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# A Parameterized NMPC Scheme for Embedded Control of Semi-active Suspension System<sup>\*</sup>

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**Abstract:** This paper proposes a parameterized NMPC scheme for control of semi-active suspension system for a quarter car vehicle model. The proposed controller is compared against a linearization based MPC to verify its performance under real-time (RT) embedded conditions. The linearization based MPC method linearizes the non-linear dynamical system/constraints at the current operating point and a linear MPC problem is solved by means of a quadratic program (QP) solver to obtain the optimal control input. The proposed parameterized NMPC method finitely parameterizes the control input and for each parameterized input, the non-linear dynamical system is numerically simulated and the optimal input is elicited from the simulations which minimizes the objective function and satisfies the system constraints. The methods were considered on the basis of RT implementable scenario and practical viability for the target system. The methods were successfully tested by means of hardware in the loop (HIL) simulations under RT conditions for given control objective on dSPACE MicroAutoBox II hardware. The results from HIL simulations exudes better performance of parameterized NMPC against linearization based MPC in terms of computational efficiency and RT feasibility of the controller.

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**Keywords:** Automotive control, Non-linear Model Predictive Control, Real-time Control, Vertical Dynamics

## 1. INTRODUCTION

Automotive suspension system is one of the crucial and an indispensable component of road vehicles and most of the automotive industries in the market are striving towards implementing high performance (comfort) and safety (road holding) oriented suspension system design. The different types of suspension systems depending upon their mode of operation could be broadly classified into a) passive, b) semi-active and c) fully-active and amongst these categories; semi-active suspension system is well pronounced in the automotive market due to its various aspects such as negligible power demand, safety characteristics, significant impact on the vehicle performance and low cost and weight of the system as mentioned in Savaresi et al. (2010). The semi-active suspension could be further classified based upon their technologies such as electrohydraulic (EH), magnetorheological (MR), electrorheological (ER) etc. The working mechanism of the semi-active suspension systems for different technologies are distinct from each

other, however, the underpinning concept of operation is the same for all classes.

One of the key challenges for the semi-active suspension system is the control system design due to the several constraints (both linear and non-linear) possessed by the system. Amongst the other physical constraints, the dissipativity constraint plays a pivotal role due to the inherent passive nature of semi-active suspension system and this property must hold for the entire period of operation. There has been several control design methods developed in the past such as Skyhook proposed in Karnopp et al. (1974), Acceleration Driven Damping (ADD) proposed in Savaresi et al. (2005), Mixed Skyhook-ADD (SH-ADD) proposed in Savaresi and Spelta (2007), LPV/H- $\infty$  based control methods proposed in Do et al. (2010), Sename et al. (2012). Despite the several mentioned control strategies provide good performance, these methods adopt the state and input constraints in an ad-hoc fashion and not completely into control design. In Nguyen et al. (2016b), a robust control approach is applied by taking into account state and input constraints into control design, however, the method has several limitations and conservativeness. This exclusion of fully incorporating the constraints into control design might deteriorate the system performance

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and not fully utilize the potential of the semi-active damper system. To circumvent this problem, model predictive control (MPC) based approach provides an elegant way of tackling the system constraints and objectives in control system design. Another key advantage of MPC based approach is the ability to incorporate future road information (road preview) into control design which could improve the performance of the system in many folds. Over the last decade, there has been several research contributions on MPC based approach for semi-active suspension systems. In Fast MPC based approach proposed in Canale et al. (2006), the optimal control input is computed offline by means of set membership approximation technique. In Hybrid MPC approach, proposed in Giorgetti et al. (2006), the system is modeled as hybrid dynamical system and the optimal control input is computed offline by solving a multi-parametric program for a mixed-integer quadratic program (MIQP). In Cseko et al. (2010), a detailed analysis of explicit MPC for semi-active suspension system is conducted. In Nguyen et al. (2016a), MPC for semi-active suspension system is implemented for full car model, where a MIQP problem is solved online, however, the sampling period is too high for practical implementation. A detailed literature review of different control strategies are presented in Tseng and Hrovat (2015) and Poussot-Vassal et al. (2012).

The main contribution of the paper is the proposed parameterized NMPC method and a comparative study between the linearization based MPC and the proposed approach in terms of computational efficiency and RT feasibility of the controller under different scenarios. The prime motivation for utilizing these methods stems from two fundamental reasons which are a) the methods must be pragmatically doable under RT conditions on the target system and b) the non-linearity in the problem formulation which arises from the dissipativity constraint of the ER damper model (mentioned in Do et al. (2010)) is ought to be included for control input design. Other state of the art NMPC controllers such as sequential quadratic programming (SQP) with real time iteration (RTI) (mentioned in Gros et al. (2016)) are also equally competing methods to address the aforementioned requirements.

This paper is organized as follows. Section 2 discusses about the system dynamics and the nonlinear ER damper model in detail. Section 3 expounds the objective and constraint requirements for the system and NMPC design. Section 4 and Section 5 discusses the linearization based MPC and parametrized NMPC methods in detail. Section 6 discusses the RT implementation. Finally, the paper is concluded with Section 7 dedicated for conclusions and future works.

## 2. MODEL FORMULATION

### 2.1 System description

The vertical dynamics model for a quarter car semi-active damper system consists of two second order differential equations defined for the sprung mass (chassis) and the unsprung mass element (wheel). The dynamics equations around the equilibrium are defined as



Fig. 1. SOBEN Semi-active suspension platform

$$\begin{aligned} m_s \ddot{z}_s &= -k_s(z_s - z_{us}) + F \\ m_{us} \ddot{z}_{us} &= k_s(z_s - z_{us}) - F - k_t(z_{us} - z_r) \end{aligned} \quad (1)$$

where,  $m_s$  and  $m_{us}$  are the sprung mass and unsprung mass respectively,  $k_s$  and  $k_{us}$  are the stiffness coefficient of damper system and the tire respectively,  $z_s$  and  $\dot{z}_s$  are the sprung mass position and velocity respectively,  $z_{us}$  and  $\dot{z}_{us}$  are the unsprung mass position and velocity respectively,  $z_r$  is the vertical road position or the road profile and  $F$  is the force exerted due to the damper system. The differential equation (1) is compactly expressed in state space form as

$$\dot{x}(t) = A_c x(t) + B_c F(t) + B_{cd} d(t) \quad (2)$$

where,  $x = [z_s \ z_{us} \ \dot{z}_s \ \dot{z}_{us}]^T$  are the system states,  $d(t) = z_r$  is the disturbance input from the road profile.  $A_c \in \mathbb{R}^{4 \times 4}$ ,  $B_c \in \mathbb{R}^{4 \times 1}$  and  $B_{cd} \in \mathbb{R}^{4 \times 1}$  are the system matrix, input matrix and disturbance matrix respectively.

### 2.2 Quasi-static nonlinear ER damper model

Despite the force term  $F$  expressed in (2) defines the input force, however, in practice, the actual input to the ER damper sub-system is a PWM signal. The force exerted by the system is modeled in a nonlinear parametric form in terms of the state and input (PWM) variables, this formulation implicitly accounts for the dissipativity constraint for the system. This quasi-static nonlinear damper model is expressed as

$$F(x, u) = f_c u \tanh(a_1 \dot{z}_{def} + a_2 z_{def}) + c_{nom} \dot{z}_{def} + k_d z_{def} \quad (3)$$

Where,  $f_c$ ,  $a_1$ ,  $a_2$  are appropriate parameters,  $z_{def} = z_{us} - z_s$  is the deflection position between the wheel and chassis,  $\dot{z}_{def} = \dot{z}_{us} - \dot{z}_s$  is the deflection velocity between the wheel and chassis,  $u$  is the PWM duty cycle input for the system such that  $u \in [u_{min}, u_{max}]$ , with  $0 \leq u_{min} < u_{max} \leq 1$  and  $c_{nom}$  is the nominal damping coefficient. The numerical values of the vehicle parameters are listed in Table 1 which is used from the INOVE test platform model. The INOVE test platform discussed in Vivas-Lopez et al. (2014) is a 1:5-scaled baja style racing car which consists of 4 controllable ER dampers and 4 DC motors to generate different road profiles for each wheel corner. The INOVE platform is shown in Fig. 1.

It is not uncommon in MPC literature to offset the PWM signal such that the input is framed in a way that  $\tilde{u} \in \mathbb{U}$ , such that  $0 \in \text{int}(\mathbb{U})$ , where  $\tilde{u} = u - u_0$  with  $u_0 = \frac{u_{max} + u_{min}}{2}$  and  $\mathbb{U} = [u_{min} - u_0, u_{max} - u_0]$ . Utilizing the above modified input onto the non-linear damper model (3) and the state space equations (2) yields a control

affine non-linear system with additive disturbance which is compactly expressed as

$$\dot{x}(t) = f(x(t)) + g(x(t))\tilde{u}(t) + B_{cd}d(t) \quad (4)$$

Table 1. Model parameters for NMPC design for quarter car model

Parameter	Symbol	Value (SI unit)
Chassis quarter car mass	$m_s$	2.27(kg)
Unsprung mass	$m_{us}$	0.25(kg)
Suspension stiffness	$k_s$	1396(N/m)
Tyre stiffness	$k_t$	12270(N/m)
Nominal damping coefficient	$c_{nom}$	23(Ns/m)
Stiffness coefficient of damper	$k_d$	186(Ns/m)
Force parameter	$f_c$	42(N)
Deflection position parameter	$a_1$	21(s/m <sup>2</sup> )
Deflection velocity parameter	$a_2$	13(1/m)
Max/Min damper force	$\bar{F}, \underline{F}$	$\pm 20(N)$
Max/Min deflection position	$z_{def}^{max}, z_{def}^{min}$	$\pm 0.025(m)$
Min PWM duty cycle	$d_{min}$	0.1
Max PWM duty cycle	$d_{max}$	0.35
Look ahead period	$T_l$	0.015(s)

### 3. NMPC DESIGN REQUIREMENTS

#### 3.1 Objective requirements

The dichotomy of the objective design for the semi-active suspension system could be both qualitatively and quantitatively classified into a) Comfort objective and b) Road Holding objective as mentioned in Savaresi et al. (2010).

**O.1** Comfort objective ( $J_t^{com}$ ): Qualitatively, the prime goal of the comfort based objective design is to guarantee the comfort for the on-board passengers. The human body is sensitive to certain frequencies and it is of paramount importance to mitigate the effects of vibrations at these spots of the spectrum. Quantitatively, this tantamount to minimizing the vertical acceleration of the chassis ( $\ddot{z}_s$ ), which is obtained from (1). The comfort objective for the given look ahead period  $T_l$  is expressed as

$$J_{T_l}^{com} = \int_0^{T_l} (\ddot{z}_s(t))^2 dt \quad (5)$$

**O.2** Road holding objective ( $J_t^{rh}$ ): Qualitatively, the prime goal of the road holding based objective design is to guarantee that the wheel is always in contact with the road. The requirement of this objective is crucial in control of longitudinal and lateral dynamics of the vehicle. Quantitatively, this objective corresponds to the requirement of minimizing the displacement between the road and the wheel ( $z_{us} - z_r$ ). The road holding objective for the given look ahead period  $T_l$  is expressed as

$$J_{T_l}^{rh} = \int_0^{T_l} (z_{us}(t) - z_r(t))^2 dt \quad (6)$$

Thus, the objective for the semi-active suspension system holistically at the current time instant  $t$  is expressed as

$$J_{T_l}^{obj} = \theta_1 J_{T_l}^{com} + \theta_2 J_{T_l}^{rh} \quad (7)$$

Where,  $\theta_1$  and  $\theta_2$  are the weighting coefficients between comfort and road holding objective and also, the coefficients form a convex combination between the two objectives such that  $\theta_1 + \theta_2 = 1$  and  $\theta_1, \theta_2 \geq 0$ .

#### 3.2 Constraint requirements

The constraints for the semi-active suspension system primarily arises from the physical limitations of the system. These are hard constraints and must be handled systematically to prevent weariness of the system components. For the NMPC design considered, six constraints are included in the problem formulation which are

**C.1** Semi-active ER damper input constraints:

(a) Max/Min damper force constraint: This forms a non-linear mixed state-input constraint such that  $F(x(t), \tilde{u}(t)) \in [\underline{F}, \bar{F}]$ , where  $\underline{F}$  and  $\bar{F}$  are the minimum and maximum saturation forces for the semi-active suspension system.

(b) Modified PWM input constraint:  $\tilde{u}(t) \in \mathbb{U}$ .

**C.2** State limitations constraints: Max/Min deflection between the chassis and wheel position: This forms a linear state constraint such that  $z_s - z_{us} \in [z_{def}^{min}, z_{def}^{max}]$ , where  $z_{def}^{min}, z_{def}^{max}$  are the minimum/maximum deflection position between the chassis and the wheel.

**C.3** Dynamics constraint: The nonlinear equality constraints due to dynamics of the system defined in (4).

**C.4** Road disturbance assumption: Under the consideration sans road preview information, it is not uncommon to presume a constant road disturbance input measured at the current time instant  $t$  (from a disturbance observer) for the entire future horizon for the NMPC problem, i.e.  $d^+ = d(t)$ .

### 4. LINEARIZATION BASED MPC DESIGN

The fundamental assumption for the linearization based MPC design is to linearize the non-linearities present in the system (i.e. constraints and dynamics) by means of first order Taylor series expansion and then, the problem is casted as a linear MPC problem, i.e. a convex QP problem. This procedure is repeated at every operating point and linear MPC is solved at every operating point in receding horizon fashion. The first order linearization of the nonlinear constraint (3) under the modified input at a given operating point  $P_i = (x_i, \tilde{u}_i)$  is expressed as

$$F_{P_i}(x_k, \tilde{u}_k) = F(P_i) + \nabla_{\tilde{u}} F|_{(P_i)} \Delta \tilde{u} + \nabla_x F|_{(P_i)} \Delta x \quad (8)$$

Where,  $\Delta x = x_k - x_i$  and  $\Delta \tilde{u} = \tilde{u}_k - \tilde{u}_i$  are the state and input deviation variables with respect to the operating point  $P_i$ . The first order linearization of the nonlinear dynamics (4) at the operating point  $P_i$  in continuous time is expressed as

$$\Delta \dot{x}(t) = A_c^i \Delta x(t) + B_c^i \Delta \tilde{u}(t) + B_{cd} d(t) \quad (9)$$

Where,  $A_c^i \in \mathbb{R}^{4 \times 4}$  and  $B_c^i \in \mathbb{R}^{4 \times 1}$  are the linearized system and input matrices at  $P_i$ . The obtained continuous time matrices are converted to discrete time matrices by means of zero order hold (ZOH) method with a sample time  $T_s$ . The discrete-time linearized state space equation at the point  $P_i$  is expressed as

$$\Delta x^+ = A_d^i \Delta x(k) + B_d^i \Delta \tilde{u}(k) + B_{dd} d(k) \quad (10)$$

Where,  $A_d^i \in \mathbb{R}^{4 \times 4}$ ,  $B_d^i \in \mathbb{R}^{4 \times 1}$  and  $B_{dd} \in \mathbb{R}^{4 \times 1}$  are the discrete-time system matrix, input matrix and disturbance matrix. The linearized MPC finite time optimal control problem (FTOCP) at the point  $P_i$  with with  $x_0 = x(0)$  and with  $d_0 = d(0)$  is casted as a convex QP problem with

horizon length  $N$  corresponding to the look ahead period  $T_l$  which is described as

$$\begin{aligned}
 J_{P_i}^* &= \min_{\tilde{u}_{0:N-1}, x_{1:N}} \sum_{k=0}^{N-1} J_k^{obj} \\
 \text{s.t.} \quad & z_s - z_{us} \in [z_{def}^{min}, z_{def}^{max}], \forall k = 1 \dots N \\
 & d_{k+1} = d_k, \forall k = 0 \dots N-1 \\
 & F_{P_i}(x_k, \tilde{u}_k) \in [\underline{F}, \overline{F}], \forall k = 0 \dots N-1 \\
 & \tilde{u}_k \in \mathbb{U}, \forall k = 0 \dots N-1 \\
 & (10), \forall k = 0 \dots N-1
 \end{aligned} \tag{11}$$

Where,  $J_{P_i}^*$  is the optimal objective function. The optimal control input at point  $P_i$  to the actual system is  $u^*(0) = \tilde{u}_0^* + u_0$  and this procedure is repeated in receding horizon policy method. For initialization of the input for the next linearization point, the solution of the previous program of (11) is utilized, i.e.  $P_{i+1} = (x_{i+1}, \tilde{u}_1^*)$ , where  $\tilde{u}_1^*$  is the solution at time step 1 at  $P_i$  point. The linearization is performed by precomputing the Jacobians a priori and is evaluated at every time instant.

## 5. PARAMETERIZED NMPC

### 5.1 Proposed approach

The proposed parametrized NMPC approach is based on simulation methods, i.e. an explicit/implicit ODE solver is utilized to simulate the non-linear system (4) to determine the evolution of the states for a set of input sequences over the horizon. The optimal input sequence is elicited from the simulations which minimizes the objective function and satisfies the constraint requirements, which are handled algorithmically. The proposed parametrized NMPC algorithm is sequentially presented as follows

#### Algorithm:

- (1) The input  $\tilde{u}(t)$  of the non-linear system (4) is finitely parameterized in time with  $N_\delta$  equidistant points over the look ahead period  $T_l$  with  $\{\delta_0 \dots \delta_{N_\delta-1}\}$  time stamps with an interval of  $\frac{T_l}{N_\delta}$  and  $T_l = \delta_{N_\delta-1}$  and in space, the set  $\mathbb{U}$  is discretized with  $N_s$  points such that  $\tilde{u} \in \{\phi_1, \dots, \phi_{N_s}\} \subset \mathbb{U}$ , where  $\phi_i$  is a discretization point in  $\mathbb{U}$ . The input sequence over the horizon is compactly represented with  $\mu(\delta_j) \{ \tilde{u}_i(\delta_j) \}_{i=1}^{N_s}, \forall j \in \{0, \dots, N_\delta - 1\}$ , i.e. at a given time instant  $\delta_j$ , there exists  $N_s$  possible input values and this spans for all given time stamps.
- (2) The explicit/implicit solver for the non-linear system in equation (4) is simulated for all input sequences along space and time under the road profile assumption mentioned in section 3.2 - C.4.
- (3) The optimal control sequence is computed with respect to the objective and constraints by plugging the simulated trajectory onto the cost function (7) and the constraint functions mentioned in section 3.2 - C.1, C.2. The constraints are handled algorithmically that if a particular input sequence violates the constraints, then the input sequence is discarded and the solver is proceeded with another control sequence until the minimum cost is obtained.

- (4) In case, if no input sequence satisfies the constraints, then the input sequence which least violates the constraints is considered as the optimal input sequence.
- (5) This procedure is repeated in receding horizon policy method at every sampling period and the optimal control input is  $u^*(0) = \tilde{u}^*(\delta_0) + u_0$ .

For the considered case of quarter car semi-active suspension system,  $N_s$  is assumed as variable (space discretization) and  $N_\delta = 1$  (time discretization) and the solver utilized is a simple fourth order explicit Runge-Kutta (RK) method with fixed integration step  $h = 1 \text{ ms}$ .

### 5.2 Analysis

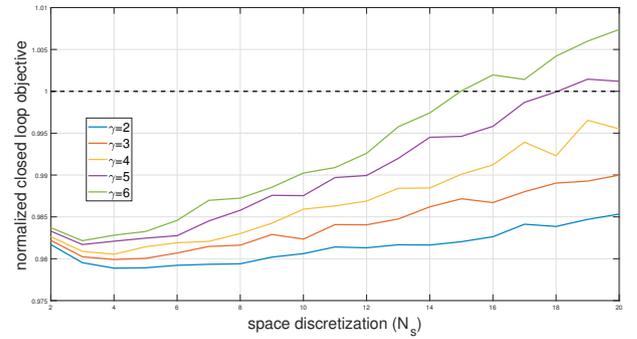


Fig. 2.  $N_s$  vs  $J_{CL}^{norm}$  for different values of computational scale factor  $\gamma$

A detailed analysis is conducted for the proposed parametrized NMPC method for the quarter car semi-active suspension system for different cases. The parameterized NMPC method is simulated in MATLAB/Simulink environment and its closed-loop performance characteristics are investigated for different complexity factors i.e. different space discretization points ( $N_s$ ) and computational time i.e. the control update period ( $\tau_u$ ). The considered acid test is for the following scenario

- A chirp road profile with a frequency sweep between  $0.1 \text{ Hz}$  to  $25 \text{ Hz}$  with an amplitude of  $1 \text{ mm}$  for a duration of  $10 \text{ s}$
- The control objective is selected to be comfort oriented design i.e.  $\theta_1 = 1$
- The control update period is a variable which is expressed with  $\tau_u = \gamma N_s T_s$ , where  $\gamma$  is the computational scale factor and  $T_s = 0.5 \times 10^{-4} \text{ s}$ .

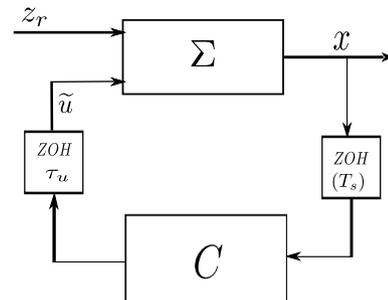


Fig. 3. Control scheme for the proposed analysis

The rationale behind this heuristic and the analysis is to comprehend the behavior of the proposed controller when executed in different computational resources for different

complexities ( $N_s$ ), control update period ( $\tau_u$ ) and also, the analysis provides insight over the optimal complexity factor  $N_s^*$  to be utilized for a given computational resource. Fig. 2 illustrates the normalized closed loop performance of the system for complexity factor ( $N_s$ ) vs normalized closed loop objective ( $J_{CL}^{norm}$ ) for different computational scale factor ( $\gamma$ ). The normalized closed loop objective ( $J_{CL}^{norm}$ ) is defined with respect to the objective of nominal passive suspension system defined as

$$J_{CL}^{norm} = \frac{J_{CL}^{obj}}{J_{CL}^{pass}} \quad (12)$$

where,  $J_{CL}^{pass}$  corresponds to the the closed loop objective for the nominal passive suspension system and  $J_{CL}^{obj}$  corresponds to the closed loop objective of the parameterized NMPC method. The curves in plot Fig. 2 illustrates the fact that the normalized closed loop objective ( $J_{CL}^{norm}$ ) for a given computational scale factor ( $\gamma$ ) declines as the complexity factor gradually increases, however as the complexity factor increases more than a certain threshold, the normalized closed loop objective ( $J_{CL}^{norm}$ ) increases due to the fact that the computational load is elevated and consequently, the control update period ( $\tau_u$ ) is increased, which results in poor performance of the controller. The abscissa of the optimal point for the curves indicates the best/optimal complexity factor  $N_s^*$  for a given computational resource or computational scale factor ( $\gamma$ ).

*Remark:* The proposed parametrized NMPC is of high interest for practical applications for a large set of semi-active dampers. Indeed it is worth noting that  $N_s$  defines the set of damping coefficients than can be used in real-time control. When that  $N_s$  tends to infinity this corresponds to a continuously-variable damper. When  $N_s = 2$  this corresponds to a 2-states damper or to a min-max suspension control approach (as for SkyHook, and ADD and SH-ADD methods). This offers a large flexibility for the implementation of several control methods for different damper types.

## 6. REAL-TIME IMPLEMENTATION

The proposed method and linearization based MPC method are implemented on RT conditions on dSPACE MicroAutoBox II hardware with MATLAB/Simulink interface. HIL tests are conducted for different scenarios and the performance characteristics of the controllers are investigated. The linearization based MPC is implemented using CVXGEN solver by Mattingley and Boyd (2012), in which the optimization problem (11) is programmed and the generated C-code is patched with Simulink using S-function builder block. The parameterized NMPC is programmed using Simulink-MATLAB function block. The generated Simulink files are deployed to the dSPACE hardware and the results were obtained from ControlDesk environment at a sampling period ( $T_s$ ) of 5 ms.

### 6.1 Computational efficiency test

To test the computational efficiency of the two methods, the maximum execution time is recorded for different complexity parameter of the controller. The complexity parameter for the linearization based MPC is selected

to be the number of Newton steps/iterations for the QP solver and for the proposed approach, the number of space discretization points ( $N_s$ ) is considered. The road profile is a chirp signal with amplitude of 1 mm and frequency sweep from 5 Hz to 25 Hz with comfort objective i.e.  $\theta_1 = 1$  (also applies for road holding objective  $\theta_2 = 0$ ).

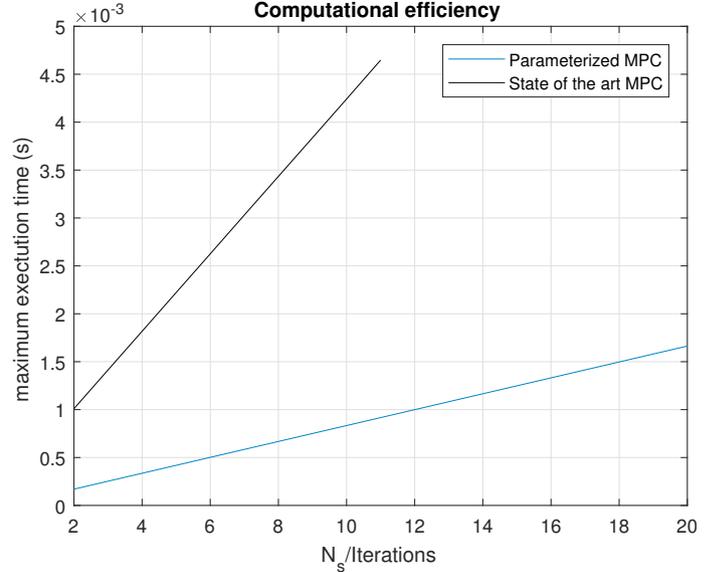


Fig. 4. Computational efficiency

Fig 4 shows a linear relationship between the complexity and the maximum execution time for both controller, however the scale of the proposed controller is roughly three times lesser than the linearization based MPC using CVXGEN. This observation leverages the plausibility to utilize smaller sampling time ( $T_s$ ) and complexity factor  $N_s$  for the proposed method.

### 6.2 Chirp test with comfort objective

The test involves a chirp road profile with amplitude of 1 mm and frequency sweep from 5 Hz to 25 Hz with comfort objective i.e.  $\theta_1 = 1$  (also applies for road holding objective i.e.  $\theta_2 = 0$ ). The complexity factor for linearization based MPC, i.e. number of iterations is 11 and for parameterized NMPC  $N_s$  is 20. The RMS values for the simulations are listed in Table 2.

For the comfort objective, the chassis acceleration is shown in Fig. 5. The dissipativity constraint due to non-linear modeling of ER damper is shown in Fig. 6.

Table 2. RMS values for comfort objective for chirp road profile

Objective	Linearized MPC	Parameterized NMPC
Comfort ( $m/s^2$ )	2.7701	2.2643

The results demonstrate better performance of the proposed approach compared to linearization based MPC for the considered scenario in RT considerations.

## 7. CONCLUSION AND FUTURE WORKS

In this work, a parameterized NMPC method is proposed for a quarter car semi-active suspension system.

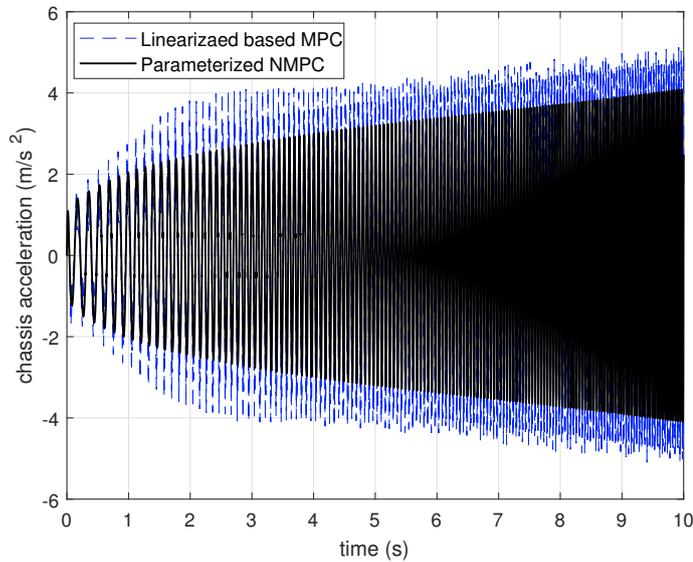


Fig. 5. Chassis acceleration

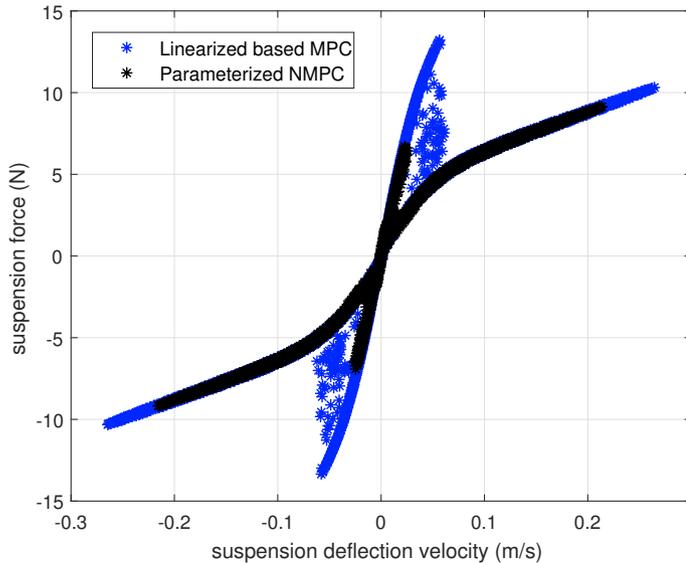


Fig. 6. Damping force vs suspension deflection velocity

RT comparison between the performance of the proposed method and linearization based MPC is conducted and the RT results and simulations exhibits better performance of the proposed method against the linearization based MPC method under different cases. The advantage of the proposed method is its simplicity and efficacy in terms of computational needs, practical viability and scalability. For the future line of work, the method is intended to be augmented to the full car suspension system by means of distributed control methods discussed in Alamir et al. (2017), which will be tested on the INOVE test platform at GIPSA lab, Grenoble.

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