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## Logarithmic mathematical morphology: a new framework adaptive to illumination changes

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## Abstract

A new set of mathematical morphology (MM) operators adaptive to illumination changes caused by variation of exposure time or light intensity is defined thanks to the Logarithmic Image Processing (LIP) model. This model based on the physics of acquisition is consistent with human vision. The fundamental operators, the logarithmic-dilation and the logarithmic-erosion, are defined with the LIP-addition of a structuring function. The combination of these two adjunct operators gives morphological filters, namely the logarithmic-opening and closing, useful for pattern recognition. The mathematical relation existing between "classical" dilation and erosion and their logarithmic-versions is established facilitating their implementation. Results in simulated and real images show that logarithmic-MM is more efficient on low-contrasted information than "classical" MM.

## Logarithmic Image Processing (LIP)

- LIP is a model based on the Transmittance Law which is adapted to process images
- LIP is consistent with the Human Visual System

Let  $f, g$  be two images defined on  $D \subset \mathbb{R}^N$  with values in  $[0, M[$ ,  $M \in \mathbb{R}$

- the transmittance is related to the grey level by

$$T_f(x) = 1 - \frac{f(x)}{M} \quad (1)$$

- the transmittance law is

$$T_{f \triangle g} = T_f \times T_g \quad (2)$$

- Equation (2) and (1) gives the sum law

$$f \triangle g = f + g - \frac{f \cdot g}{M} \quad (3)$$

- From equation (3), a LIP-negative function is deduced

$$\triangle f = \frac{-f}{1-f/M} \quad (4)$$

- From equations (4) and (3), the difference law is deduced

$$f \triangle g = \frac{f-g}{1-g/M} \quad (5)$$

### A strong physical property

The negative values  $\triangle f$ , with  $f > 0$ , are light intensifiers that can be used to compensate the light attenuation due a variation of exposure-time (or light intensity).



$\triangle 100 =$



## Mathematical Morphology (MM)

Let  $f : D \rightarrow [0; M]$  be a grey level image and  $b : D_b \rightarrow [0, M]$  a structuring function (invariant under translation (in  $D$ )), the dilation  $\delta$  and erosion  $\varepsilon$ , are usually defined in the lattice  $(\mathbb{R}^D, \leq)$  by :

$$\delta_b(f)(x) = \vee \{f(x-h) + b(h), h \in D_b\}$$

$$\varepsilon_b(f)(x) = \wedge \{f(x-h) - b(h), h \in D_b\}$$

### Adjunction

$(\varepsilon_b, \delta_b)$  is an adjunction because it satisfies

$$\forall f, g \in \mathbb{R}^D, \quad \delta_b(f) \leq g \Leftrightarrow f \leq \varepsilon_b(g)$$

As  $(\varepsilon_b, \delta_b)$  forms an adjunction:

- $\gamma_b = \delta_b \varepsilon_b$  is an opening (increasing, anti-extensive and idempotent)
- $\varphi_b = \varepsilon_b \delta_b$  is a closing (increasing, extensive and idempotent)

## Logarithmic-Mathematical Morphology

Let  $\bar{\mathcal{F}} = [-\infty, M]^D$  be the lattice of functions with values in  $[-\infty, M]$

Let  $f \in \bar{\mathcal{F}}$  be a function and  $b \in ]-\infty, M[$  a structuring function, the mappings defined in  $\bar{\mathcal{F}}$  by

$$\delta_b^{\triangle}(f)(x) = \vee \{f(x-h) \triangle b(h), h \in D_b\}$$

$$\varepsilon_b^{\triangle}(f)(x) = \wedge \{f(x-h) \triangle b(h), h \in D_b\}$$

form an adjunction  $(\varepsilon_b^{\triangle}, \delta_b^{\triangle})$ , with  $\varepsilon_b^{\triangle}$  an erosion and  $\delta_b^{\triangle}$  a dilation.

$\varepsilon_b^{\triangle}$  is called a logarithmic-erosion and  $\delta_b^{\triangle}$  a logarithmic-dilation.

As  $(\varepsilon_b^{\triangle}, \delta_b^{\triangle})$  is an adjunction,

- $\gamma_b^{\triangle} = \delta_b^{\triangle} \varepsilon_b^{\triangle}$  is an opening called logarithmic-opening
- $\varphi_b^{\triangle} = \varepsilon_b^{\triangle} \delta_b^{\triangle}$  is a closing called logarithmic-closing

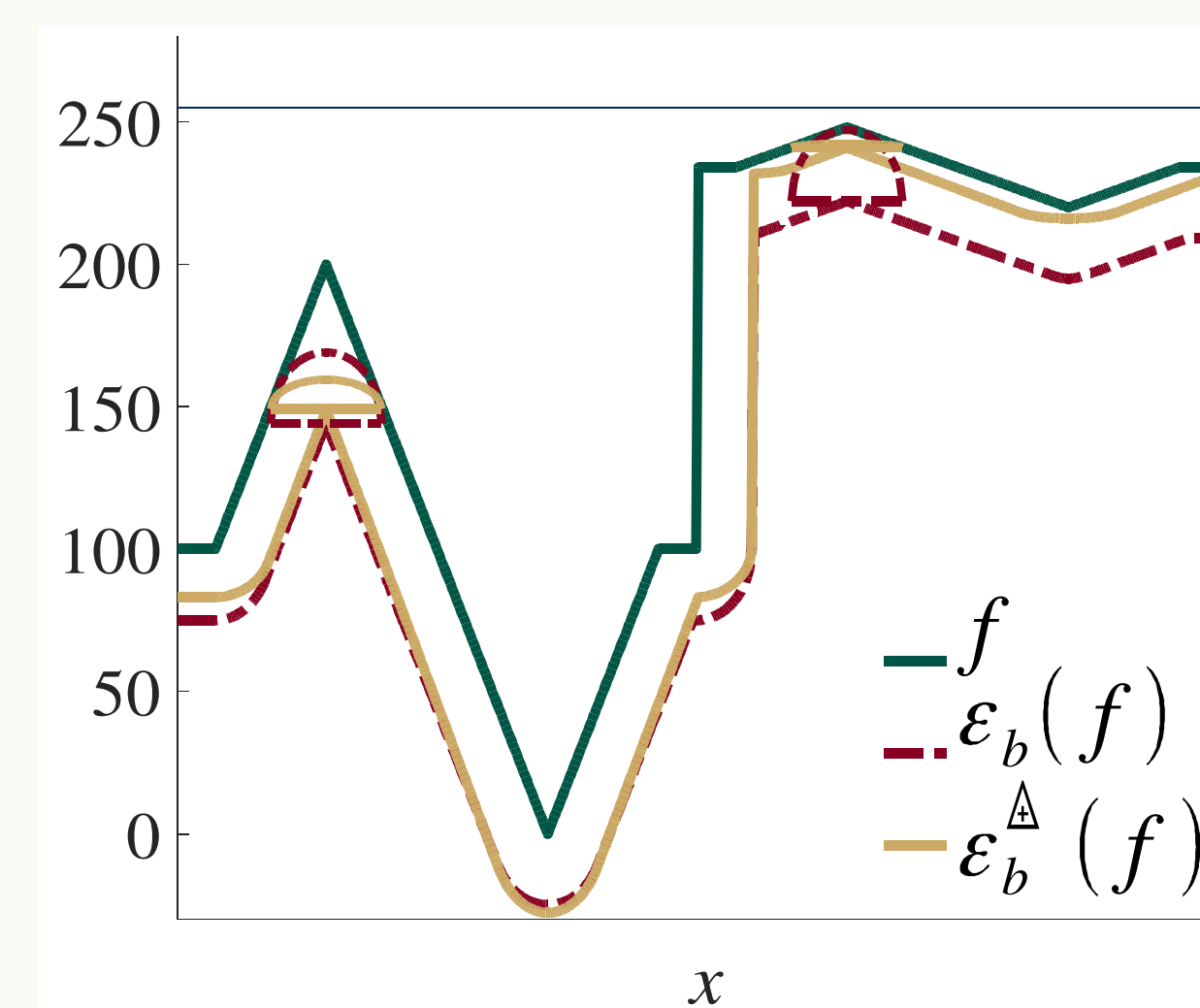
### Relation with classical Mathematical Morphology

The logarithmic-dilation  $\delta_b^{\triangle}$  and the logarithmic-erosion  $\varepsilon_b^{\triangle}$  are related to the dilation  $\delta_b$  and erosion  $\varepsilon_b$  by:

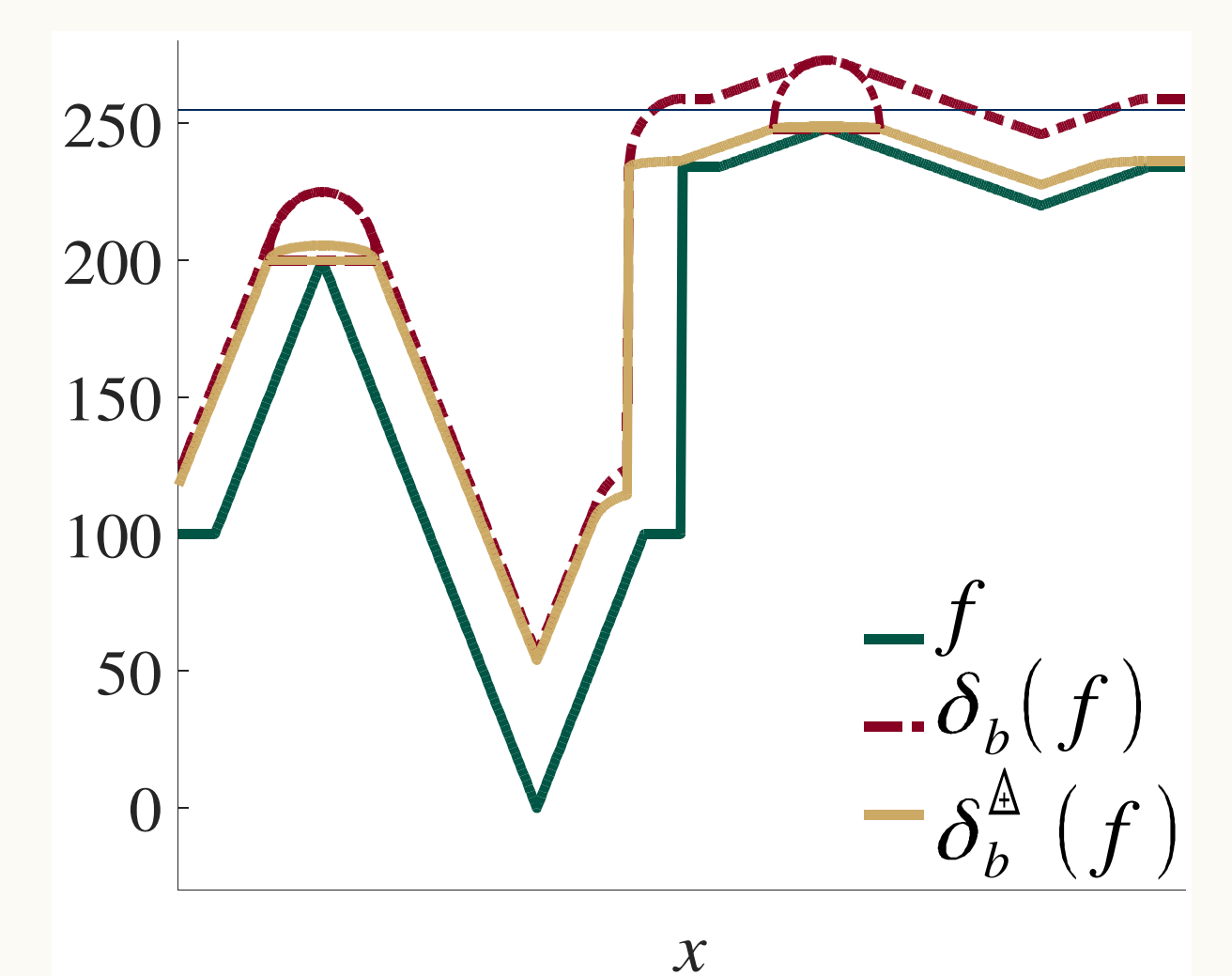
$\delta_b^{\triangle}(f)(x) = M(1 - \exp(-\delta_b \hat{f}))$  and  $\varepsilon_b^{\triangle}(f)(x) = M(1 - \exp(-\varepsilon_b \hat{f}))$   
where  $\hat{f} = -\ln(1 - f/M)$ ,  $\hat{f} \in \mathbb{R}$ .

## Results

### Adaptivity to illumination changes



(a) Erosions  $\varepsilon_b$  and  $\varepsilon_b^{\triangle}$



(b) Dilations  $\delta_b$  and  $\delta_b^{\triangle}$

Fig. Comparison between the classical MM and the Logarithmic-MM

### Robustness to illumination changes caused by variation of the camera exposure-time

- Complementation of the images  $f^c = (M-1) - f$
- Morphological gradient  $q_b(f^c) = \delta_b(f^c) - \varepsilon_b(f^c)$
- Logarithmic-morphological gradient  $q_b^{\triangle}(f^c) = \delta_b^{\triangle}(f^c) \triangle \varepsilon_b^{\triangle}(f^c)$



Bright image  $f$  (1/40s)



Gradient  $q_b(f^c)$



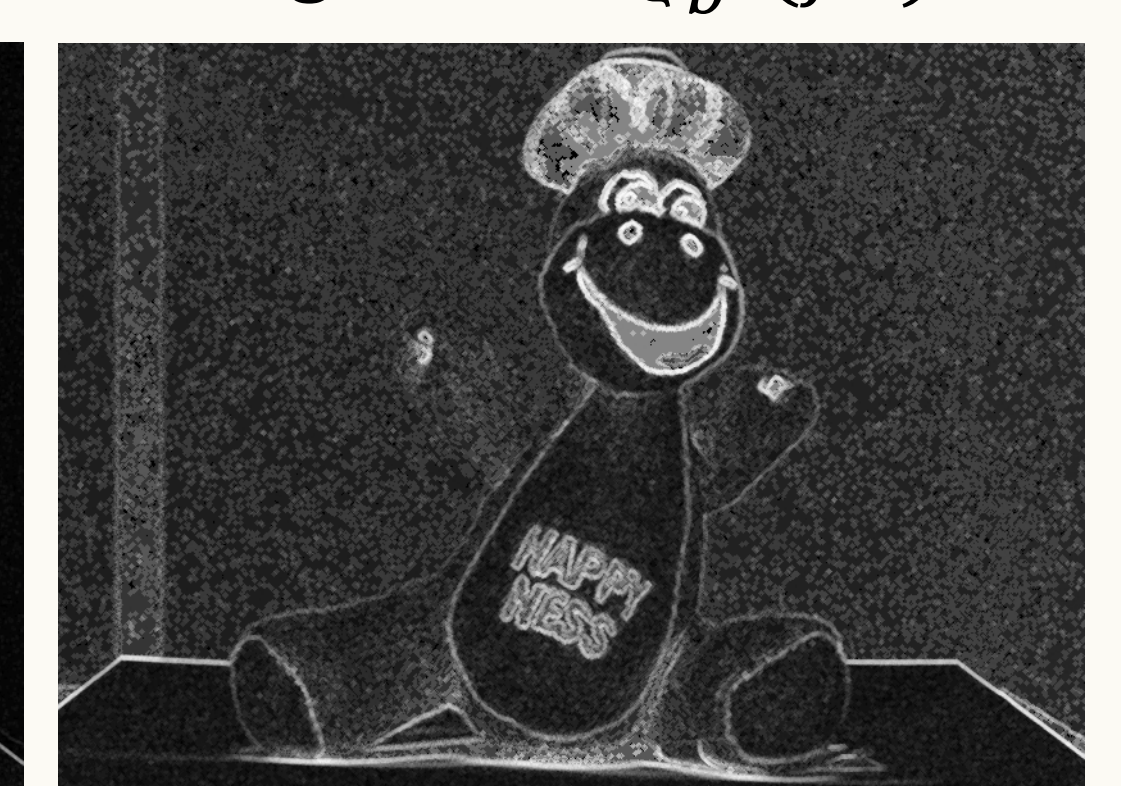
L-gradient  $q_b^{\triangle}(f^c)$



Dark image  $f^d$  (1/800s)



Gradient  $q_b((f^d)^c)$



L-gradient  $q_b^{\triangle}((f^d)^c)$

## Conclusion

The logarithmic MM operators are adaptive to illumination changes caused by a variation of the camera-exposure time or the source intensity.