

Constraint regularization for anon-smooth damage model

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Introduction

On damage mechanics

- ▶ Description by a scalar field d , localization
- ▶ Regularization by ∇d (Frémond Nedjar, Marigo, ...)
- ▶ Integral-type non local models (Pijaudier-Cabot, Jirasek, Havlasek,...)
- ▶ Phases field (Karma, Miehe ...)
- ▶ Model based on moving surface or layers

Some classical laws

Materials with simple dissipation (A. Ehrlacher)

Description based on energy

- ▶ Free energy

$$w_{rev}(\boldsymbol{\varepsilon}, d) = \frac{1}{2}g(d)\boldsymbol{\varepsilon} : \mathbb{C}_o : \boldsymbol{\varepsilon}$$

- ▶ Dissipation function on the actual state ($c \neq 0$, Frémond, Nedjar, ..., Marigo, .. Lorentz)

$$w_s(d) = Y_c d + H_d(d) + \frac{1}{2}c(\nabla d)^2$$

Introduction of a total potential

$$\mathcal{E}(\mathbf{u}, d) = \int_{\Omega} w_{rev} + w_s \, d\Omega - \int_{\partial\Omega} \mathbf{T} \cdot \mathbf{u} \, dS$$

Equilibrium state and evolution law

Variations of \mathcal{E} with respect to \mathbf{u} leads to stress field

$$\boldsymbol{\sigma} = g(d)\mathbb{C}_o : \boldsymbol{\varepsilon}$$

momentum conservation

$$\operatorname{div} \boldsymbol{\sigma} = 0, \quad [\boldsymbol{\sigma}] \cdot \mathbf{n} = 0$$

and boundary conditions

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{T}$$

Due to condition $\dot{d} \geq 0$ an inequality is obtained

Dissipation - Normality rule

Driving force Y for d given by

$$Y(\varepsilon, d) = -\frac{1}{2} g'(d) \varepsilon : \mathbb{C}_o : \varepsilon$$

Variations of \mathcal{E} with respect to d

- ▶ local terms

$$G = Y - G_c(d) = Y - Y_c - H'_d(d) + c\Delta d \leq 0$$

For point where $Y(\varepsilon) = Y_c + H'_d(0)$, damage d is continuous.

- ▶ surface terms : $c \llbracket \nabla d \rrbracket_{\Gamma} \cdot \mathbf{n} = 0$
Boundary natural conditions : $\nabla d \cdot \mathbf{n} = 0$ over $\partial\Omega$

Local normality rule

$$Y - G_c(d) \leq 0; \quad \dot{d} \geq 0, \quad (Y - G_c(d)) \dot{d} = 0$$

Dissipation is then positive

$$D_m = \int_{\Omega} Y \dot{d} \, d\Omega = \int_{\Omega} G_c(d) \dot{d} \, d\Omega$$

This describes the classical theories of damage

- ▶ local theory with $c = 0$
- ▶ introduction of gradient with $c \neq 0$
- ▶ phase fields is similar: $w_s = W_c \left(\frac{d^2}{\eta} + \eta \ell^2 (\nabla d)^2 \right)$
 $g(d) = (1 - d)^\alpha, \quad \alpha > 2, \quad \eta \ll 1 .$

Some fundamental questions

- ▶ existence and uniqueness
- ▶ localization and mesh dependency $c = 0$
- ▶ natural boundary condition ($\mathbf{n} \cdot \nabla d = 0$) : physical meaning
- ▶ transition : initiation - propagation - rupture
- ▶ integral type damage non local : problems on boundary
- ▶ determination of characteristics ($c, \eta \dots$)

We choose another point of view

Interfaces and moving domains

Irreversible change of materials characteristics

Sharp Transition : moving interface and strong discontinuities.

Material 1 is transformed in material 2 through a moving surface.

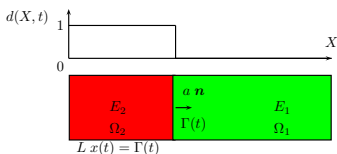
$$F = \int_a^b f(x, t) dx = \int_a^{\Gamma(t)} f(x, t) dx + \int_{\Gamma(t)}^b f(x, t) dx$$

$$\dot{F} = \int_a^b \dot{f} dx + (f(\Gamma^+, t) - f(\Gamma^-, t))\dot{\Gamma}$$

$$[[f]]_{\Gamma} = f(\Gamma^+, t) - f(\Gamma^-, t)$$

Moving interface \Rightarrow discontinuities and jump conditions (Hadamard).

Analytic solutions for particular cases but numerical simulation are not easy.



Two linear elastic materials : x proportion of phase 2
 Equilibrium:

$$\sigma = E_1 \varepsilon_1 = E_2 \varepsilon_2 = \Sigma, \quad u(0) = 0$$

Energy :

$$W(\sigma, x) = -\frac{1}{2} \sigma^2 \left(\frac{x}{E_2} + \frac{1-x}{E_1} \right)$$

Energy release rate:

$$G = -\frac{\partial W}{\partial x} = \frac{1}{2} \sigma^2 \left(\frac{1}{E_2} - \frac{1}{E_1} \right); \quad D_m = G \dot{x}$$

A new approach : continuous transition

STEP I : A layer where the damage varies continuously
description of diffuse Interface or graded damage.

- ▶ d varies from 0 to 1
- ▶ Free energy : $w(\boldsymbol{\varepsilon}, d) = g(d)\frac{1}{2}\boldsymbol{\varepsilon} : \mathbb{C}_o : \boldsymbol{\varepsilon}$
- ▶ Driving force Y associated to d :

$$Y = -\frac{\partial w}{\partial d}$$

- ▶ g continuous, no strong discontinuities (at order 1).

The dissipation is local

$$d_m = Y\dot{d}$$

STEP II : Connect the evolution \dot{d} to a moving layer !

Main idea : prescribed the shape of d inside the layer.

d is defined by distance function ϕ from the undamaged material .

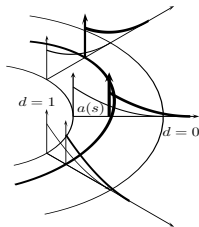
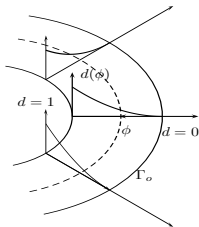
$$\phi \geq 0,$$

$$0 \leq d(\phi) \leq 1,$$

$$d' > 0$$

$$\phi \geq l_c, d(\phi) = 1$$

$$\|\nabla\phi\| = 1$$



$$\phi(\mathbf{X}, t) = l = \phi(\mathbf{X}_o(s) + l\mathbf{n}, t), \quad \dot{\phi} - a(\mathbf{X})\nabla\phi \cdot \mathbf{n} = 0$$

$$a(\mathbf{X}) = a(\mathbf{X}_o) = a(s), \quad \mathbf{X}_o(s)$$

Graded damage and level-set (Thick Level-Set)

d is defined by distance function ϕ from the sound material.

Γ_o where $d = O^+$, $\phi(\mathbf{X}_o, t) = 0$

Iso- $\phi = \ell$ is also iso- d with normal vector $\mathbf{n} = \nabla\phi$

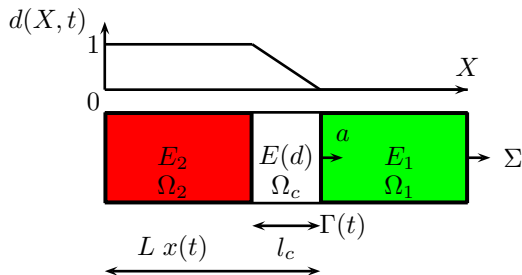
$$\phi(\mathbf{X}, t) = \ell = \phi(\mathbf{X}_o(S) + \ell\mathbf{n}, t), \quad \dot{\phi} - a(\mathbf{X})\nabla\phi \cdot \mathbf{n} = 0$$

$\|\nabla\phi\| = 1$, implies that $a(\mathbf{X}) = a(\mathbf{X}_o(s)) = a(s)$.

Dissipation determines the driving force associated to $a(s)$

$$D_m = \int_B \mathbf{Y} \dot{d} \, d\Omega = \int_{\Gamma} \left(\int_{\ell} \mathbf{Y} \, d'(\phi) \, j(z, s) \, dz \right) a(s) \, ds = \int_{\Gamma} G(s) a(s) \, ds$$

Extension of uni-axial bar



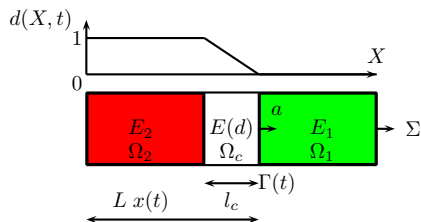
$$D_m = \int_{\ell} Y \dot{d} \, dz$$

Following the motion of the layer we have :

$$f(x, t) = f_a(x - at \mathbf{n}, t) \quad \dot{f} = -a \nabla f \cdot \mathbf{n} + \mathcal{D}_{\phi} f$$

With local quasi-steady state $\mathcal{D}_{\phi} f = 0$

Uniaxial bar



Free energy: $W = \frac{1}{2}E(d)\varepsilon^2$, $\boldsymbol{\sigma} = E(d)\boldsymbol{\varepsilon} = \Sigma$, $Y = -\frac{1}{2}E'\varepsilon^2$

Matching conditions : $E(0) = E_1, E(1) = E_2$

Dissipation ($d(\phi) = d_o, d'(\dot{\phi} + a\nabla\phi \cdot \mathbf{n}) = 0$)

$$D_m = a \int_{l_c} \frac{1}{2}\Sigma^2 \left(-\frac{E'}{E^2}\right) d' dz = a\Sigma^2 \int_1^0 \left(-\frac{E'}{2E^2}\right) dd = [W]a$$

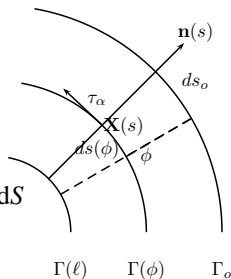
Geometry of the layer

Curvature:

$$\nabla \mathbf{n} \cdot \boldsymbol{\tau}_\alpha = -\kappa_\alpha^\beta \boldsymbol{\tau}_\beta$$

$$dS_\phi = \det(\mathbf{I} - \phi \kappa) = j(\phi, s) dS$$

$$\int_B f d\Omega = \int_\Gamma \left(\int_\ell f j(\phi, s) d\phi \right) dS = \int_\Gamma f_S dS$$



Generalized energy release rate

$$D_m = \int_B \mathbf{Y} \dot{d} d\Omega = \int_\Gamma \left(\int_\ell \mathbf{Y} d'(\phi) j(\phi, \mathbf{X}_o(s)) d\phi \right) a(s) ds$$

$$\bar{G}(s, t) = \int_{\ell(s)} \mathbf{Y}(\boldsymbol{\varepsilon}, d) d' j(\phi, s) d\phi$$

Properties

- ▶ Normality rule on Y implies normality rule on \bar{G}

$$(Y - Y_c(d))\dot{d} = 0 \Rightarrow \left(\bar{G} - \int_{\ell} Y_c(d) d' j(\phi) d\phi \right) a = 0$$

$$\bar{G} \leq \bar{G}_c(s) = \int_{\ell} Y_c(d) d' j d\phi$$

- ▶ On the driving force associated to $a(X_o(s))$

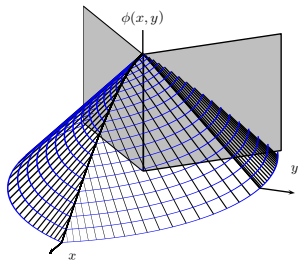
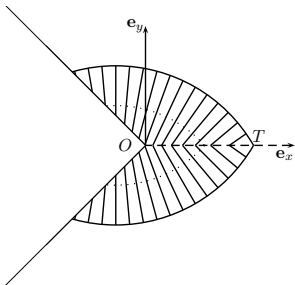
$$\bar{Y}(s, t) = \frac{\bar{G}}{\int_{\ell(s)} d' j(\phi, s) d\phi} = \frac{\int_{\ell(s)} Y d' j d\phi}{\int_{\ell(s)} d' j d\phi} \leq \bar{Y}_c(\ell(s))$$

During propagation we have

$$\bar{Y}(s, t) = \bar{Y}_c(\ell(s))$$

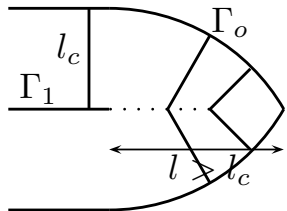
At initiation $\lim_{t \rightarrow 0^+} \bar{Y} = Y(x, t)$

- ▶ then $Y_c(0)$ is the critical value at initiation.
- ▶ $0 \leq \ell \leq l_c$, l is an increasing function.



$\nabla d \cdot \mathbf{n} \neq 0$ on the boundary

On the skeleton : discontinuities of ∇d .



Evolution of damage

$$\mathbf{X}_o(s, t) \in \Gamma_o(t)$$

$$\bar{Y}(s, t) = \frac{\int_o^{\ell(s)} Y(\boldsymbol{\varepsilon}, d) d' j(\phi) d\phi}{\int_o^{\ell(s)} d' j(\phi, s) d\phi} \leq \bar{Y}_c(\ell(s))$$

$$\lim_{t \rightarrow 0^+} \bar{Y} = Y(\boldsymbol{\varepsilon}, 0) \leq Y_c(0^+)$$

- ▶ $\ell = 0$, $Y \leq Y_c$, no damage
- ▶ Initiation.
At point x , $Y(x, t) = Y_c(0^+)$,
- ▶ $\bar{Y}(\ell(s), t) = \bar{Y}_c(\ell(s))$ implicit equation of a moving surface.
Extension of damaged zone $\dot{\ell} \geq 0$, $0 \leq \ell \leq l_c$
- ▶ $\ell = l_c$, layer is moving $a(s)\mathbf{n}$

Models of fracture and damage

Linear Frac. Mech.	Total damage	Graded Damage
$W(\mathbf{u}, l)$ $\mathcal{G} = -\frac{\partial W}{\partial l}$ $\mathcal{G} \leq G_c$ $a \geq 0$ $a \mathcal{D}_l(\mathcal{G} - G_c) = 0.$	$W(\mathbf{u}, \Gamma)$ $\mathcal{G}(s) = -\frac{\partial W}{\partial \Gamma}$ $\mathcal{G} \leq G_c$ $a(s) \geq 0$ $a(s) \mathcal{D}_\phi(\mathcal{G} - G_c) = 0.$	$W(\mathbf{u}, \phi)$ $\bar{G} = -\frac{\partial W}{\partial \phi}$ $\bar{G} \leq \bar{G}_c$ $a(s) \geq 0, \dot{\phi} + a \nabla \phi \cdot \mathbf{n} = 0$ $a(s) (\mathcal{D}_\phi \bar{G} - \mathcal{D}_\phi \bar{G}_c) = 0.$

Solution of evolution \mathbf{v}, a satisfies a variational inequality

$$\frac{\partial F}{\partial \mathbf{v}}(\mathbf{v} - \mathbf{v}^*) + \frac{\partial F}{\partial a}(a - a^*) + \left(\frac{\partial F}{\partial a} \cdot \frac{d(a - a^*)}{ds} \right) \geq 0$$

Analysis of stability and bifurcation (existence and uniqueness)
(CS 1985-2012)

New approach

Convex analysis with internal constrain

- ▶ d satisfies $0 \leq d \leq 1$,

$$g_1(d) = d(d - 1) \leq 0,$$

- ▶ gradient is bounded:

$$g_2(d, \nabla d) = \|\nabla d\| - f(d) \leq 0.$$

$\mathcal{C}_g = \{(d, \nabla d) | g_2(d, \nabla d) \leq 0\}$ is a convex set
if f is concave, $f(0) > 0$.

- ▶ For each convex function $g_i(d, \nabla d)$

$$\lambda_i \geq 0, \quad g_i(d, \nabla d) \leq 0, \quad \lambda_i g_i(d, \nabla d) = 0.$$

unilateral constrain with non dissipation.

Energy-Variation-Dissipation

$$\mathcal{E}(\mathbf{u}, d, \lambda_1, \lambda_2) = \int_{\Omega} w(\boldsymbol{\varepsilon}, d) \, d\Omega - \int_{\partial\Omega_T} T^d \cdot \mathbf{u} \, dS + \int_{\Omega} \sum_i \lambda_i g_i \, d\Omega.$$

Variations with respect to displacement

$$\frac{\partial \mathcal{E}}{\partial \mathbf{u}} \cdot \delta \mathbf{u} = 0, \quad \forall \delta \mathbf{u} \in \{\mathbf{v} \mid \mathbf{v}(x) = 0, x \in \partial\Omega_u\}.$$

$$0 = \operatorname{div} \boldsymbol{\sigma}, \quad \text{over } \Omega, \quad \mathbf{n} \cdot \boldsymbol{\sigma} = T^d, \quad \text{on } \partial\Omega_T.$$

Variations of λ_i

$$\frac{\partial \mathcal{E}}{\partial \lambda_1} \cdot \delta \lambda_1 = \int_{\Omega} d(d-1) \delta \lambda_1 \, d\Omega = 0.$$

$$\frac{\partial \mathcal{E}}{\partial \lambda_2} \cdot \delta \lambda_2 = \int_{\Omega} (f(d) - \|\nabla d\|) \delta \lambda_2 \, d\Omega = 0.$$

Ω is decomposed into parts where the constraints are reached

$$\lambda_2 \geq 0, \quad \Omega_2^+ = \{\mathbf{x} \mid \|\nabla d\| - f(d) = 0\}$$

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial d} \cdot \delta d &= \int_{\Omega} \frac{\partial w}{\partial d} \cdot \delta d + \lambda_1 (2d - 1) \delta d - \lambda_2 (f' \delta d - \nabla d \cdot \frac{\nabla \delta d}{\|\nabla d\|}) \, d\Omega \\ &= - \int_{\Omega} \mathcal{G} \cdot \delta d \, d\Omega + \int_S \left[\lambda_2 \frac{\nabla d}{\|\nabla d\|} \right]_S \cdot \mathbf{n} \, \delta d \, dS. \end{aligned}$$

Volume $g_2(\nabla d, d) = 0$

$$\mathcal{G} = - \frac{\partial w}{\partial d} + \lambda_2 f' + \operatorname{div} \left(\lambda_2 \frac{\nabla d}{\|\nabla d\|} \right) - \lambda_1 (2d - 1).$$

Surface of discontinuities

$$\left[\lambda_2 \frac{\nabla d}{\|\nabla d\|} \right]_S \cdot \mathbf{n} = 0. \rightarrow \lambda_2 = 0.$$

Boundary conditions

$$\lambda_2 \nabla d \cdot \mathbf{n} = 0, \text{ sur } \partial\Omega, \rightarrow \lambda_2 = 0.$$

Dissipation and evolution law

$$0 \in -\mathcal{G} + \partial \mathbf{D}(\dot{d})$$

Properties

- $d > 0$, $\lambda_1 = 0$, loading case

$$\mathcal{G} = \mathbf{Y} + \frac{1}{f} \operatorname{div}(\lambda_2 \nabla d) = \mathbf{Y} + \frac{1}{f} (\nabla \lambda_2 \cdot \nabla d + \lambda_2 \Delta d) = Y_c$$

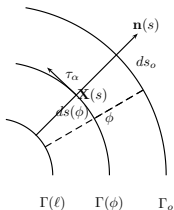
$\lambda_2 = 0$ over $\partial\Omega_d$ and $\lambda_2 \geq 0$, $\|\nabla d\| = f(d)$

$\nabla \lambda_2 \cdot \nabla d \geq 0$ over $d = 0$

$\nabla \lambda_2 \cdot \nabla d \leq 0$ over $d = d_m$

$\mathbf{Y} \leq Y_c$ at point $d = 0$

$\mathbf{Y} \geq Y_c$ at point d_m



Averaging on cones $\omega \in \Omega_d$, $\partial\Omega_d$ with normal to iso- d and tangent vector τ_α to iso- d

$$\int_{\omega} \operatorname{div}(\lambda_2 \nabla d) \, d\omega = \int_{\partial\Omega_d \cap \partial\omega} \lambda_2 \nabla d \cdot \mathbf{n} \, dS + \int_{\partial\omega_T} \lambda_2 \nabla d \cdot \tau_\alpha(m) \, dS_\alpha = 0$$

$$\int_{\mathcal{Z}} (\mathbf{Y} - Y_c) f \, j(\phi, s) \, d\phi = 0, \quad \|\nabla d\| = f.$$

Uniaxial Bar in extension

$$\lambda_i \geq 0, \quad g_i \leq 0, \quad \lambda_i g_i = 0$$

$$w(\varepsilon, d) = (1-d) \frac{E}{2} \varepsilon^2 + \frac{1}{2} H d^2, \quad f(d) = \frac{1}{l_c}.$$

- ▶ Step of linear elasticity $g_1 = 0, g_2 < 0, \Sigma = (1-d)E\varepsilon$

$$\mathcal{G} = \frac{1}{2} E \varepsilon^2 + \lambda_1 \leq Y_c$$

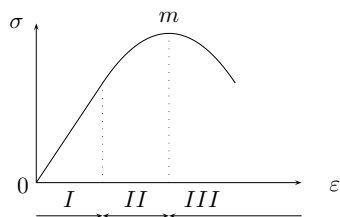
$$\varepsilon_c, \quad \Sigma_c = E\varepsilon_c, \quad \lambda_1 = 0, \quad Y_c = \frac{1}{2} \Sigma_c \varepsilon_c$$

- ▶ Uniform damage : $g_2 < 0, g_1 \neq 0, \lambda_1 = \lambda_2 = 0$

$$\dot{d} > 0, \quad \mathcal{G} = \frac{1}{2} \frac{\Sigma^2}{E(1-d)^2} - Hd = Y_c$$

$H = \alpha Y_c : \alpha > 2$ possible solution $\varepsilon \leq \varepsilon_m, \alpha < 2$ impossible.

Uniaxial bar in extension



Step I : Elasticity

Step II :Uniform damage

Localization at point m : Σ_m, ε_m

Step III : graded damage model $\|\nabla d\| - \frac{1}{l_c} = 0$

Uniaxial bar in extension

Graded damage : initiation

$g_2 = 0$ defines the domain $]0, \ell[$, $\ell \leq l_c$

$$d = \frac{\ell - x}{l_c} + d_m \leq 1, \quad \dot{d} = \frac{\dot{\ell}}{l_c} \quad d_o = d(0) > d_m.$$

Release rate of energy and dissipation

$$\mathcal{G} = \frac{\Sigma_l^2}{2E(1-d)^2} - \frac{1}{l_c} \frac{d\lambda_2}{dx} - Hd = Y_c.$$

At point $x = l$

$$\left[\lambda_2 \frac{\nabla d}{\|\nabla d\|} \right]_S = 0, \quad \rightarrow \lambda_2 = 0.$$

idem at $x = 0$.

Transform variable x in d , integration from d_m to d_o ,

$$\frac{\Sigma_l^2}{2E} \left(\frac{1}{1-d_o} - \frac{1}{1-d_m} \right) = Y_c \left(d_o - d_m + \frac{\alpha}{2} (d_o^2 - d_m^2) \right).$$

$$\frac{1}{2E} (\Sigma_l^2 - \Sigma_m^2) = -\alpha Y_c \left(\frac{\ell}{l_c} \right)^2 (1 - d_m) \leq 0.$$

$$\frac{1}{l_c^2} \frac{\lambda_2(d)}{d - d_m} = -\frac{d - d_o}{2(1-d)} \left(2\frac{\ell}{l_c} - \frac{x}{l_c} \right) > 0.$$

$$\frac{\Sigma_l^2}{2E(1-d_m)^2} + \frac{1}{l_c^2} \frac{d\lambda_2}{dd}(d_m) = Y_c(1 + \alpha d_m),$$

$$\frac{\Sigma_l^2}{2E(1-d_o)^2} + \frac{1}{l_c^2} \frac{d\lambda_2}{dd}(d_o) = Y_c(1 + \alpha d_o).$$

The definition of the generalized release rate is recovered by integration

$$\bar{g} = \int_l (Y - Hd) \frac{d\phi}{l_c} = \int_l Y_c \frac{d\phi}{l_c}.$$

Regularization of convex indicator of \mathcal{C}_1 et \mathcal{C}_2 (Moreau-Yoshida) :

$$I^\eta(d, \nabla d) = \frac{1}{2\eta} \left| f(d) - \|\nabla d\| \right|_-^2 + \frac{1}{2\eta} |d|_-^2 + \frac{1}{2\eta} |d - 1|_+^2,$$

with $|x|_- = \sup \{0, -x\}$, $|x|_+ = \sup \{0, x\}$, $\eta > 0$ small. Approximation gives the definition of d .

$$\begin{aligned} \frac{\partial I^\eta}{\partial d} &= -\frac{1}{\eta} \left| f(d) - \|\nabla d\| \right|_- \frac{df}{dd} - \frac{|d|_-}{\eta} + \frac{|d - 1|_+}{\eta}, \\ \frac{\partial I^\eta}{\partial \nabla d} &= -\frac{1}{\eta} \left| f(d) - \|\nabla d\| \right|_- \left(-\frac{\nabla d}{\|\nabla d\|} \right), \end{aligned}$$

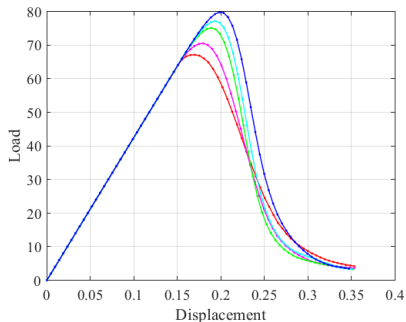
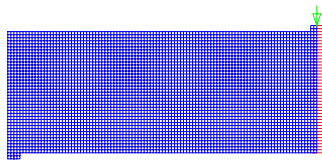
Then

$$-Y - \frac{\partial I^\eta}{\partial d}(d, \nabla d) - \operatorname{div} \left(\frac{\partial I^\eta}{\partial \nabla d}(d, \nabla d) \right) + \partial \mathbf{D}(\dot{d}) \ni 0.$$

No use of level-set : the numerical cost must be studied

First Numerical example

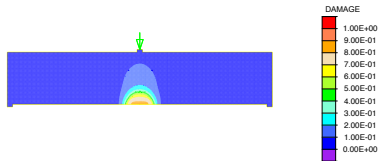
A beam in flexion.



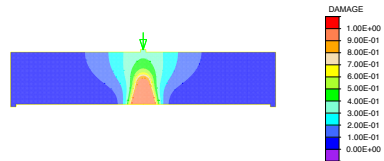
A point load in the middle the global response with different l_c .

More l_c smaller more brittle is the behaviour.

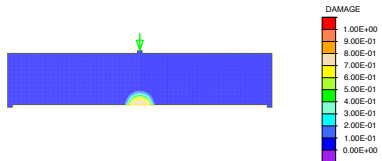
Results using different regularisation supplementary term $\frac{1}{2}c \|\nabla d\|^2$.



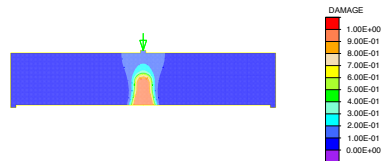
Time = 2.00E-01



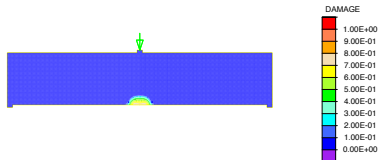
Time = 2.40E-01



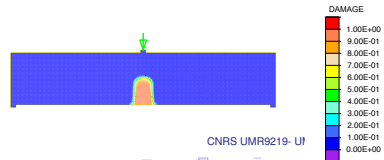
Time = 2.00E-01



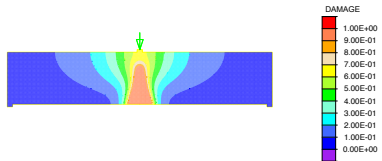
Time = 2.40E-01



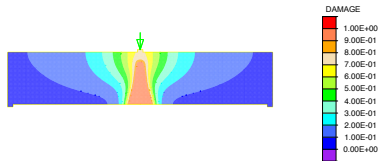
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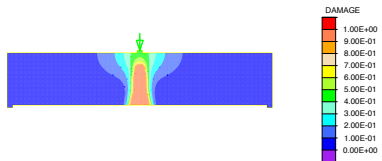
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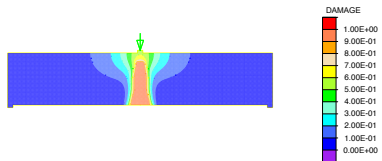
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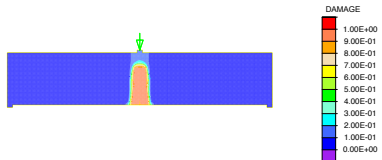
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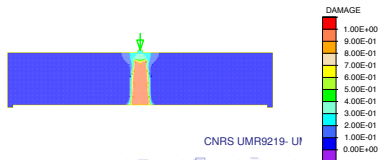
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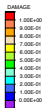
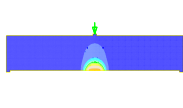
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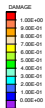
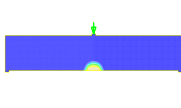
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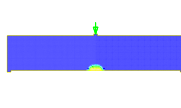
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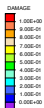
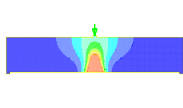
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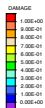
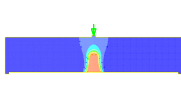
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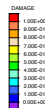
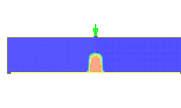
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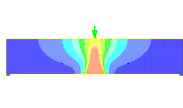
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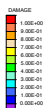
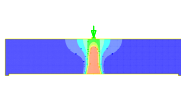
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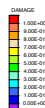
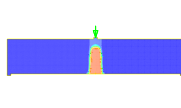
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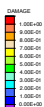
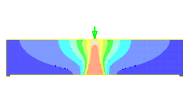
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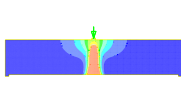
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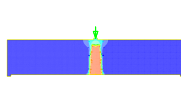
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Conclusion

- ▶ The graded damage model (TLS) proposes a new description of damage
- ▶ The propagation is determined by local quantities defined on surface
- ▶ The thickness l_c and the damage function $d(\phi)$ are characteristics.
- ▶ Level-set can be used (Sethian, Allaire-Jouve, Moës, Chevaugeon, Salzman, ...)
- ▶ The convex analysis with constrain on ∇d gives same description than TLS with local fields (no level-set) and exhibits properties of the solution. Classical finite element methods can be used.
- ▶ The natural boundary conditions are not required is the main difference with regularization by ∇d .

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