



HAL
open science

Expression for the radius r of an AR(2) process used to model a wireless communication channel

Ali Houssam El Hussein, Laurent Ros, Eric Pierre Simon

► To cite this version:

Ali Houssam El Hussein, Laurent Ros, Eric Pierre Simon. Expression for the radius r of an AR(2) process used to model a wireless communication channel. 2018. hal-01952796

HAL Id: hal-01952796

<https://hal.science/hal-01952796>

Preprint submitted on 12 Dec 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Expression for the radius r of an AR(2) process used to model a wireless communication channel

Ali Houssam EL HUSSEINI*, Laurent ROS†, Eric Pierre SIMON*,

*University of Lille, UMR 8520 - IEMN, F-59655 Villeneuve d'Ascq, France

†Univ. Grenoble Alpes, CNRS Grenoble INP‡, GIPSA-lab 38000, Grenoble, France

‡ Institute of Engineering Univ. Grenoble Alpes

Email: ali.elhousseini@ed.univ-lille1.fr, eric.simon@univ-lille1.fr, laurent.ros@gipsa-lab.grenoble-inp.fr

I. INTRODUCTION

This report applies to the context of the Kalman filter (KF) estimation of a flat fading channel with slow fading; i.e., normalized Doppler frequencies less than 10^{-2} . Note that this context corresponds to many practical applications, including vehicular applications. The principle of the channel estimation is to track the complex baseband equivalent flat fading coefficient, called the channel complex gain (CG), which will be denoted by α .

The design of the KF requires a linear recursive state-space model to be selected for the parameter to be tracked; i.e., α here. In this report, the selected state-space model is the conventional second order autoregressive model AR(2) [1]–[3]. Note that because the true CG does not follow a linear recursive model, the AR(2) model is considered to be an approximation.

The aim of this report is to obtain, in this context, the expression for the pole radius r of an AR(2) as a function of the state noise variance and the resonance frequency of the AR(2).

A. True channel model

We consider the estimation of a flat Rayleigh fading channel. The observation is that ¹

$$y_{(k)} = \alpha_{(k)} + w_{(k)}, \quad (1)$$

where k is the time index, $w_{(k)}$ is a zero-mean additive white circular complex Gaussian noise with variance σ_w^2 , and $\alpha_{(k)}$ is a zero-mean correlated circular complex Gaussian channel gain with variance σ_α^2 . Note that this report is not limited to the conventional Jakes' Doppler spectrum for α [5].

B. Approximated channel model

The second order autoregressive AR(2) process $\tilde{\alpha}_{(k)}$ that is used to approximate the true CG $\alpha_{(k)}$ is:

$$\tilde{\alpha}_{(k)} = a_1 \tilde{\alpha}_{(k-1)} + a_2 \tilde{\alpha}_{(k-2)} + u_{(k)} \quad (2)$$

where $u_{(k)}$ is a white circular complex Gaussian state noise with variance $\sigma_u^2 = R_{\tilde{\alpha}}[0] - a_1 R_{\tilde{\alpha}}[1] - a_2 R_{\tilde{\alpha}}[2]$, with $R_{\tilde{\alpha}}[p]$ the autocorrelation coefficient of $\tilde{\alpha}$ for a lag p . This expression is calculated using Yule-Walker equations [6]. It is classically assumed that $R_{\tilde{\alpha}}[0] = \sigma_\alpha^2$ (i.e., $\tilde{\alpha}$) has the same power as α . These equations also give $R_{\tilde{\alpha}}[1] = \frac{a_1 R_{\tilde{\alpha}}[0]}{1 - a_2}$ and $R_{\tilde{\alpha}}[2] = a_1 R_{\tilde{\alpha}}[1] + a_2 R_{\tilde{\alpha}}[0]$. Using these expressions, we obtain σ_u^2 only as a function of a_1 and a_2 :

$$\sigma_u^2 = \sigma_\alpha^2 \frac{(1 + a_2)(1 - a_1 - a_2)(1 + a_1 - a_2)}{(1 - a_2)} \quad (3)$$

By passing to the z -transformation of the equation (2), the transfer function of the AR(2) model is:

$$H(z) = \frac{1}{1 - a_1 z^{-1} - a_2 z^{-2}} \quad (4)$$

¹ Model (1) assumes that symbols are normalized and known (or decided), in addition to the flat fading assumption. Although this model is admittedly simplistic, it can be applied to different (more involved) contexts, such as pilot-aided multi-carrier systems in frequency-selective wireless channels [4].

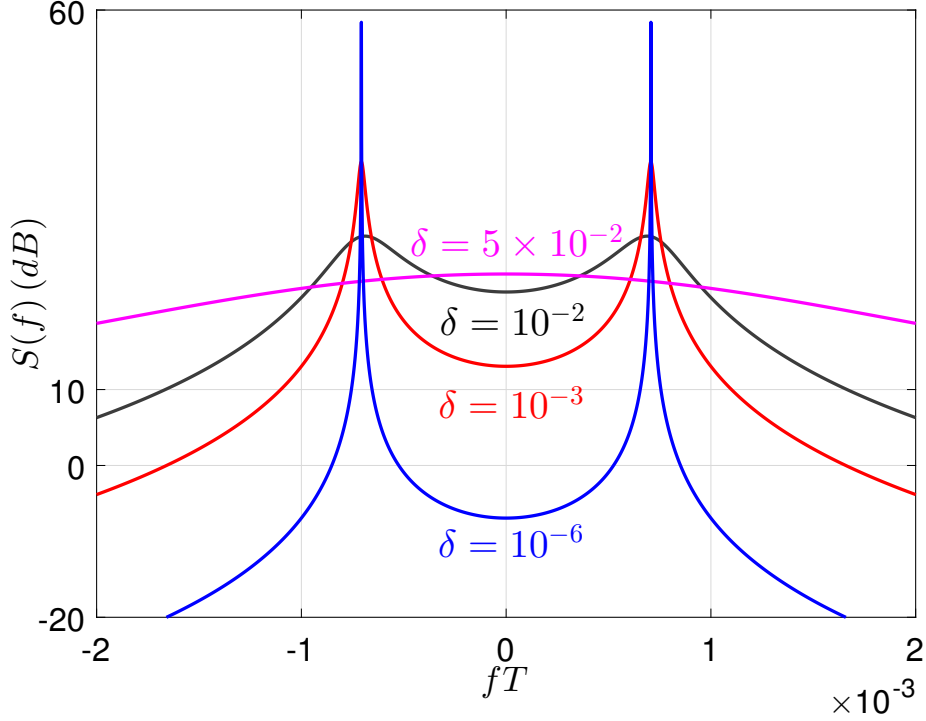


Fig. 1: PSD of the AR(2) model for different values of δ and $f_d T = 10^{-3}$.

This transfer function is also often expressed in the literature with a set of complex conjugate poles in the z -plane at $z_1 = r \cdot e^{-j2\pi f_{AR(2)} T}$ and $z_2 = r \cdot e^{+j2\pi f_{AR(2)} T}$, where $r \in [0, 1[$ is the radius of the poles, and $f_{AR(2)} T \in [0, 1[$ is the normalized resonance frequency of the AR(2) process [7]:

$$\begin{aligned} H(z) &= \frac{1}{(1 - z_1 z^{-1})(1 - z_2 z^{-1})} \\ &= \frac{1}{1 - 2r \cos(2\pi f_{AR(2)} T) z^{-1} + r^2 z^{-2}} \end{aligned} \quad (5)$$

By comparing equations (4) and (5) we have :

$$a_1 = 2r \cos(2\pi f_{AR(2)} T) \quad a_2 = -r^2 \quad (6)$$

In our context of slow fading, $f_{AR(2)} T$ is chosen such that $f_{AR(2)} T \ll 1$.

Let us define $\delta = 1 - r$, which will be used in the following calculations.

An example of the PSDs of the AR(2) process $S(f) = \sigma_u^2 |H(e^{j2\pi f T})|^2$ is given in Fig. 1 for four different values of $\delta \ll 1$. This figure shows that the AR(2) gives a wide variety of PSD shapes, depending on the value of δ , when assuming $\delta \ll 1$ (i.e., r close to one).

In the next section, analytical expression for r is given in terms of σ_u and $f_{AR(2)} T$.

II. APPROXIMATION OF σ_u^2 AND EXPRESSION FOR r

A. Approximation of σ_u^2

First, let us sum up the assumptions corresponding to our slow fading context : $f_{AR(2)} T \ll 1$, with $\delta \ll 1$. In this section, an approximate expression for σ_u^2 is given. Replacing r by $1 - \delta$ in the expression (6) for a_2 yields

$$a_2 = -(1 - \delta)^2 = -(1 - 2\delta + \delta^2) \simeq -(1 - 2\delta). \quad (7)$$

because $\delta^2 \ll \delta \ll 1$. We have

$$1 + a_2 \simeq 2\delta \quad (8)$$

$$1 - a_2 \simeq (1 - \delta). \quad (9)$$

By inserting these equations into the expression for σ_u^2 defined in (3),

$$\begin{aligned}\sigma_u^2 &\simeq \sigma_\alpha^2 \frac{2\delta}{2(1-\delta)} (2(1-\delta) - 2(1-\delta) \cos(2\pi f_{\text{AR}(2)} T)) \\ &\times (2(1-\delta) + 2(1-\delta) \cos(2\pi f_{\text{AR}(2)} T)) \\ &\simeq 4\sigma_\alpha^2 \delta(1-\delta)(1 - \cos(2\pi f_{\text{AR}(2)} T))^2.\end{aligned}\quad (10)$$

However, $\cos(2\pi f_{\text{AR}(2)} T)^2 \simeq \left(1 - \frac{(2\pi f_{\text{AR}(2)} T)^2}{2}\right)^2 \simeq 1 - (2\pi f_{\text{AR}(2)} T)^2$ since $(2\pi f_{\text{AR}(2)} T)^4 \ll (2\pi f_{\text{AR}(2)} T)^2 \ll 1$, so (10) leads to

$$\sigma_u^2 \simeq 4\sigma_\alpha^2 r(1-r)(2\pi f_{\text{AR}(2)} T)^2. \quad (11)$$

B. Expression for r

In this section, the expression for r is given in terms of σ_u^2 and $f_{\text{AR}(2)} T$. Using the expression for σ_u^2 defined in (11):

$$\frac{\sigma_u^2}{4\sigma_\alpha^2 (\omega_{\text{AR}(2)} T)^2} \simeq (1-r)r, \quad (12)$$

where $\omega_{\text{AR}(2)} T = 2\pi f_{\text{AR}(2)} T$.

So r is one solution of the second degree equation

$$r^2 - r + \frac{\sigma_u^2}{4\sigma_\alpha^2 (\omega_{\text{AR}(2)} T)^2} \simeq 0. \quad (13)$$

By solving this equation, we found

$$r \simeq \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{\sigma_u^2}{\sigma_\alpha^2 (\omega_{\text{AR}(2)} T)^2}}. \quad (14)$$

Due to the fact that $\frac{\sigma_u^2}{\sigma_\alpha^2 (\omega_{\text{AR}(2)} T)^2} \simeq 4(1-r)r$ and $r \simeq 1$, we have

$$r \simeq 1 - \frac{\sigma_u^2}{4\sigma_\alpha^2 (\omega_{\text{AR}(2)} T)^2}. \quad (15)$$

REFERENCES

- [1] A. H. El Hussein, E. P. Simon, and L. Ros, "Optimization of the second order autoregressive model AR(2) for Rayleigh-Jakes flat fading channel estimation with Kalman filter," in *Digital Signal Processing*. London, United Kingdom: IEEE, Aug. 2017. [Online]. Available: <https://hal.archives-ouvertes.fr/hal-01576842>
- [2] M. Baruah, A. Misra, and K. K. Sarma, "AR-model of faded MIMO set-up optimized for ITU channels," in *2015 2nd International Conference on Signal Processing and Integrated Networks (SPIN)*, Feb 2015, pp. 242–247.
- [3] L. Lindbom, A. Ahlen, M. Sternad, and M. Falkenstrom, "Tracking of time-varying mobile radio channels. II. A case study," *IEEE Transactions on Communications*, vol. 50, no. 1, pp. 156–167, Jan 2002.
- [4] H. Shu, L. Ros, and E. P. Simon, "Simplified Random-Walk-Model-Based Kalman Filter for Slow to Moderate Fading Channel Estimation in OFDM Systems," *IEEE Trans. Signal Process.*, vol. 62, Aug 2014.
- [5] R. H. Clarke, "A statistical theory of mobile radio reception," *Bell Labs Technical Journal*, (1968).
- [6] G. Eshel, "The yule walker equations for the AR coefficients," *Internet resource*, vol. 2, p. 68-73., 2003.
- [7] L. Lindbom, "Simplified Kalman estimation of fading mobile radio channels: high performance at LMS computational load," in *IEEE International Conference on Acoust. Speech Signal Process.*, vol. 3, April 1993, pp. 352–355 vol.3.