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DEFORMATION THEORY OF HOLOMORPHIC CARTAN GEOMETRIES

INDRANIL BISWAS, SORIN DUMITRESCU, AND GEORG SCHUMACHER

ABSTRACT. Introducing the deformation theory of holomorphic Cartan geometries, we compute infinitesimal automorphisms and infinitesimal deformations. We also prove the existence of a semi-universal deformation of a holomorphic Cartan geometry.

1. INTRODUCTION

Let G be a complex Lie group and $H < G$ a complex Lie subgroup with Lie algebras \mathfrak{g} , and \mathfrak{h} respectively. The quotient map $G \rightarrow G/H$ defines a holomorphic principal H -bundle. Moreover, on the total space of this principal bundle, namely G , we have a tautological \mathfrak{g} -valued holomorphic 1-form (the Maurer-Cartan form), constructed by identifying \mathfrak{g} with the right-invariant vector fields. This 1-form is an isomorphism of vector bundles and its restriction to the fibers of $G \rightarrow G/H$ coincide with the Maurer-Cartan form of H .

A holomorphic Cartan geometry of type (G, H) on a compact complex manifold X is infinitesimally modeled on this. More precisely, on a holomorphic principal H -bundle E_H over X there is a holomorphic 1-form A with values in the Lie algebra \mathfrak{g} of G that induces an isomorphism from the tangent bundle of E_H to the trivial bundle on E_H with fiber \mathfrak{g} . This isomorphism is required to be H -invariant and on each fiber of E_H it should be the Maurer-Cartan form [Sh]. A fundamental result of E. Cartan shows that the obstruction for A to satisfy the Maurer-Cartan equation of G is a curvature tensor which vanishes if and only if (E_H, A) is locally isomorphic to the H -principal bundle $G \rightarrow G/H$ endowed with the Maurer-Cartan form [Sh].

Our aim here is to introduce the deformation theory of holomorphic Cartan geometries on a compact complex manifold. We compute the tangent cohomology of a holomorphic Cartan geometry (E_H, A) in degree zero and one. Infinitesimal automorphisms of a holomorphic Cartan geometry (E_H, A) consist of all the $ad(E_H)$ -valued holomorphic vector fields ϕ satisfying the condition that the Lie derivative $L_\phi(A)$ vanishes. The space of infinitesimal deformations of (E_H, A) fits into a short exact sequence that we construct in the fourth section. The construction of the semi-universal deformation is worked out in the last section. Using the same methods a semi-universal deformation can also be constructed when the principal H -bundle and the underlying compact complex manifold are both moving.

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2. DEFORMATIONS OF HOLOMORPHIC CARTAN GEOMETRIES – DEFINITION

We will denote by G a connected complex Lie group, and by $H < G$ a closed connected complex subgroup. Furthermore X is a compact complex manifold. Let E_H denote a holomorphic principal H -bundle over X . Let $E_G = E_H \times_H G$ be the holomorphic principal G -bundle on X obtained by extending the structure group of E_H using the inclusion of H in G . We denote by T_M the holomorphic tangent bundle of a complex manifold M .

The action of the group H on E_H produces an action of H on the tangent bundle T_{E_H} : for $p \in E_H$, a tangent vector $v \in T_p(E_H)$ and $h \in H$ we have $v \cdot h = R_{h*}v$, where R_h is the right multiplication by h . For $\gamma \in \mathfrak{g} = \text{Lie}(G)$ we have $h \cdot \gamma = \text{ad}(h^{-1}(\gamma))$.

Let $\pi : E_H \rightarrow X$ be the projection, and $\pi_* : T_{E_H} \rightarrow T_X$ the induced map. The adjoint bundle $\text{ad}(E_H)$ is defined as $E_H \times_H \mathfrak{h}$, where $\mathfrak{h} = \text{Lie}(H)$. The space of vertical tangent vectors $\ker \pi_*$ is invariant under the action of H , and we have

$$\text{ad}(E_H) = \ker(\pi_*)/H. \quad (2.1)$$

Given a section of $\text{ad}(E_H)$ we will use the same notation for its pull-back to $\ker(\pi_*) \subset T_{E_H}$.

Definition 2.1. A *holomorphic Cartan geometry* of type (G, H) on X is a pair (E_H, A) , where E_H is a holomorphic principal H -bundle on X , and A is a \mathbb{C} -linear holomorphic isomorphism of vector bundles

$$A : T_{E_H} \xrightarrow{\sim} E_H \times \mathfrak{g}$$

over E_H such that

- (i) A is H -invariant, and
- (ii) the restriction of A to fibers of $E_H \rightarrow X$ is equal to the Maurer-Cartan form.

An *isomorphism* of holomorphic Cartan geometries $\Phi : (E_H, A) \rightarrow (F_H, B)$ is given by a holomorphic isomorphism $\phi : E_H \rightarrow F_H$ of principal bundles that takes A to B so that

$$\begin{array}{ccc} T_{E_H} & \xrightarrow{A} & E_H \times \mathfrak{g} \\ \swarrow & \downarrow \phi_* & \downarrow \phi \times \text{id}_{\mathfrak{g}} \\ X & & F_H \times \mathfrak{g} \\ \nwarrow & \downarrow B & \\ T_{F_H} & \xrightarrow{B} & F_H \times \mathfrak{g} \end{array} \quad (2.2)$$

is a commutative diagram.

We note that the above isomorphism A induces an isomorphism

$$A_H : T_{E_H}/H \xrightarrow{\sim} (E_H \times \mathfrak{g})/H, \quad (2.3)$$

where T_{E_H}/H is, by definition, the Atiyah bundle $\text{At}(E_H)$, which fits into the Atiyah sequence

$$0 \rightarrow \text{ad}(E_H) \rightarrow \text{At}(E_H) \rightarrow T_X \rightarrow 0$$

(see [At]). We also have $(E_H \times \mathfrak{g})/H \simeq \text{ad}(E_G)$, where the quotient is for the conjugation action mentioned earlier.

The isomorphism A_H in (2.3) induces a holomorphic connection on E_G [Sh], [BD, (2.8)], which in turn produces a holomorphic connection on $ad(E_G)$. Therefore, we have a holomorphic differential operator

$$D : ad(E_G) \longrightarrow \Omega_X^1 \otimes ad(E_G) \quad (2.4)$$

of order one.

Let us define now deformations of holomorphic Cartan geometries.

Definition 2.2. Let (E_H, A) be a principal H -bundle together with a holomorphic Cartan geometry.

- (i) Let S be a complex space, and let \mathcal{E}_H be a holomorphic principal H -bundle on $X \times S$. Let $\mathcal{E}_{H,s} = \mathcal{E}|_{X \times \{s\}}$ for $s \in S$. A holomorphic family of principal H -bundles with holomorphic Cartan geometries over S consists of a principal H -bundle \mathcal{E}_H over $X \times S$ together with a linear isomorphism \mathcal{A} over S

$$\begin{array}{ccc} T_{\mathcal{E}_H} & \xrightarrow[\sim]{\mathcal{A}} & \mathcal{E}_H \times \mathfrak{g} \\ & \searrow & \downarrow \\ & & S \end{array} \quad (2.5)$$

such that the restrictions \mathcal{A}_s of \mathcal{A} to fibers $T_{\mathcal{E}_{H,s}}$ over $s \in S$ define holomorphic Cartan geometries.

- (ii) Let (S, s_0) be a complex space with a distinguished point (or a germ of a complex space). A deformation of (E, A) over a space (S, s_0) consists of
- (a) a holomorphic family $(\mathcal{E}_H, \mathcal{A})$ of holomorphic Cartan geometries over S , and
 - (b) an isomorphism $\Phi : (E_H, A) \xrightarrow{\sim} (\mathcal{E}_{H,s_0}, \mathcal{A}_{s_0})$.
- (iii) Deformations are identified with their restrictions to any neighborhood of s_0 in S .
- (iv) Two deformations of (E_H, A) over a space (S, s_0) are called isomorphic, if (after replacing S by a neighborhood of s_0 , if necessary) there is an isomorphism of the respective families that induces the identity of (E_H, A) via the maps Φ of the above type.

3. DEFORMATIONS OF HOLOMORPHIC CARTAN GEOMETRIES

The tangent cohomology of E_H is equal to $H^\bullet(X, ad(E_H))$, where $ad(E_H)$ is the adjoint bundle of E_H with fiber \mathfrak{h} [Don]. In particular in degrees 0, 1, and 2 we have infinitesimal automorphisms, infinitesimal deformations, and the space containing obstructions respectively.

3.1. Infinitesimal automorphisms of holomorphic Cartan geometries. We begin with tangent cohomology $T^0(E_H, A)$ of degree zero for holomorphic Cartan geometries. Let (E_H, A) be a holomorphic Cartan geometry with A taking values in \mathfrak{g} . From (2.1) it follows that holomorphic section ψ of $ad(E_H)$ over $U \subset X$ gives a H -invariant holomorphic vector field $\tilde{\psi}$ over $E_H|_U$ which is vertical for the projection π . We will identify ψ with $\tilde{\psi}$. By L_ψ we denote the Lie derivative with respect to this vector field $\tilde{\psi}$.

Proposition 3.1. *The space of infinitesimal automorphisms is equal to*

$$T^0(E_H, A) = H^0(X, ad(E_H))_A = \{\psi \in H^0(X, ad(E_H)) \mid L_\psi(A) = 0\}. \quad (3.1)$$

Proof. We consider (2.2) for $F_H = E_H$. Now the infinitesimal action of the group of vertical automorphisms of the given principal bundle gives rise to the following diagram. We denote by $\psi \in H^0(X, ad(E_H))$ an infinitesimal automorphism of E_H . We have the following diagram of homomorphisms on E_H :

$$\begin{array}{ccc} TE_H & \xrightarrow{A} & E_H \times \mathfrak{g} \\ L_\psi \downarrow & & \downarrow \psi \\ TE_H & \xrightarrow{A} & E_H \times \mathfrak{g} \end{array} \quad (3.2)$$

Diagram (3.2) is interpreted as follows. As mentioned before, holomorphic sections of $ad(E_H)$ are H -invariant, vertical sections of the holomorphic tangent bundle TE_H . We apply A to a holomorphic section v of TE_H ; then ψ is applied to $A(v)$, which is simply the Lie bracket on \mathfrak{g} because ψ is a function on E_H with values in \mathfrak{h} . On the other hand, ψ acts on holomorphic sections of $ad(E_H)$ by applying the Lie derivative L_ψ , which is the infinitesimal version of the adjoint action.

The infinitesimal automorphism ψ is compatible with the holomorphic Cartan geometry, if (3.1) commutes for all sections v of $ad(E_H)$, i.e.

$$A(L_\psi(v)) = \psi(A(v)) = L_\psi(A(v)) + A(L_\psi(v))$$

for all v . Therefore, (3.1) commutes if and only if $L_\psi(A) = 0$. \square

3.2. Infinitesimal deformations of holomorphic Cartan geometries. Let $\mathbb{C}[\epsilon] = \mathbb{C}[t]/(t^2)$, so that $\mathbb{C}[\epsilon] = \mathbb{C} \oplus \epsilon \cdot \mathbb{C}$ holds with $\epsilon^2 = 0$. The space $D = (\{0\}, \mathbb{C}[\epsilon])$ is also called double point with $\mathcal{O}_D = \mathbb{C}[\epsilon]$. The tangent space T_{S, s_0} of an arbitrary complex space S at a point $s_0 \in S$ can be identified with the space of all holomorphic mappings $D \rightarrow S$ such that the underlying point 0 is mapped to s_0 . A deformation of H -principal bundles (with holomorphic Cartan geometries) over $(D, 0)$ is called an infinitesimal deformation of principal bundles (of holomorphic Cartan geometries).

We assume that an (isomorphism class of an) infinitesimal deformation \mathcal{E}_H of E_H is given, corresponding to an element of $H^1(X, ad(E_H))$ [Don]. This gives rise to exact sequences

$$\begin{array}{ccccccc} 0 & \longrightarrow & \epsilon \cdot \mathbb{C} & \longrightarrow & \mathbb{C} \oplus \epsilon \cdot \mathbb{C} & \longrightarrow & \mathbb{C} \longrightarrow 0 \\ & & \downarrow & & \downarrow & & \downarrow \\ 0 & \longrightarrow & TE_H & \longrightarrow & T_{\mathcal{E}_H} & \longrightarrow & TE_H \longrightarrow 0 \\ & & \downarrow \sim A & & \downarrow \mathcal{A}_{ij} & & \downarrow \sim A \\ 0 & \longrightarrow & E_H \times \mathfrak{g} & \longrightarrow & \mathcal{E}_H \times \mathfrak{g} & \longrightarrow & E_H \times \mathfrak{g} \longrightarrow 0, \end{array} \quad (3.3)$$

where the isomorphism \mathcal{A} is still to be constructed.

Let us cover X by a system \mathfrak{U} of contractible, Stein, open subsets U_i such that over any U_i an isomorphism \mathcal{A}_i does exist. The difference of any two maps \mathcal{A}_i and \mathcal{A}_j determines a cocycle $\mathcal{A}_{ij} \in Z^1(\mathfrak{U}, Hom(T_{E_H}, E_H \times \mathfrak{g})^H)$ in the space of H -invariant homomorphisms. Suppose that \mathcal{A}_{ij} is a coboundary. Then the morphisms \mathcal{A}_i can be changed to define the desired (global) map \mathcal{A} . It can be verified immediately that the cohomology class of \mathcal{A}_{ij} is uniquely determined by the infinitesimal deformation of the principal H -bundle E_H and the holomorphic Cartan geometry A .

All the above morphisms $\mathcal{A}_i, \mathcal{A}_{ij}$ can be chosen as H -invariant. Then the obstructions are cohomology classes of $\text{Hom}(At(E_H), ad(E_G))$.

Proposition 3.2. *Let (E_H, A) be a holomorphic Cartan geometry of type (G, H) on X . Let \mathcal{E}_H be an isomorphism class of infinitesimal deformations of E_H . Then the obstructions to extend the holomorphic Cartan geometry A to \mathcal{E}_H are in*

$$H^1(E_H, \text{Hom}(T_{E_H}, E_H \times \mathfrak{g})^H)$$

or equivalently in

$$H^1(X, \text{Hom}(At(E_H), ad(E_G))).$$

We denote by $H^1(X, ad(E_H))_A$ the space of isomorphism classes of infinitesimal deformations of E_H such that the Cartan geometry A of the central fiber can be extended, so $H^1(X, ad(E_H))_A \subset H^1(X, ad(E_H))$. Furthermore we denote by $T^1(E_H, A)$ the space of isomorphism classes of infinitesimal deformations of (E_H, A) .

If we assume that a holomorphic deformation $(\mathcal{E}_H, \mathcal{A})$ of the holomorphic Cartan geometry (E_H, A) does exist, then \mathcal{A} is unique up to an H -invariant morphism $T_{E_H} \rightarrow E_H \times \mathfrak{g}$. This implies the following result.

Theorem 3.3 (Infinitesimal deformations). *Let (E_H, A) be a holomorphic Cartan geometry. Then there is an exact sequence*

$$0 \rightarrow H^0(E_H, \text{Hom}(T_{E_H}, E_H \times \mathfrak{g})^H) \rightarrow T^1(E_H, A) \rightarrow H^1(X, ad(E_H))_A \rightarrow 0,$$

where $H^0(E_H, \text{Hom}(T_{E_H}, E_H \times \mathfrak{g})^H)$ can be identified with $H^0(X, \text{Hom}(At(E_H), ad(E_G)))$.

3.3. Semi-universal deformation of principal bundles. We begin the construction with a semi-universal deformation of E_H . A deformation over a complex space S with base point s_0 is given by a holomorphic family \mathcal{E}_H of principal H bundles over $X \times S$, together with an isomorphism $\Xi : E_H \rightarrow \mathcal{E}_H|_{X \times \{s_0\}}$. Semi-universality amounts to the following conditions (cf. also the more general Definition 2.2):

- (i) (*Completeness*) for any deformation $\underline{\xi}$ of E_H over a space (W, w_0) , given by a holomorphic family $\mathcal{E}_{H,W} \rightarrow X \times W$ together with an isomorphism of the above type, there is a base change morphism $f : (W, w_0) \rightarrow (S, s_0)$ such that (after replacing the base space with open neighborhoods of the base points, if necessary) the pull back $f^*\underline{\xi} = (id_X \times f)^*(\mathcal{E}_H)$ and $\mathcal{E}_{H,W}$ are isomorphic with isomorphism inducing the identity map over $X \times \{w_0\}$.
- (ii) Let $(W, w_0) = (D, 0)$ be as in (i). Then any such base change f is uniquely determined by the given deformation. The set of isomorphism classes of deformations over $(D, 0)$ is called ‘‘tangent space of the deformation functor’’ or first tangent cohomology associated to the given deformation problem. For deformations of a holomorphic principal H -bundle on X this space is known to be equal to $H^1(X, ad(E_H))$. (See [Don]; see also [BHH].)

For the sake of completeness we mention the Kodaira-Spencer map:

Given a deformation $\underline{\xi}$ over (W, w_0) we identify a tangent vector v of W at w_0 with a holomorphic map $f : (D, 0) \rightarrow (W, w_0)$. Then the Kodaira-Spencer map $\rho : T_{W, w_0} \rightarrow H^1(X, ad(E_H))$ maps $\underline{\xi}$ to the isomorphism class of $f^*\underline{\xi}$. We already computed the tangent cohomologies of order zero and one.

4. SEMI-UNIVERSAL DEFORMATION FOR HOLOMORPHIC CARTAN GEOMETRIES

4.1. **Morphisms of vector bundles.** Let again X be a compact complex manifold, S a complex space and E_1, E_2 holomorphic vector bundles on $X \times S$.

Definition 4.1. Let \mathbf{An}_S be the category of complex analytic spaces over S and \mathbf{Sets} the category of sets. The objects of \mathbf{An}_S are complex space $W \rightarrow S$ and the morphisms compatible holomorphic mappings.

We have the following functor

$$F : \text{Hom}(E_1, E_2)_S : \mathbf{An}_S \rightarrow \mathbf{Sets}$$

(i) Any $f : W \rightarrow S$ is assigned to the set

$$F(f) = F(f : W \rightarrow S) = \text{Hom}((id_X \times f)^* E_1, (id_X \times f)^* E_2),$$

(ii) given a holomorphic map α

$$\begin{array}{ccc} W_1 & \xrightarrow{\alpha} & W_2 \\ & \searrow f_1 & \downarrow f_2 \\ & & S \end{array}$$

we set $E_{i,W_i} = (id_X \times f_i)^* E_i$ for $i = 1, 2$. Then $F(\alpha)$ assigns to

$$\kappa : \begin{array}{ccc} E_{1,W_2} & \xrightarrow{\beta} & E_{2,W_2} \\ & \searrow & \downarrow \\ & & X \times W_2 \end{array}$$

the pull back $g^* \kappa$ via g of κ :

$$g^* \kappa : \begin{array}{ccc} E_{1,W_1} & \xrightarrow{(id_X \times \alpha)^* \beta} & E_{2,W_1} \\ & \searrow & \downarrow \\ & & X \times W_1 \end{array}$$

We will use the representability of the morphism functor. In the algebraic case it was shown by Grothendieck in [Gr, EGAI 7.7.8 and 7.7.9]. In the analytic case the corresponding theorem for complex spaces is due to Douady [Dou, 10.1 and 10.2]. For coherent (locally free sheaves) \mathcal{M} , and \mathcal{N} over a space $\mathcal{X} \rightarrow S$, one considers morphisms of the simple extensions $\mathcal{X}[\mathcal{M}] \rightarrow \mathcal{X}[\mathcal{N}]$.

Fact. The functor F is representable by a complex space $g : R \rightarrow S$. There is a universal object in $F(g)$

$$v : \begin{array}{ccc} E_{1,R} & \xrightarrow{\gamma} & E_{2,R} \\ & \searrow & \downarrow \\ & & X \times R \end{array}$$

such that any other object over $\tilde{R} \rightarrow S$ is isomorphic to $\tilde{g}^* v$, where $\tilde{g} : \tilde{R} \rightarrow R$ is a holomorphic map over S .

4.2. Application to holomorphic Cartan geometries.

Theorem 4.2. *Let X be a compact complex manifold, $H < G$ connected complex Lie groups, and (E_H, A) a holomorphic Cartan geometry on X of type (G, H) . Then (E_H, A) possesses a semi-universal deformation.*

Proof. We begin with a semi-universal deformation of E_H :

$$\begin{array}{ccccc} E_H & \xrightarrow{\sim} & \mathcal{E}_H|_{X \times \{s_0\}} & \hookrightarrow & \mathcal{E}_H \\ & \searrow & \downarrow & & \downarrow \\ & & X \times \{s_0\} & \hookrightarrow & X \times S, \end{array}$$

and look at the holomorphic vector bundles

$$\begin{array}{ccc} At(\mathcal{E}_H) & & ad(\mathcal{E}_G) \\ & \searrow & \swarrow \\ & X \times S & \end{array}$$

to which we apply the representability of the homomorphism functor. We have the following universal homomorphism η . (It can be checked easily that At and ad commute with base change).

$$\begin{array}{ccccccc} & & \eta & & & & \\ & \xrightarrow{\quad} & & \xrightarrow{\quad} & & \xrightarrow{\quad} & \\ At(\mathcal{E}_{H,R}) & \xrightarrow{\quad} & ad(\mathcal{E}_{G,R}) & & At(\mathcal{E}_H) & & ad(\mathcal{E}_G) \\ & \searrow & \swarrow & & \searrow & & \swarrow \\ & X \times R & \xrightarrow{id_X \times h} & X \times S & & & \\ & \downarrow & & \downarrow & & & \\ & R & \xrightarrow{h} & S & & & \end{array}$$

Now the holomorphic Cartan geometry $A_H : At(E_H) \xrightarrow{\sim} ad(E_G)$ amounts to a point $r_0 \in R$, which is mapped to s_0 under $R \rightarrow S$ (observing the deformation theoretic isomorphisms of the given objects and distinguished fibers of the semi-universal families). We take the connected component of R through r_0 and restrict it to an open neighborhood, where the universal morphism is an isomorphism. Going through the construction, we see that the restricted family yields a semi-universal deformation of the holomorphic Cartan geometry (E_H, A) . \square

REFERENCES

[At] Atiyah, M. F.: *Complex analytic connections in fibre bundles*, Trans. Amer. Math. Soc. **85** (1957), 181–207.
 [BD] Biswas, I. and Dumitrescu, S.: *Branched Holomorphic Cartan Geometries and Calabi–Yau manifolds*, Inter. Math. Res. Not. (to appear), arXiv:1706.04407.
 [BHH] Biswas, I., Heu, V. and Hurtubise, J.: *Isomonodromic deformations of logarithmic connections and stability*, Math. Ann. **366** (2016), 121–140.
 [BSch] Biswas, I. and Schumacher, G.: *Kähler structure on moduli spaces of principal bundles*. Differ. Geom. Appl. **25** 136–146 (2007).
 [Don] Donin, I. F.: *Construction of a versal family of deformations for holomorphic bundles over a compact complex space*, Math. USSR Sb. **23** (1974), 405–416.

- [Dou] Douady, A.: *Le problème des modules pour les sous-espaces analytiques compacts d'un espace analytique donné*, Ann. Inst. Fourier **16** (1966), 1–95.
- [Gr] Grothendieck, A.: *Éléments de géométrie algébrique III, Étude cohomologique des faisceaux cohérents*. Publications Mathématiques, No. **11**, Paris, 1961.
- [Sh] Sharpe, R. W.: *Differential geometry. Cartan's generalization of Klein's Erlangen program*, Graduate Texts in Mathematics, 166. Springer-Verlag, New York, 1997.

SCHOOL OF MATHEMATICS, TATA INSTITUTE OF FUNDAMENTAL RESEARCH, HOMI BHABHA ROAD,
MUMBAI 400005, INDIA

E-mail address: `indranil@math.tifr.res.in`

UNIVERSITÉ CÔTE D'AZUR, CNRS, LJAD, FRANCE

E-mail address: `dumitres@unice.fr`

FACHBEREICH MATHEMATIK UND INFORMATIK, PHILIPPS-UNIVERSITÄT MARBURG, LAHNBERGE,
HANS-MEERWEIN-STRASSE, D-35032 MARBURG, GERMANY

E-mail address: `schumac@mathematik.uni-marburg.de`