sum_{\bar{v}\in\mathcal{V}} \xi_{hv}^{\bar{h}\bar{v}} - \xi_h^v - \sum_{\bar{h}\in\bar{\mathcal{H}}}\sum_{\bar{v}\in\mathcal{V}} \xi_{hv}^{\bar{h}\bar{v}} = 0 \quad \forall h\in\mathcal{H},\ \forall v\in\mathcal{V} \tag{30}$$

$$\xi_h^v \leq Q\zeta_v^h \quad \forall h\in\mathcal{H},\ \forall v\in\mathcal{V} \tag{31}$$

$$\xi_{hv}^{\bar{h}\bar{v}} \in \mathbb{N}, \quad \forall h,\bar{h}\in\mathcal{H},\ \forall v,\bar{v}\in\mathcal{V} \tag{32}$$

$$\xi_h^v, \zeta_v^h \in \mathbb{N}, \quad \forall h\in\mathcal{H},\ \forall v\in\mathcal{V} \tag{33}$$

where

- $\xi_{hv}^{\bar{h}\bar{v}}$ represents the total volume of orders for period h and mode v treated on period $\bar{h}$ by mode $\bar{v}$
- ξ_h^v represents the total volume of orders for period h and mode v that is not postponed or reassigned
- ζ_h^v represents the total number of required vehicles for mode v in period h

and D_{hv} is the total number of packages which should be prepared on period h and delivered by mode v.

Proposition 3. *Model (RPSP II) is a relaxation of model (PSP II).*

Proof. From Equations (4), multiplying both terms by vol_{hd}, we obtain:

$$\sum_{\bar{h}\in\bar{\mathcal{H}}}\sum_{v\in\mathcal{V}} vol_{hd} x_{hd}^{\bar{h}v} = vol_{hd} \quad \forall h\in\mathcal{H},\ \forall d\in D_h$$

and summing up on the demands $d\in\mathcal{D}_h$ we obtain:

$$\sum_{d\in\mathcal{D}_h}\sum_{\bar{h}\in\bar{\mathcal{H}}}\sum_{v\in\mathcal{V}} vol_{hd} x_{hd}^{\bar{h}v} = \sum_{d\in\mathcal{D}_h} vol_{hd} = \sum_{v\in\mathcal{V}}\sum_{\substack{d\in\mathcal{D}_h \\ v_{hd}=v}} vol_{hd} \quad \forall h\in\mathcal{H}$$

$$\sum_{v\in\mathcal{V}}\sum_{\substack{d\in\mathcal{D}_h \\ v_{hd}=v}}\sum_{\bar{h}\in\bar{\mathcal{H}}}\sum_{\bar{v}\in\mathcal{V}} vol_{hd} x_{hd}^{\bar{h}\bar{v}} = \sum_{d\in\mathcal{D}_h} vol_{hd} = \sum_{v\in\mathcal{V}}\sum_{\substack{d\in\mathcal{D}_h \\ v_{hd}=v}} vol_{hd} \quad \forall h\in\mathcal{H}$$

and, for $v\in\mathcal{V}, h\in\mathcal{H}$ let us define $\xi_{v\bar{v}}^{h\bar{h}} = \sum_{\substack{d\in\mathcal{D}_h \\ v_{hd}=v}} vol_{hd} x_{hd}^{\bar{h}\bar{v}}$ as the total demand of day h assigned to mode v that is delivered on period $\bar{h}$ by mode $\bar{v}$. We then have

$$\sum_{v\in\mathcal{V}}\sum_{\bar{h}\in\bar{\mathcal{H}}}\sum_{\bar{v}\in\mathcal{V}} \xi_{v\bar{v}}^{h\bar{h}} = \sum_{d\in\mathcal{D}_h} vol_{hd} = \sum_{v\in\mathcal{V}}\sum_{\substack{d\in\mathcal{D}_h \\ v_{hd}=v}} vol_{hd} = \sum_{v\in\mathcal{V}} D_v^h = D^h \quad \forall h\in\mathcal{H} \tag{34}$$

where D_v^h and D^h are respectively the total demand of day h originally associated with mode v and the total demand of day h. Note that since, for each $h\in\mathcal{H}$, for each $d\in\mathcal{D}_h$, for each $v\in\mathcal{V}$ and for each $\bar{h}\in\bar{\mathcal{H}}$ there exists only one variable $x_{hd}^{\bar{h}v}$ equal to one, we can write Equations (34) as:

$$\sum_{\bar{h}\in\bar{\mathcal{H}}}\sum_{\bar{v}\in\mathcal{V}} \xi_{v\bar{v}}^{h\bar{h}} = \sum_{\substack{d\in\mathcal{D}_h \\ v_{hd}=v}} vol_{hd} = D_v^h \quad \forall h\in\mathcal{H},\ \forall v\in\mathcal{V}$$

$$D_v^h - \sum_{\bar{h}\in\bar{\mathcal{H}}}\sum_{\bar{v}\in\mathcal{V}} \xi_{v\bar{v}}^{h\bar{h}} = 0 \quad \forall h \in \mathcal{H},\ \forall v \in \mathcal{V} \tag{35}$$

From Equations (6), summing on $t \in \mathcal{T}$ we obtain:

$$\sum_{t\in\mathcal{T}}\sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}}\sum_{d\in\mathcal{D}_{h-\bar{h}}} f_{(h-\bar{h})d}^{\bar{h}vt} + \sum_{t\in\mathcal{T}} k_h^{vt} = \sum_{\substack{t\in\mathcal{T}\\ t>0}} k_h^{v(t-1)} + Q\sum_{t\in\mathcal{T}} u_h^{vt} \quad \forall h \in \mathcal{H},\ \forall v \in \mathcal{V}$$

$$\sum_{t\in\mathcal{T}}\sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}}\sum_{d\in\mathcal{D}_{h-\bar{h}}} f_{(h-\bar{h})d}^{\bar{h}vt} + k_h^{vT} = Q\sum_{t\in\mathcal{T}} u_h^{vt} \quad \forall h \in \mathcal{H},\ \forall v \in \mathcal{V}$$

$$\sum_{t\in\mathcal{T}}\sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}}\sum_{d\in\mathcal{D}_{h-\bar{h}}} f_{(h-\bar{h})d}^{\bar{h}vt} \leq Q\sum_{t\in\mathcal{T}} u_h^{vt} \quad \forall h \in \mathcal{H},\ \forall v \in \mathcal{V}$$

Let us now define $\xi_v^h = \sum_{t\in\mathcal{T}}\sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}}\sum_{d\in\mathcal{D}_{h-\bar{h}}} f_{(h-\bar{h})d}^{\bar{h}vt}$ and $\zeta_v^h = \sum_{t\in\mathcal{T}} u_h^{vt}$. Then we obtain:

$$\xi_v^h \leq Q\zeta_v^h \quad \forall h \in \mathcal{H},\ \forall v \in \mathcal{V} \tag{36}$$

that are Constraints (31). ξ_v^h represents the total volume of packages that must be prepared in day h and delivered by mode v after postponing and re-affecting operations. ζ_v^h represents the number of vehicles needed to transport the ξ_v^h packages.

From the definition of ξ_v^h we have:

$$\xi_v^h = \sum_{t\in\mathcal{T}}\sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}}\sum_{d\in\mathcal{D}_{h-\bar{h}}} f_{(h-\bar{h})d}^{\bar{h}vt}, \quad \forall h \in \mathcal{H}, \forall v \in \mathcal{V}$$

$$\xi_v^h = \sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}}\sum_{\bar{v}\in\mathcal{V}}\sum_{\substack{d\in\mathcal{D}_{h-\bar{h}}\\ \bar{v}=v_{hd}}}\sum_{t\in\mathcal{T}} f_{(h-\bar{h})d}^{\bar{h}vt}, \quad \forall h \in \mathcal{H}, \forall v \in \mathcal{V}$$

$$\xi_v^h = \sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}}\sum_{\bar{v}\in\mathcal{V}} \xi_{\bar{v}v}^{\bar{h}h}, \quad \forall h \in \mathcal{H}, \forall v \in \mathcal{V}$$

$$\sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}}\sum_{\bar{v}\in\mathcal{V}} \xi_{\bar{v}v}^{\bar{h}h} - \xi_v^h = 0, \quad \forall h \in \mathcal{H}, \forall v \in \mathcal{V} \tag{37}$$

where we have defined $\xi_{\bar{v}v}^{\bar{h}h} = \sum_{\substack{d\in\mathcal{D}_{h-\bar{h}}\\ \bar{v}=v_{hd}}}\sum_{t\in\mathcal{T}} f_{(h-\bar{h})d}^{\bar{h}vt}$, that represents all packages originally assigned to day $\bar{h}-h$ and mode $\bar{v}$ that are prepared on day h (i.e., are postponed by $\bar{h}$) and mode v.

Summing Equations (35) and (37) we obtain:

$$D_v^h + \sum_{\substack{\bar{h}\in\bar{\mathcal{H}}\\ h-\bar{h}\geq 0}}\sum_{\bar{v}\in\mathcal{V}} \xi_{\bar{v}v}^{\bar{h}h} - \xi_v^h - \sum_{\bar{h}\in\bar{\mathcal{H}}}\sum_{\bar{v}\in\mathcal{V}} \xi_{v\bar{v}}^{h\bar{h}} = 0 \quad \forall h \in \mathcal{H},\ \forall v \in \mathcal{V} \tag{38}$$

that are Constraints (30). All the other constraints in the model (PSP II) are relaxed.

□

From Proposition 3 it follows that the value of the optimal solution of (RPSP II) is a lower-bound for (PSP II).

The model (RPSP II) is a special case of the multi-commodity capacitated network design problem where only one commodity has to be routed on the network, and capacities have to be respected or installed to satisfy the demand (see, for example, Gendron et al. [11]). In particular, (RPSP II) is equivalent to

$$\text{(AF-RPSP II)} \quad \min \sum_{v\in\mathcal{V}} \sum_{\bar{v}\in\mathcal{V}} \sum_{h\in\mathcal{H}} \sum_{\bar{h}\in\mathcal{H}} p^{\bar{h}}_{v\bar{v}} \xi^{\bar{h}\bar{v}}_{hv} + c^{truck} \sum_{v\in\mathcal{V}} \sum_{h\in\mathcal{H}} \zeta^{v}_{h} \tag{39}$$

$$\mathbf{A}\boldsymbol{\xi} = \mathbf{b} \tag{40}$$

$$\mathbf{0} \leq \boldsymbol{\xi} \leq c(\boldsymbol{\zeta}) \tag{41}$$

$$\boldsymbol{\zeta}, \boldsymbol{\xi} \in \mathbb{N} \tag{42}$$

Model (AF-RPSP II) defines an arc-flow problem on an oriented graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ where $\mathcal{N} = \{s,t\} \cup \mathcal{N}^h_v$, and $\mathcal{N}^h_v$ contains a node n^h_v for each pair (v,h), $v \in \mathcal{V}$, $h \in \mathcal{H}$ and

$$\mathcal{A} = \{(s,i) | i \in \mathcal{N}^h_v\} \cup \{(i,t) | i \in \mathcal{N}^h_v\} \cup \{(i,j) | i,j \in \mathcal{N}^h_v, i \neq j\}$$

$\mathbf{A}$ is the adjacency matrix of graph $\mathcal{G}$. Vectors $\mathbf{b}$ and c are as follows:

$$b_i = \begin{cases} -\sum_{v\in\mathcal{V}} \sum_{h\in\mathcal{H}} D^h_v & \text{if } i = s \\ \sum_{v\in\mathcal{V}} \sum_{h\in\mathcal{H}} D^h_v & \text{if } i = t \\ 0 & \text{otherwise} \end{cases}$$

and,

$$c_a = \begin{cases} D^h_v & \text{if } a = (s, n^h_v) \\ Q\zeta^h_v & \text{if } a = (n^h_v, t) \\ \sum_{v\in\mathcal{V}} \sum_{h\in\mathcal{H}} D^h_v & \text{if } a = (n^h_v, n^{h_1}_{v_1}), i \neq s,t \end{cases}$$

The model (RPSP II) is solved with a commercial solver, and the optimal solution value gives a lower-bound for phase II. With respect to our testbed, the size of instances remains small, and optimal solutions are obtained almost instantly. Each time a feasible solution for model (PSP II) is identified and its value is equal to the lower-bound, the solution of model (PSP II) is stopped.

4.4.2 Lower-bound for phase I

The model (RPSP II) determines orders that require postponement or reassignment to minimize the number of vehicles, and it computes the resulting penalties. A valid lower-bound for phase I is obtained accordingly: an estimation of postponed orders over the horizon is computed to minimize the number of required workers. The problem can be formulated as an arc-flow problem similar to (AF-RPSP II). Due to similarities shared between both constructions we omit the details here.

4.4.3 Order aggregation for phase III

Phase II determines the quantities of orders assigned to each mode in each period. Based on these decisions, phase III looks for a packages loading plan, in other words, the quantities loaded at each slot and the required trucks movements, which minimize the docks occupation. To speed up the solution of phase III model, we *aggregate* orders. We group orders that

- have the same release date,
- have to be processed on the same period $h \in \mathcal{H}$,
- have to be delivered using the same mode $v \in \mathcal{V}$

as a single order whose volume is the sum of individual order volumes. Orders in $\mathcal{D}_h$ postponed by $\bar{h} > 0$ are included in the volume of the single order created for period $h+\bar{h}$ associated with a release date equal to zero. Since these orders were available $\bar{h}$ periods before, they are available at the beginning of period $h + \bar{h}$.

This aggregation can be performed since no postponement or reassignment is allowed during this phase. Postponement or reassignment must be done on the total volume of an order. Aggregation is not possible when postponement or reassignment is admissible since the packages associated with each order have to be known. This information is lost in case of aggregation.

Formally, for each $h \in \mathcal{H}$, for each $v \in \mathcal{V}$ and for each $t \in \mathcal{T}$ we define a unique order D^t_{hv} with a volume $vol_{D^t_{hv}}$ defined as follows:

$$vol_{D^t_{hv}} = \begin{cases} \sum_{\substack{d\in\mathcal{D}_h \\ r_{hd}=t}} vol_{hd}x^{0v}_{hd} + \sum_{\substack{d\in\mathcal{D}_{h-\bar{h}} \\ h-\bar{h}\geq 0,\ \bar{h}>0}} vol_{(h-\bar{h})d}x^{\bar{h}v}_{(h-\bar{h})d} & \text{if } t = 0 \\ \sum_{\substack{d\in\mathcal{D}_h \\ r_{hd}=t}} vol_{hd}x^{0v}_{hd} & \text{if } t > 0 \end{cases} \tag{43}$$

When $t = 0$, the volume of D^t_{hv} corresponds to the sum of volumes of all orders released exactly at $t = 0$ on the period h and processed on the same period, plus the volume of all the orders released during period $h-\bar{h}$ and postponed by $\bar{h}$ periods. When an order is postponed to a given period h, it is known at the beginning of period h. When $t > 0$, the volume D^t_{hv} corresponds to the sum of volumes of all orders released exactly at $t > 0$ on the same period h. Moreover, D^t_{hv} is characterized by its mode $v_{D^t_{hv}} = v$, and its release date $r_{D^t_{hv}} = t$.

Let $\bar{\mathcal{D}}_h$ denote the set of all aggregated orders. These orders are the input of the model solved by the commercial solver in phase III. Let us indicate with $\bar{f}^{\bar{h}vt}_{hd}$ the variables corresponding to orders in $\bar{\mathcal{D}}_h$. A solution of phase III determines an operational planning for orders $d \in \bar{\mathcal{D}}_h$. The solution of the problem in terms of variables $f^{\bar{h}vt}_{hd}$ can easily be obtained by applying a greedy algorithm using the values of variables $\bar{f}^{\bar{h}vt}_{hd}$.

5 Computational results

This section discusses the efficiency of the three-phase procedure we developed for the PSP. First we describe the instances we created from data provided by an industrial partner (Section 5.1). The results on these instances are reported in Section 5.2.1. Sensitivity analyses of the three-phase algorithm with respect to slight modification of instances and with respect to different penalty profiles are reported, respectively in Sections 5.2.2 and 5.2.3. In Section 5.2.4 we assess the performance of the lower-bounds introduced in Sections 4.4.1 and 4.4.2. Finally, in Section 5.2.5 we compare the three-phase algorithm with the solution of the (PSP) using a commercial solver.

5.1 Instance generation

Since the PSP is a new problem, we have to generate a set of instances to test the algorithm described in Section 4. The instances are based on real data provided by a logistics company operating in the e-commerce sector.

Each working day is identified by a "profile", i.e. a number of orders processed during that day. We define three profiles, named *low*, *normal*, *high*, respectively characterized by 1000, 3000, 5000 orders.

A list of common data is shared among the different instances. Specifically, we consider a three-day horizon, $\mathcal{H} = \{0, 1, 2\}$, whereas the order process can be postponed by one day, i.e., $\bar{\mathcal{H}} = \{0, 1\}$. Orders are received only during days 0 and 1. The third day is only used if the whole demand cannot be prepared during days 0 and 1. Each day consists of two shifts including eight time slots each.

Two delivery modes are available, the *express* mode and the *standard* mode. Trucks associated with the express mode leave the warehouse earlier than the other trucks, which are scheduled to leave at the end of the last shift. As an example, the express mode departure time is slot 12. This means packages can be loaded into vehicles until slot 11. Trucks have a capacity of 1300 packages and a fixed cost of 650 Euros for both modes.

For permanent workers, the productivity is set to 40 packages per time slot, and the cost to 185 Euros. Temporary workers produce up to 30 packages per time slot and cost 210 Euros. Temporary workers are hired for at least one shift. We limit the number of permanent workers to 15 for each shift of each day.

The number of available docks is set to 10. The penalties for a postponement or a reassignment are as follows:

$$p_{hd}^{\bar{h}v} = \begin{cases} 0 & \text{if } \bar{h} = 0 \text{ and } v = v_{hd}, \\ 1 & \text{if } \bar{h} = 0 \text{ and } v \neq v_{hd} \text{ or } \bar{h} = 1 \text{ and } v = v_{hd}, \\ 2 & \text{if } \bar{h} = 1 \text{ and } v \neq v_{hd}, \\ \infty & \text{otherwise.} \end{cases} \tag{44}$$

We consider nine types of instances associated with all possible profile combinations for day 0 and day 1 chosen among low, normal and high profiles. For each type of instance, five instances are generated randomly fixing the values of the release date, the number of packages that constitute an order as well as their delivery mode. Order volumes are uniformly drawn among values $\{1, 2, 3\}$ and release dates are drawn uniformly among the slots of the day. Modes are initially assigned to orders according to a uniform distribution.

5.2 Discussion

The algorithm was implemented in `C++` with Visual Studio environment. The models presented in Sections 4.1–4.3 were solved with Cplex 12.6. All tests were performed on an Intel® Core™ i7-4600U CPU 2.10 GHz. We allowed a maximum computation time of 30 minutes for each phase of the algorithm. Moreover, the resolution in phases I and II is stopped if the optimality gap with respect to the lower bound provided by Cplex 12.6 or with respect to the lower bounds computed as explained in Sections 4.4.2 and–4.4.1 is less than 2%. For all computational experiments the value of α in the objective function (1) is set to 1.

5.2.1 Results on the basic instances

First, we ran our three-phase algorithm on one instance of each type described in Section 5.1. Detailed results are reported in Table 2. Column *Instance* reports the name of the instance type as a couple corresponding to the profiles of day 0 and 1. For each instance, we report results in four lines: the first three lines correspond to each of the algorithm phases. The fourth line gives the total cost and the CPU time values.

Column *Phase* indicates the considered phase of the algorithm. Column *Cost* reports the cost of the objective function for each phase as well as the total solution cost. Note that the objective function at phase I takes into account the penalties that occur when setting the workforce. Then the sum of workers cost is only included in the value of the final solution. The penalties are computed again in phase II and then contribute effectively to the final solution cost. Columns *workers Per* and *workers Temp* indicate respectively the number of permanent and temporary workers. Note that these columns are empty for phase II and phase III, as they are determined during phase I. Column *Pen* reports the value of the sum of all postponement and reassignment penalties. Column *Truck* indicates the number of trucks needed to deliver the orders. Phase III does not modify the values in these columns, and the corresponding slots are left blank. Column *Dock-slots* reports the total number of time slots during which docks are occupied. The solution of the phase I model does not provide the values of *Truck* and *Dock-slots* columns (variables u_h^{vt} and w_h^{vt} are not present in model PSP I) while in phase II this term is not minimized. As a consequence the value is reported only for phase III, when it is optimized. Since we provide to the solver specific lower bounds in phase I and II, the reported gap is calculated using either the value of the linear relaxation or the corresponding lower-bound (see Section 4.4). The gap is reported only when it is strictly positive. When the slot is empty, an optimal solution (for that phase) is identified. Finally, column *Time* provides the CPU time in seconds.

For one type of instances and each of the three phases, a (local) optimal solution is obtained. For other eight types, phase II fails to reach the optimal solution within 30 minutes of CPU time. However, the optimality gap is less than 2% for six of them. Phase II reveals to be the bottleneck of the procedure.

For five types of instances, phase I suggests hiring temporary workers even if the total availability of permanent workers for the first two days has not been used. We recall that 15 permanent workers are available for each shift, for a total of 60 workers for day-0 and day-1. The penalty scheme considered guides the optimization through solutions that favor temporary workers hiring, rather than order postponement.

For seven types of instances, phase II can reduce the number of trucks that was first determined in phase I. This is possibly due to the reassignment strategy. Note that the increase in penalty costs is always lower than the savings due to unused trucks. This result highlights the potential benefits of incorporating postponement and reassignment into the process planning. Finally, phase III always reduces the number of dock-slots which is crucial for handling high activity peaks.

5.2.2 Algorithm behavior analysis on Normal-Low type instances

We ran our algorithm on five different instances of the type Normal-Low. Since e-commerce enterprises often experience the same sequence of day profiles, but with different orders quantities, we selected the Normal-Low sequence to analyze the sensitivity of the algorithm. Table 3 presents detailed results on the five runs. Column headings correspond to those reported in Table 2. It can be seen that the results for different instances are equivalent. It is worthwhile to note that the solution time for the three phases does not vary significantly among runs. We can conclude that our solution algorithm is not deeply impacted by the structure of the instance solved.

We made the same analysis for the other instance types solving each time five instances, and we ended up each time with the same conclusions. Thus we omit to report detailed results on these instances.

Instance	Phase	Cost	Workers		Pen	Truck	Dock-slots	Gap	Time
			Per	Temp					
	I	2405 + 75	13	0	75	4			11
Low-Low	II	2677			77	4		1.95%	13
	III	35					35		1
		5117							25
	I	4810 + 75	26	0	75	8			4
Low-Normal	II	4665			115	7		1.63%	163
	III	37					37		3
		9512							170
	I	7290+75	36	3	75	11		0.43%	18
Low-High	II	6775			275	10		1.73%	80
	III	51					51		4
		14116							102
	I	4810	26	0	0	8			14
Normal-Low	II	4961			411	7		0.02%	177
	III	38					38		7
		9809							198
	I	7030	38	0	0	12			19
Normal-Normal	II	6994			494	10		0.09%	1074
	III	42					42		8
		14066							1101
	I	9485	49	2	0	14		0.58%	16
Normal-High	II	8766			316	13		0.01%	481
	III	41					41		150
		18292							647
	I	7290 + 2	36	3	2	10		1.40%	31
High-Low	II	6539			39	10		0.60%	148
	III	44					44		14
		13837							193
	I	9485+1	49	2	1	14		0.15%	33
High-Normal	II	8807			356	13		0.71%	1261
	III	47					47		15
		18339							1307
	I	12150	60	5	0	17		1.80%	34
High-High	II	10560			160	16			1758
	III	44					44		16
		22754							1808

Table 2: Computational results on the basic instances

Phase	Cost	Workers		Pen	Truck	Dock	Gap	Time
		Per	Temp					
I	4810	26	0		8			13
II	4893			343	7			117
III	38					38		6
	9741							136
I	4810	26	0		8			14
II	4851			301	7			68
III	35					35		6
	9696							88
I	4810	26	0		8			14
II	4919			369	7		1.23%	64
III	35					35		7
	9764							85
I	4810	26	0		8		1.50%	13
II	4871			321	7			77
III	34					34		6
	9715							96
I	4810	26	0		8			14
II	5022			472	7		1.23%	132
III	34					34		6
	9866							152

Table 3: Algorithm performance on 5 instances with a Normal-Low demand profile

5.2.3 Analysis of penalty schemes

In this section we compare the results obtained according to different penalty schemes for the postponement and reassignment policies. In Table 4 we report results obtained when penalty values in Equation (44) are divided by 10. The algorithm is run on one instance of each type.

There are two main differences with the results reported in Table 2. The first is related to the number of temporary workers while the second one concerns the optimality gap. For the High-High instance, the obtained solution postpones the orders processed to the third day (day 2) with a consequent use of 3 permanent workers. Note that 3 permanent workers guarantee a production (40 packages per slot * 8 slots * 3 equals to 960) equal to 4 temporary workers (30*8*4), but cost 555 instead of 840, leaving room for a large postponement that is favored by the low penalization scheme considered.

Moreover larger optimality gaps are obtained in phase II. An explanation could be the following. Let us consider two orders d_1 and d_2 for the same day, with the same volume and the same release date. Let us suppose to have in hand the complete planning. Exchanging production of d_1 with d_2 would provide an equivalent planning. This leads to equivalent solutions which the solver needs to consider to prove optimality. When penalty is low, this symmetry is also present for the postponement and reassignment policies, making computation even harder.

Table 5 reports results when the penalty scheme proposed in Equation (44) is modified to move orders from the standard to the express delivery mode for free (even if associated with postponement). On one hand, earlier deliveries increase the company's image. On the other hand, a postponement coupled with a change to a faster mode leads to on-time deliveries. Similar observations as those formulated for Table 4 can be drawn. Low-cost reassignments and postponements make disadvantageous to hire temporary workers and increase solution symmetry. The latter leads to significant optimality gaps that are reported in the table.

5.2.4 Lower-bound effectiveness

In Table 6 we report the deviations of the lower-bounds defined in Sections 4.4.1–4.4.2 on the instances considered in Table 2. Columns *Instance* and *Phase* are self-explanatory. Column $Cplex_{gap}$ reports the gap value between upper- and lower-bounds provided by Cplex 12.6 when the solution of the corresponding phase is stopped. Column LB_{gap} indicates the gap value of the lower-bound computed solving the related arc-flow problem. When the gap is null, the cell is left empty.

An empty cell (a zero gap value) or a value lower than 2% in column LB_{gap} associated with a value grater than 2% in column $Cplex_{gap}$ certifies the effectiveness of the lower-bound used to stop the corresponding model solution. It can be seen that the lower-bound for the phase I (LB1) allows for an earlier stop of the computation 3 times, while the lower-bound for phase II (LB2) does it in 8 cases. When the optimal values are not reached, LB2 provides a better optimality gap compared to the one given by $Cplex_{gap}$ on all instances except the Low-Low type ones.

5.2.5 Comparison with a commercial solver

Last we report on the comparison between our algorithm and the commercial solver Cplex 12.6. The result on the complexity of the PSP suggests that only small size instances can be solved to optimality.

In Table 7 we compare the performances of the three-phase method against Cplex 12.6 on the same instances as those used to obtain results reported in Table 2. In Table 7 columns *Cplex* report the results obtained by the Cplex 12.6, while columns *Three-phase* report the results obtained by the proposed algorithm. Columns *CPU* report the computational time in seconds.

Instance	Phase	Cost	Workers		Pen	Truck	Dock	Gap	Time
			Per	Temp					
Low-Low	I	2405 + 9.2	13	0	9.2	4		0.07%	5
	II	2641				4		1.58%	45
	III	36					36		1
		5082							51
Low-Normal	I	4810+25	26	0	25	10		0.36%	9
	II	5224.1			24.1	8		6.07%	1800
	III	52					52		4
		10086.1							1813
Low-High	I	7030 +72.5	38	0	72.5	12		0.92%	7
	II	7235.1			85.1	11		6.09%	1800
	III	67					67		3
		14332.1							1810
Normal-Low	I	4810.0	26	0	0	8			19
	II	4679.3			129.3	7		1.92%	154
	III	40					40		8
		9529.3							181
Normal-Normal	I	7030+30.4	38	0	30.4	12		0.03%	22
	II	6605.6			105.6	10		0.73%	1016
	III	39					39		10
		13674.6							1048
Normal-High	I	9435 +49.1	51	0	49.1	16		0.52%	32
	II	9849.5			99.5	15		16.00%	1800
	III	56					56		11
		19340.5							1843
High-Low	I	7030 +48.9	38	0	48.9	11		0.24%	53
	II	6574.8			74.8	10		1.15%	643
	III	44					44		21
		13648.8							717
High-Normal	I	9435+48.1	51	0	48.1	14		0.51%	42
	II	9164.0			64.0	14		8.07%	1800
	III	45					45		38
		18644.0							1880
High-High	I	11865+124.3	63	0	124.3	18		1.94%	100
	II	11255.7			205.7	17		8.06%	1800
	III	47					47		55
		23167.7							1955

Table 4: Computational results with reduced penalties

Instance	Phase	Cost	Workers		Pen	Truck	Dock	Gap	Time
			Per	Temp					
Low-Low	I	2405 +12	13	0	12	6		0.50%	48
	II	3457			207	5		2.31%	46
	III	40					40		1
		5902							95
Low-Normal	I	4810	26	0	0	8			5
	II	5200			0	8		1.97%	720
	III	43					43		2
		10053							727
Low-High	I	7030	38	0	0	11			60
	II	7150			0	11		1.94%	552
	III	48					48		2
		14228							614
Normal-Low	I	4810	26	0	0	8			13
	II	5200			0	8		12.09%	1800
	III	38					38		7
		10048							1820
Normal-Normal	I	7030	38	0	0	12			18
	II	7150			0	11		9.73%	1800
	III	46					46		5
		14226							1823
Normal-High	I	9620	52	0	0	15		1.96%	16
	II	8450			0	13			1050
	III	49					49		5
		18119							1071
High-Low	I	7030	38	0	0	11			262
	II	6500			0	10			521
	III	40					40		13
		13570							796
High-Normal	I	9620	52	0	0	14		1.96%	35
	II	8567			117	14		1.38%	1434
	III	46					46		14
		18233							1483
High-High	I	12025	65	0	0	17		3.06%	52
	II	11370			320	17		9.33%	1800
	III	52					52		16
		23447							1868

Table 5: Computational results when the delivery service is modified free of charge.

Instance	Phase	$Cplex_{gap}$	LB_{gap}
Low-Low	I	5.19%	
	II	1.95%	2.96%
Low-Normal	I	4.36%	
	II	4.49%	1.63%
Low-High	I	0.43%	3.66%
	II	4.35%	1.73%
Normal-Low	I	2.87%	
	II	12.87%	0.02%
Normal-Normal	I	0.81%	
	II	13.77%	0.09%
Normal-High	I	1.19%	0.58%
	II	8.83%	0.01%
High-Low	I	1.86%	1.40%
	II	3.03%	0.60%
High-Normal	I	0.15%	0.54%
	II	8.38%	0.71%
High-high	I	1.80%	2.83%
	II	4.85%	

Table 6: Lower-bound effectiveness

	Three-phase		Cplex		Gap	
Instance	Cost	Cpu	Cost	Cpu	Cost	Cpu
Low-Low	5117	25	5097	705	0.39%	-96.45%
Low-Normal	9512	174	9511	3600	0.01%	-95.17%
Low-High	14116	102	14062	3600	0.38%	-97.17%
Normal-Low	9809	198	9801	3600	0.08%	-94.50%
Normal-Normal	14066	474	14295	3600	-1.60%	-86.83%
Normal-High	18292	647	18299	3600	-0.04%	-82.03%
High-Low	13873	193	13803	3600	0.51%	-94.64%
High-Normal	18339	1307	21101	3600	-13.09	-63.69%
High-High	22754	1808	24548	3600	-7.31%	-49.78%

Table 7: Comparison with Cplex 12.6

	Three-phase		Cplex		Gap	
Instance	Cost	Cpu	Cost	Cpu	Cost	Cpu
Normal-Normal	13874	335	14079	3600	-1.46%	-90.69%
	14262	694	14607	3600	-2.36%	-80.72%
	14205	397	14369	3600	-1.14%	-88.97%
	13968	744	13983	3600	-0.11%	-79.83%
	14290	617	14695	3600	-2.76%	-82.86%
Normal-High	18531	948	18616	3600	-0.46%	-73.67%
	18464	1500	19048	3600	-3.07%	-58.33%
	18671	532	18678	3600	-0.04%	-85.22%
	18440	1312	18482	3600	-0.23%	-63.56%
	18427	1121	18646	3600	-1.17%	-68.86%
High-Normal	18330	1867	19171	3600	-4.39%	-48.14%
	18156	1626	18783	3600	-3.34%	-54.83%
	18290	386	18302	3600	-0.07%	-89.28%
	18295	471	18329	3600	-0.19%	-86.92%
	18495	1753	18843	3600	-1.85%	-51.31%
High-High	23553	1874		3600		-47.94%
	22671	1804	23375	3600	-3.01%	-49.89%
	22957	1995	26022	3600	-11.78%	-44.58%
	24037	1952		3600		-45.78%
	23821	1884	24428	3600	-2.48%	-47.67%

Table 8: Comparison with Cplex 12.6 on instances not including a low demand day

Columns *Cost* report the value of the solution obtained. Finally, columns *Gap* report the gap between both solutions. Negative gaps correspond to better solutions obtained by the three-phase algorithm. A time limit of 1 hour of computation is given to Cplex 12.6. Note that the three-phase algorithm never runs for more than 1808 seconds. On the other hand, we allow 30 minutes for each phase of our algorithm even if only phase-II could use the whole allowed amount of time. Thus, giving to Cplex 12.6 1 hour of computation time allows for a fair comparison.

We can notice from Table 7 that when one of the days has a Low profile, Cplex 12.6 is competitive with respect to the quality of the solution obtained. For the Low-Low instance, it can even find the optimal solution. On the other hand, the three-phase heuristic systematically provides better results on instances that consider days characterized by Normal and High productivity.

To assess the efficiency of our procedure we run the three-phase procedure on 5 other instances for each combination of days with Normal and High profiles. Table 8 reports the results obtained. The three-phase heuristic always provides a better solution and is always quicker. In two cases, Cplex 12.6 cannot even find a solution after one hour of computational time, while the three-phase heuristic provides one in a little longer than half an hour.

It can then be stated that Cplex 12.6 can be competitive as long as the instance is easy. On the other hand, when solving instances characterized by Normal or High activities, the three-phase algorithm becomes necessary to obtain good quality results in reasonable computational times.

	Three-phase		Cplex		Gap	
Instance	Cost	Cpu	Cost	Cpu	Cost	Cpu
Low-Low	5124	13	5099	1121	0.49%	-98.84%
	5121	15	5094	702	0.53%	-97.86%
	5104	15	5079	547	0.49%	-97.26%
	5093	14	5066	1094	0.53%	-98.72%
	5103	13	5075	523	0.55%	-97.51%

Table 9: Comparison with optimal solutions on Low-Low instances

5.2.6 Quality of the solution on Low-Low instances

To better evaluate the performance of the three-phase heuristic, we compare the results obtained with optimal solutions provided by the commercial solver Cplex 12.6. Optimal solutions can systematically be obtained (within a reasonable amount of CPU time) for Low-Low type instances. We thus ran our three-phase heuristic and Cplex 12.6 on 5 instances with a Low-Low profile. Results are reported on Table 9.

It can be noticed that the three-phase heuristic can always identify near-optimal solutions with a maximal optimality gap of 0.55%. Moreover, the heuristic procedure allows solutions to be obtained in computational times that are almost two orders of magnitude lower than those of the commercial solver.

6 Conclusions

In this paper, we introduced the Packaging and Shipping Problem (PSP) arising in e-commerce logistics. It consists in determining the number of employees required to process a set of orders in a multi-day horizon setting. In addition, an operational planning has to be produced as well as a loading of the packages into trucks for deliveries that can be performed with different modes. We considered two strategies in order to obtain overall solutions with a lower cost: *reassignment* and *postponement*. The first strategy consists in changing the delivery mode chosen by the customer to another available to decrease operational costs. The second strategy consists in processing the orders in a subsequent day rather than the day of arrival. These strategies generate penalties, but they can lead to hire fewer employees or to use fewer trucks and, as a result, to savings for the company.

We proposed a mathematical model for the PSP and proved that the PSP is NP-hard. It is then unlikely the PSP can be efficiently solved to optimality within a reasonable time regardless the size of the instances (unless $\mathcal{P} = \mathcal{NP}$). We then proposed a three-phase matheuristic approach that allows us to deal with large real-life instances. Our approach exploits the structure of the PSP by sequentially solving three sub-problems to construct a feasible solution. We first take the tactical decisions, fixing the workforce for each day, and consequently, we determine the operational planning. Moreover, our approach is enhanced with speed up techniques based on lower-bounds for the subproblems.

We created a set of instances for the PSP based on data provided by our industrial partner. Instances with up to 5000 orders per day are then solved by the three-phase procedure that we proposed. Results show the efficiency of the method which can provide high-quality solutions in a reasonable amount of time and performs significantly better than the commercial solver whenever sequences of days with normal or high production activities are considered.

Future work could consider the stochastic nature of the problem. In this paper, we consider all order information to be deterministic. In real life, total demand is only forecast for the following days and, consequently, is subject to variations. Since we have interaction between decisions of consecutive periods, future demand uncertainty should be taken into account in the decision-making process. We plan to apply a rolling horizon based procedure that corresponds well to the dynamics of data acquisition and decision-making in e-fulfillment and to investigate appropriate stochastic optimization techniques.

Acknowledgement

This work was supported by the French National Research Agency (ANR-Agence Nationale de la Recherche) and was part of project RESPET - Efficient network managing for the door-to-door transportation services (gestion de RésEaux de Services Porte à porte Efficace pour le Transport de marchandises). Thanks are also due to the referees for their valuable comments.

APPENDICES

A Complexity of the PSP

Proposition 1. *The Packaging and Shipping Problem (PSP) is $\mathcal{NP}$-hard.*

Proof. We prove the $\mathcal{NP}$-hardness of (PSP) by reduction from the knapsack problem (KP). Given a knapsack with volume B and a set $\mathcal{N}$ of N items, indexed from 1 to N, each with a volume b_i and a value c_i, the KP consists in selecting a subset $\bar{\mathcal{N}}$ of $\mathcal{N}$ under the budget constraint which imposes that the total volume is less than or equal to B, and that $\sum_{i\in\bar{\mathcal{N}}} c_i$ is maximized. It can be formulated as follows:

$$\text{(KP)} \quad \max \quad \sum_{i=1}^{N} c_i x_i \tag{45}$$

$$s.t. \quad \sum_{i=1}^{N} b_i x_i \leq B \tag{46}$$

$$x_i \in \{0,1\}, \forall i \in \mathcal{N} \tag{47}$$

where binary variable x_i is equal to 1 if the i-th item has been selected, and zero otherwise. The objective function (45) is to maximize the value of the selected items. Constraint (46) is the budget constraint, while Constraints (47) impose variables to be binary.

For each instance of the KP we construct the following PSP instance. For each item $i \in \mathcal{N}$, we construct an order d, such that $d \in \mathcal{D}_0$ (i.e., it is an order associated with the first period of the horizon), $vol_{0d} = b_i$, $r_{0d} = 0$, and is assigned to a unique mode v. The horizon is made of two periods, i.e., $\mathcal{H} = \{0,1\}$. Orders received the period 0, can be processed during period 1, i.e., $\bar{\mathcal{H}} = \{0,1\}$. Each period is covered by a unique shift indexed with zero, i.e., $\mathcal{S} = \{0\}$. Only one slot is associated with the shift, $\mathcal{T} = \{0\}$. No order is received during period 1. For sake of simplicity, in this section we omit the index related to the period and the shift as well as the mode index.

At most one permanent worker is available for each shift, namely, $e_0^{\max} = e_1^{\max} = 1$ with a null cost. The productivity is set to $\sum_{i\in\mathcal{N}} vol_i$ for the permanent worker working during period 0 (this worker can process all orders arrived in period 0), and B for the permanent worker of period 1. On the other side, temporary workers have a null productivity and their cost is fixed to a strict positive constant, i.e., 1. By construction, reassignment is not possible (only one mode is available). Postponing order d to period 1 generates a penalty

$$p_d^{\bar{h}} = \begin{cases} \tilde{c}_d = -\frac{c_d}{vol_d} & \text{if } \bar{h} = 1, \\ 0 & \text{if } \bar{h} = 0. \end{cases} \tag{48}$$

The cost of a truck is set to zero, i.e., $c^{truck} = 0$. truck capacity is set to $\sum_{i \in \mathcal{N}} vol_i$: a truck can contain all the orders received in period 0. It is supposed that only one dock is available, $N_{\max}$ is set to 1. Other time related parameters like the shift starting period are trivially fixed. This transformation of a KP instance into a PSP instance is polynomial in time and takes $O(|\mathcal{N}|)$ operations.

For the obtained instance, the model (PSP) is reduced to (49)–(67). Note that variables related to the truck management are not present in the objective function. Then, we can suppose that variables w, u and y fixed to 1, which leads to Constraints (54)–(58).

$$\min \sum_{d \in \mathcal{D}_0} \sum_{\bar{h} \in \bar{\mathcal{H}}} p_d^{\bar{h}} \, vol_d \, x_d^{\bar{h}} + \sum_{h \in \mathcal{H}} (c_h^{per} z_h^{per} + c_h^{temp} z_h^{temp}) \tag{49}$$

$$\text{s.t.} \sum_{\bar{h} \in \bar{\mathcal{H}}} f_d^{\bar{h}} = vol_d \quad \forall d \in \mathcal{D}_0 \tag{50}$$

$$\sum_{\bar{h} \in \bar{\mathcal{H}}} x_d^{\bar{h}} = 1 \quad \forall d \in D_0 \tag{51}$$

$$f_d^{\bar{h}} \leq vol_d \, x_d^{\bar{h}} \quad \forall d \in D_0, \ \forall \bar{h} \in \bar{\mathcal{H}} \tag{52}$$

$$\sum_{\substack{\bar{h} \in \bar{\mathcal{H}} \\ h - \bar{h} = 0}} \sum_{d \in \mathcal{D}_0} f_d^{\bar{h}} + k_h = Q u_h \quad \forall h \in \mathcal{H} \tag{53}$$

$$u_h \leq N_{\max} \, y_h \quad \forall h \in \mathcal{H} \tag{54}$$

$$y_h \leq u_h \quad \forall h \in \mathcal{H} \tag{55}$$

$$Q u_h \leq Q w_h + Q(1 - y_h) \quad \forall h \in \mathcal{H} \tag{56}$$

$$Q u_h \leq Q w_h \quad \forall h \in \mathcal{H} \tag{57}$$

$$w_h \leq N_{\max} \quad \forall h \in \mathcal{H} \tag{58}$$

$$\sum_{\substack{\bar{h} \in \bar{\mathcal{H}} \\ h - \bar{h} = 0}} \sum_{d \in \mathcal{D}_0} f_d^{\bar{h}} \leq prod_h^{per} z_h^{per} + prod_h^{temp} z_h^{temp} \quad \forall h \in \mathcal{H} \tag{59}$$

$$z_h^{per} \leq 1 \quad \forall h \in \mathcal{H} \tag{60}$$

$$z_h^{temp} \leq z_h^{per} \quad \forall h \in \mathcal{H} \tag{61}$$

$$x_d^{\bar{h}} \in \{0, 1\} \ \forall d \in \mathcal{D}_0, \ \forall \bar{h} \in \bar{\mathcal{H}} \tag{62}$$

$$y_h \in \{0, 1\} \ \forall h \in \mathcal{H} \tag{63}$$

$$z_h^{per}, z_h^{temp} \in \mathbb{N} \ \forall h \in \mathcal{H} \tag{64}$$

$$f_d^{\bar{h}} \in \mathbb{N} \ \forall d \in \mathcal{D}_0, \ \forall \bar{h} \in \bar{\mathcal{H}} \tag{65}$$

$$w_h, k_h, u_h \in \mathbb{N} \ \forall h \in \mathcal{H} \tag{66}$$

Moreover, since periods are constituted by only one period, variables k_h become useless and Constraints (53) can be replaced by

$$\sum_{\substack{\bar{h} \in \bar{\mathcal{H}} \\ h - \bar{h} = 0}} \sum_{d \in \mathcal{D}_0} f_d^{\bar{h}} \leq Q \quad \forall h \in \mathcal{H} \tag{67}$$

Due to construction, Constraints (67) are trivially satisfied (a truck can contain the full orders received in period 0). Finally, due to construction, we are sure that in the optimal solution only one fix worker works each period ($z_0^{per} = z_1^{per} = 1$), while no temporary workers will be hired ($z_0^{temp} = z_1^{temp} = 0$). Constraints (60)–(61) are trivially satisfied. Then the model reduces to

$$\min \sum_{d \in \mathcal{D}_0} \sum_{\bar{h} \in \bar{\mathcal{H}}} p_d^{\bar{h}}\, vol_d\, x_d^{\bar{h}} \tag{68}$$

$$\text{s.t.} \sum_{\bar{h} \in \bar{\mathcal{H}}} f_d^{\bar{h}} = vol_d \quad \forall d \in \mathcal{D}_0 \tag{69}$$

$$\sum_{\bar{h} \in \bar{\mathcal{H}}} x_d^{\bar{h}} = 1 \quad \forall d \in D_0 \tag{70}$$

$$f_d^{\bar{h}} \leq vol_d\, x_d^{\bar{h}} \quad \forall d \in D_0,\ \forall \bar{h} \in \bar{\mathcal{H}} \tag{71}$$

$$\sum_{\substack{\bar{h} \in \bar{\mathcal{H}} \\ h - \bar{h} = 0}} \sum_{d \in \mathcal{D}_0} f_d^{\bar{h}} \leq prod_h^{per} \quad \forall h \in \mathcal{H} \tag{72}$$

$$x_d^{\bar{h}} \in \{0,1\}\ \forall d \in \mathcal{D}_0,\ \forall \bar{h} \in \bar{\mathcal{H}} \tag{73}$$

$$f_d^{\bar{h}} \in \mathbb{N}\ \forall d \in \mathcal{D}_0,\ \forall \bar{h} \in \bar{\mathcal{H}} \tag{74}$$

Replacing $p_d^{\bar{h}}$ with the expression given in Equation (48), the objective function (68) is

$$\sum_{d \in \mathcal{D}_0} \sum_{\bar{h} \in \bar{\mathcal{H}}} p_d^{\bar{h}}\, vol_d\, x_d^{\bar{h}} = \sum_{d \in \mathcal{D}_0} (p_d^0\, vol_d\, x_d^0 + p_d^1\, vol_d\, x_d^1) = \sum_{d \in \mathcal{D}_0} -\frac{c_d}{vol_d}\, vol_d\, x_d^1 = \sum_{d \in \mathcal{D}_0} -c_d\, x_d^1$$

and Constraints (72) decompose into

$$\sum_{d \in \mathcal{D}_0} f_d^0 \leq prod_0^{per} \leq \sum_{d \in \mathcal{D}_0} vol_d \tag{75}$$

$$\sum_{d \in \mathcal{D}_0} f_d^1 \leq prod_1^{per} \leq B \tag{76}$$

Constraint (75) is always satisfied and can be removed. The model becomes

$$\min \sum_{d \in \mathcal{D}_0} -c_d\, x_d^1 \tag{77}$$

$$\text{s.t.} \sum_{\bar{h} \in \bar{\mathcal{H}}} f_d^{\bar{h}} = vol_d \quad \forall d \in \mathcal{D}_0 \tag{78}$$

$$\sum_{\bar{h} \in \bar{\mathcal{H}}} x_d^{\bar{h}} = 1 \quad \forall d \in D_0 \tag{79}$$

$$f_d^{\bar{h}} \leq vol_d\, x_d^{\bar{h}} \quad \forall d \in D_0,\ \forall \bar{h} \in \bar{\mathcal{H}} \tag{80}$$

$$\sum_{d \in \mathcal{D}_0} f_d^1 \leq B \tag{81}$$

$$x_d^{\bar{h}} \in \{0,1\}\ \forall d \in \mathcal{D}_0,\ \forall \bar{h} \in \bar{\mathcal{H}} \tag{82}$$

$$f_d^{\bar{h}} \in \mathbb{N}\ \forall d \in \mathcal{D}_0,\ \forall \bar{h} \in \bar{\mathcal{H}} \tag{83}$$

Note that Constraints (80) are never strict, and inequalities can be changed to

$$f_d^{\bar{h}} = vol_d\, x_d^{\bar{h}} \quad \forall d \in D_0,\ \forall \bar{h} \in \bar{\mathcal{H}}$$

(perfect relation between variables x and f is due to the fact that periods are made by only one time slot). Then the model is equivalent to

$$\text{(PSP-KP)} \quad \max \sum_{d \in \mathcal{D}_0} c_d \, x_d^1 \tag{84}$$

$$\text{s.t.} \quad \sum_{\bar{h} \in \bar{\mathcal{H}}} x_d^{\bar{h}} = 1 \quad \forall d \in D_0 \tag{85}$$

$$\sum_{d \in \mathcal{D}_0} vol_d x_d^1 \leq B \tag{86}$$

$$x_d^{\bar{h}} \in \{0,1\} \; \forall d \in \mathcal{D}_0, \; \forall \bar{h} \in \bar{\mathcal{H}} \tag{87}$$

It is trivial to see that problems defined by models (PSP-KP) and (KP) have the same optimal solution, and this concludes the proof.

□

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