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# Fracture Mechanics and Spall Detachment in Cyclic Hertzian Contacts: Theory and Computer Simulation

M.-C. Dubourg, F. Colin and B. Villechaise

A semi-analytical two-dimensional linear elastic model of multiple bent fatigue cracks is developed for contact loadings, taking into account friction between crack faces [1,2,3]. The model rests on the dislocation theory and on the contact problem solution between crack faces as a unilateral contact with friction. This model is used to predict particle detachment and crack propagation and direction under non proportional mixed mode loading conditions. A typical configuration that may lead to spall detachment is analysed.

## 1. INTRODUCTION

Conditions which lead to particle and spall detachment under contact loadings - fretting or rolling contact - in combination with uniform bulk tension are investigated through fracture mechanics. Spall detachment is analysed as the meeting of fatigue cracks. These cracks are subjected to non proportional (ratios of stress intensity factors  $k_I/k_{II}$  are not constant during the cycle) mixed mode loading, modes I + II in that case. For that type of loading, cracks usually branch, i.e. grow in a different direction from the initial one.

The model of fatigue crack behaviour used in this analysis was presented earlier [1-3]. It includes here crack propagation and crack path criteria. Cracks are modelled by continuous distributions of dislocations following Keer et al [4], Comninou [5], Hills and Comninou [6]. However, a single formulation for stress and displacement fields along the crack is used here [7] for generality. The originality of the model lies in the treatment of the contact problem with friction which uses Kalker's [8] two-body unilateral solution. Thus no limiting assumptions concerning the distribution of stick,

slip and open zones along crack faces - that govern the stress field and thus the SIFs - are needed. Thus bent cracks in any number, length and orientation, whatever the contact conditions along their faces are, can be modelled [2,3,7]. Conditions that lead to their propagation are studied and the direction of propagation is determined.

## Nomenclature

$\mu, \nu$	shear modulus, Poisson's ratio
$k$	$3-4\nu$ or $(3-\nu)/(1+\nu)$ for plane strain or stress respectively
$P(y), Q(y)$	distributed normal and tangential traction per unit length
$a$	half width of the loading zone
$f_s$	tangential to normal load ratio
$\sigma^T, \sigma^{MC}, \sigma^F$	resultant, continuum, crack stress fields
$\delta u_n, \delta u_t$	normal and tangential relative displacements between crack faces
$\delta u_x, \delta u_y$	relative displacements expressed in the reference axis
$P$	number of cracks

$\Gamma_q$	potential displacement zone for crack labelled q
$b_{qx}, b_{qy}$	dislocation distributions along crack q
$n_q, t_q$	local reference axis related to crack q
$\beta_q$	orientation of a straight crack q
$\beta_{qr}$	orientation of the r <sup>th</sup> segment of a bent crack q
$f_q$	coefficient of friction of crack q
$K_{ij}^x, K_{ij}^y$	stress kernels
$K_I, K_{II}$	stress intensity factors (SIF) in mode I and II respectively
$dy_q$	crack q location with respect to the origine

## 2. MODELLING

Theory is recalled briefly, details are given elsewhere [2,3,7].

### 2.1 Boundary conditions and assumptions

The following assumptions are made : quasi-static, two-dimensional, linear elastic conditions prevail, bodies are both homogeneous and isotropic, the cracked region is analysed as a half-plane. This last assumption is justified as the dimensions of the contact zone are small next to those of the bodies in contact. Coulomb friction is assumed between crack faces. Surface breaking or embedded, straight or kinked cracks are considered. They are identified as shown in fig. 1. A normal and tangential pressure distribution is applied over a section of the half-plane edge. They are also related through Coulomb's law. ( $q(y) = fs * p(y)$ ).

Uniform bulk tension  $\sigma$  can be superimposed. During the analysis of a full cycle, crack face friction introduces hysteresis, and an incremental formulation which takes into account the actual loading history must be developed. Boundary conditions for the (i+1) step, over the surface (relations 1 to 4)

and along crack faces (relations 5 to 13), are presented in tables I and II respectively.

### 2.2 Crack model

Stress fields are the sum of the homogeneous half-plane response to both the applied tractions over the surface and the bulk tension, and of the crack response or displacement discontinuities (slip or opening) along crack faces. In the half-plane response, corresponding stress and displacement fields are derived from Airy stress functions associated to concentrated normal and tangential loads. For the crack response, analytical expressions for stress and displacement fields exist in the literature [4-6]. Displacement jumps along crack faces are modelled with continuous distributions of dislocations  $b_{qx}$  and  $b_{qy}$ . A single formulation for stress and displacement expressions for a point  $M_i$  belonging to the crack labelled q are (cf. fig. 2):

$$\sigma_{nqtq}^F(x, y) = \frac{2\mu}{\pi(k+1)} \sum_{q=1}^p \int_{\Gamma_q} \left[ b_{qx}(\xi) K_{nqtq}^x(x, y, \eta, \xi) + b_{qy}(\xi) K_{nqtq}^y(x, y, \eta, \xi) \right] d\xi \quad (14)$$

$$\delta u_{nq}(x, y) = \delta u_y \cos \beta_q - \delta u_x \sin \beta_q \quad (15)$$

$$\delta u_{tq}(x, y) = \delta u_x \cos \beta_q + \delta u_y \sin \beta_q \quad (16)$$

with,

$$\delta u_x(x, y) = \frac{1}{2\pi(k+1)} \int_{\Gamma_q} b_{qx}(\xi) d\xi, \quad (17)$$

$$\delta u_y(x, y) = \frac{1}{2\pi(k+1)} \int_{\Gamma_q} b_{qy}(\xi) d\xi \quad (18)$$

Surface boundary conditions step i+1	
$y1 + \Delta y < y < y2 + \Delta y$	$y > y2 + \Delta y , y < y1 + \Delta y$
<b>(1)</b> $\sigma_{xx}^T(0,y) = - ( p(y) + \Delta p )$ <b>(2)</b> $\sigma_{xy}^T(0,y) = - ( q(y) + \Delta q )$	<b>(3)</b> $\sigma_{xx}^T(0,y) = 0$ <b>(4)</b> $\sigma_{xy}^T(0,y) = 0$

Table I

Interfacial boundary conditions for M(x,y) step i+1			
Open zone	Contact zone		
<b>(5)</b> $\sigma_{nqnq}^T(M) = 0$	<b>(8)</b> $\delta u_{nq}^{i+1}(M) = 0$	<b>(9)</b> $\sigma_{nqnq}^T M \leq 0$	
	<b>(6)</b> $\sigma_{nqtq}^T(M) = 0$	Contact stick zone	Contact slip zone
<b>(7)</b> $\delta u_{nq}^{i+1}(M) > 0$	<b>(10)</b> $\delta u_{tq}^{i+1}(M) = 0$	<b>(12)</b> $ \sigma_{nqtq}^T(M)  = f  \sigma_{nqnq}^T(M) $	
	<b>(11)</b> $ \sigma_{nqtq}^T(M)  < f  \sigma_{nqnq}^T(M) $	<b>(13)</b> $\delta u_{tq}^{i+1}(M) \cdot \sigma_{tq}^T(M) \leq 0$	

Table II

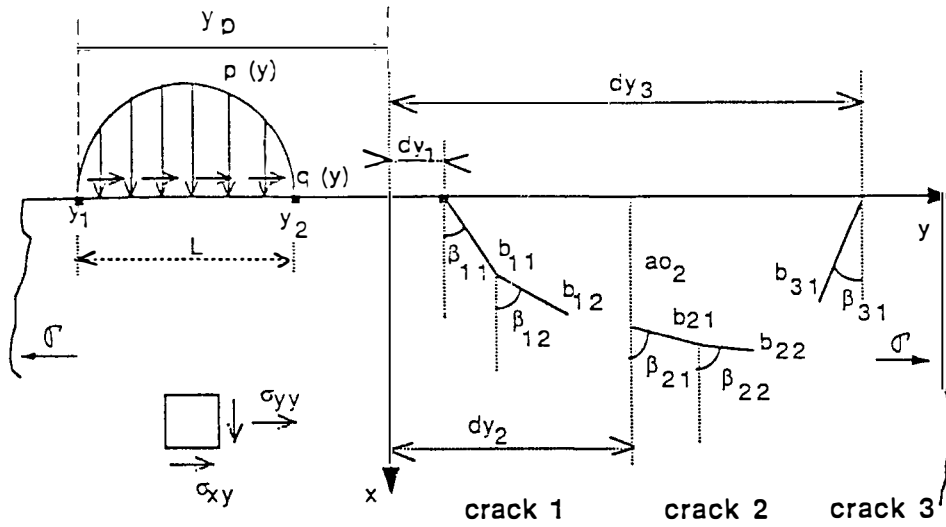


Fig. 1. Multiple bent cracks under loading ( $p = 3$ )

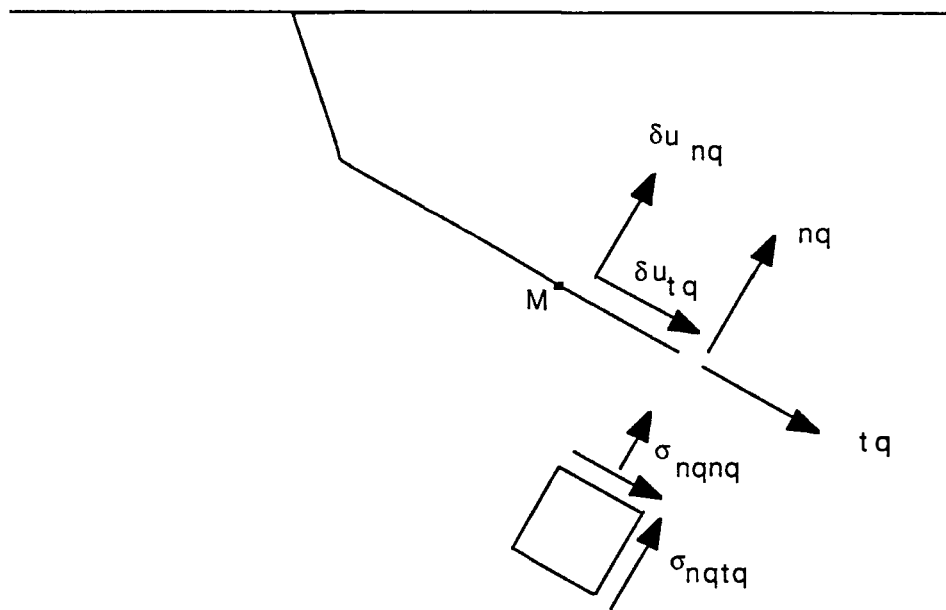


Fig. 2. Notations employed for crack labelled  $q$  ( $1 \leq q \leq p$ )

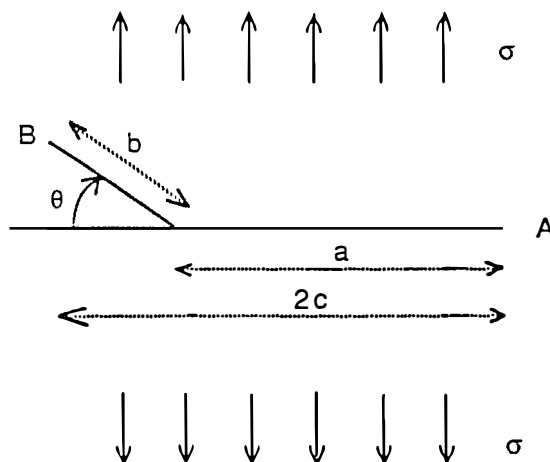


Fig. 3. Bent crack under uniform tension [11]

The expressions of the stress kernels  $K_{ij}^x$ ,  $k_{ij}^y$  are given elsewhere [2].

The displacement field is made unique by imposing a geometrical cut that fits with the crack line (cf. fig. 2). Numerical solutions are employed to solve these singular integrals [9]. Integration points  $r_i$  and collocation points  $s_k$  are chosen [9]. Discretized form of equations (14) to (16) are thus available. SIFs at each crack tip are determined using Krenk's interpolation formulas [10].

The model is adapted to the determination of SIFs at the tip of bent or kinked cracks. The crack profile is described using straight segments. Then a spline interpolation is performed. Dislocations are then distributed continuously along crack faces as mentioned above for straight cracks [7]. Comparisons with exact solutions for straight cracks with branches of various lengths and orientations (cf. fig. 3), [11] indicate that the solution is numerically correct (cf. fig. 4) : 2% maximum differences are observed for kink angles up to  $45^\circ$ . For small branch ( $b/a = 0.05$ ) results are within to 5 % of those given by Abe et al [12] (12 segments are used to describe the crack profile).

### 2.3 The contact problem

The crack contact problem is solved as a unilateral contact problem with friction following Kalker [8]. Difficulties are met because different boundary conditions formulated both in terms of equations and inequalities relate to zones whose dimensions are not known. An iterative process is used. The crack is assumed to be completely closed and adherent at start. Initially [1] the Panagiotopoulos process was employed. It consists in solving iteratively the normal and the tangential problem until convergence occurs. The normal problem is the determination of the contact and open

zones along crack faces ; the tangential problem is concerned with the stick and slip zone determination along the contact zones. A new process, called "kombi", developed by Kalker, which has proved to be completely reliable, is now employed. It is slower than the Panagiotopoulos process but converges when the other one does not.

### 3. **SPALL DETACHMENT AND FRACTURE MECHANICS**

As all elements for fatigue crack behaviour analysis are assembled, it is possible now to study crack propagation and direction and see if spall detachment can be predicted. Criteria for crack propagation and direction are first presented. Then they are incorporated in the fatigue crack behaviour model. Finally an applications is presented.

#### 3.1 Criteria

Criteria advanced for proportional loadings state that the crack bifurcates in a direction where  $K_I^*$  is maximum like for instance the maximum tangential stress criterion proposed by Erdogan and Sih [13], the maximum strain energy density concept criterion [14] or the maximum energy release rate criterion [16]. Cracks subjected to non proportional mixed mode loading behave differently, as the direction in which  $K_I^*$  is maximum is varying during the load cycle. New criteria are thus needed. It has been stated [16] that cracks develop by local mode I. Thus crack growth occurs when  $K_I$  is larger than the threshold value  $K_{Ith}$ , and in either the direction where  $K_I^*$  or  $\Delta K_I^*$  is maximum, or in the direction which gives the maximum crack growth rate. Here  $K_I^*$  is the SIF calculated at the tip of the branch segment. The choice of criteria depends on the material sensitivity to the R ratio ( $R = \sigma_{min} / \sigma_{max}$ ). For materials with low Mode I crack growth sensitivity to R ratio, crack growth path is predicted by the  $\Delta K_I^*$  criterion or by the maximum crack growth rate criterion. For

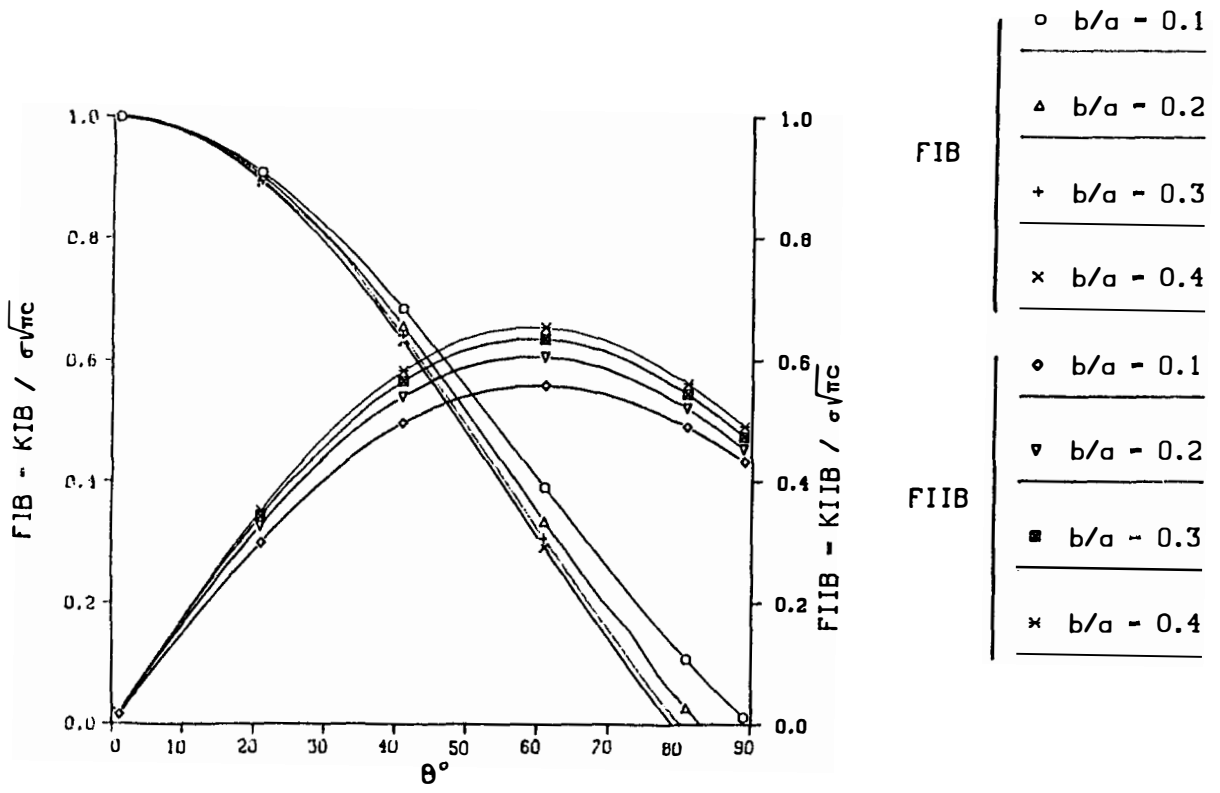


Fig. 4. SIF for a bent crack at tip B under uniform tension

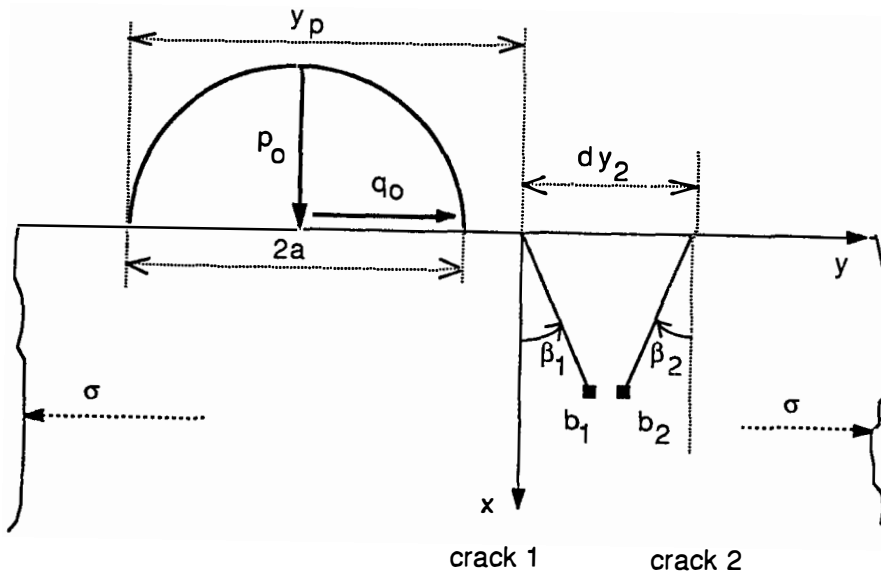


Fig. 5. Two cracks under combined tension and contact loading

material with high sensitivity ratio, the maximum crack growth rate gives good results when compared with experimental observations [16].

### 3.2 Method employed

Multiple bent or straight cracks situated in an half plane, are submitted to a load cycle. As propagation of a crack labelled  $q$  is feasible ( $K_I > K_{Ith}$ ) for a given step  $i$  of the load cycle, this crack will bifurcate in a direction  $\Theta$ . Thus a branched segment of finite length  $b$  ( $b/a = 0.1$ , cf. fig. 3) is considered (it is observed that crack growth direction for cracked specimens is constant versus the branched length [16]) and SIFs are then computed numerically as a function of the orientation  $\Theta$  of the branched segment. As the SIFs at the tip of the branched crack are dependant from relative displacements and stresses calculated along the initial crack profile at the step  $i$ , a redistribution of these data along branched crack is performed in the following way :

- first, integration and collocation points are distributed along the modified crack profile;

- then, stresses, relative displacements and distribution of dislocations are associated to these points in the following way:

- \* along the finite branched : stresses corresponding to the half-plane response at the  $i^{th}$  step are determined for each collocation point ;  $b_{qx}$  and  $b_{qy}$  are set equal to zero and the relative displacements too;

- \* along the initial crack profile : continuum stresses and displacement jumps, distribution of dislocations determined respectively at collocation and integration points for the  $i^{th}$  step are linearly redistributed at new collocation and integration points.

The contact problem is solved and the new distribution dislocation  $b_{qx}$  and  $b_{qy}$  that satisfy the boundary conditions along the modified crack profile are determined ; the SIF  $K_I^*$  is calculated. The growth path is defined by the direction where  $K_I^*$  or  $\Delta K_I^* = K_I^*_{max} - K_I^*_{min}$  is maximum, ( $K_I^*_{max}$  and  $K_I^*_{min}$  being respectively maximum and minimum values greater than the threshold value obtained during the load cycle) depending on the criterion selected. The criterion based on the maximum crack growth rate is not used here due to a lack of experimental data.

### 4. APPLICATION

A moving hertzian load, (width  $2a=440 \mu m$ , maximum pressure  $p_0 = 840 \text{ MPa}$ ), is applied over the surface of a half-plane in combination with a uniform bulk tension  $\sigma$ . This half-plane exhibits two surface breaking cracks. The position of the load is defined with respect of the position of the trailing edge  $yp$  (cf. fig.5). The crack coefficient of friction and mode I threshold are respectively equal to 0.1 and 1 MPavm. 30 discretisation points are taken along crack faces. The data are :

$$\begin{aligned} b_1 &= 110 \mu m & \beta_1 &= 65^\circ \\ b_2 &= 110 \mu m & \beta_2 &= -65^\circ \\ f_s &= -0.05 & \sigma &= 500 \text{ MPa} \\ dy_1 &= 0. & dy_2 &= 220 \mu m \end{aligned}$$

490 steps of displacement are observed between  $yp = -600 \mu m$  and  $yp = 440 \mu m$ . SIFs variations for both cracks versus  $yp$  in mode I and II are presented in figures 6 and 7 respectively.

The variations of SIFs calculated with combined contact and tensile loads are quite different from those found under only one form of loading [2]. In this application, the load travels from left to right. At first, tension dominates the scene: both cracks are completely open.



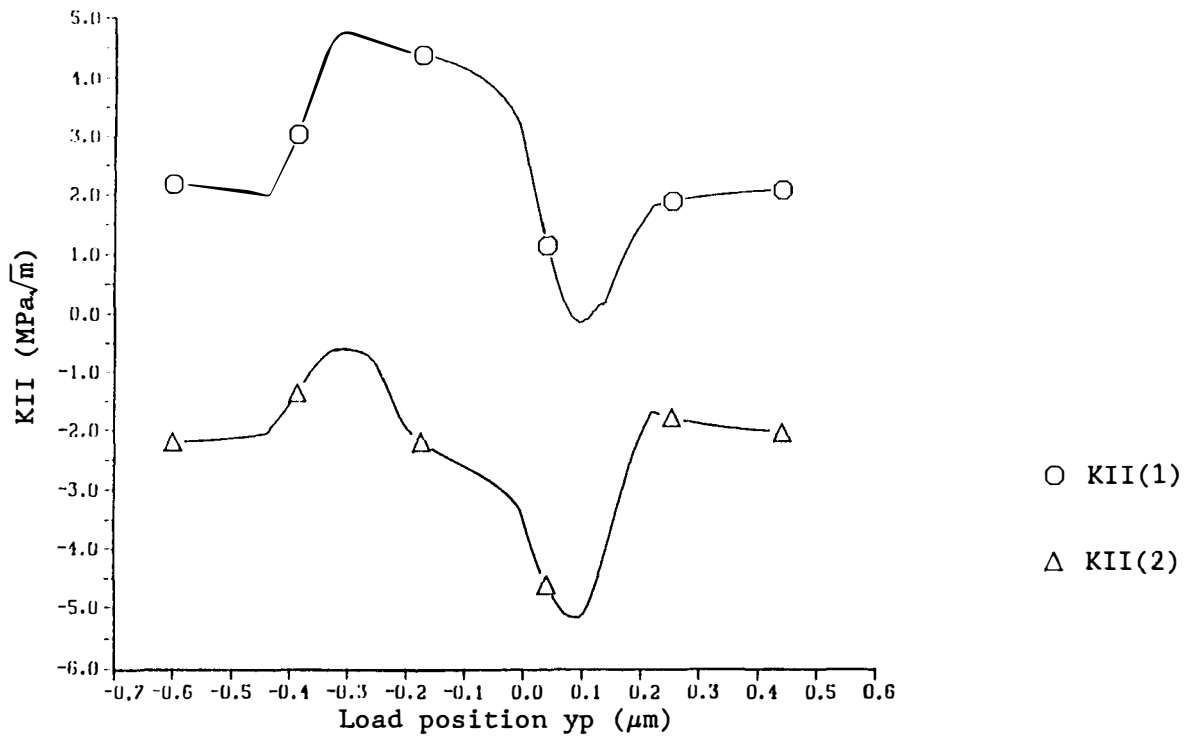


Fig. 6. Variations of  $K_{II}$  for cracks 1 and 2 versus  $y_p$

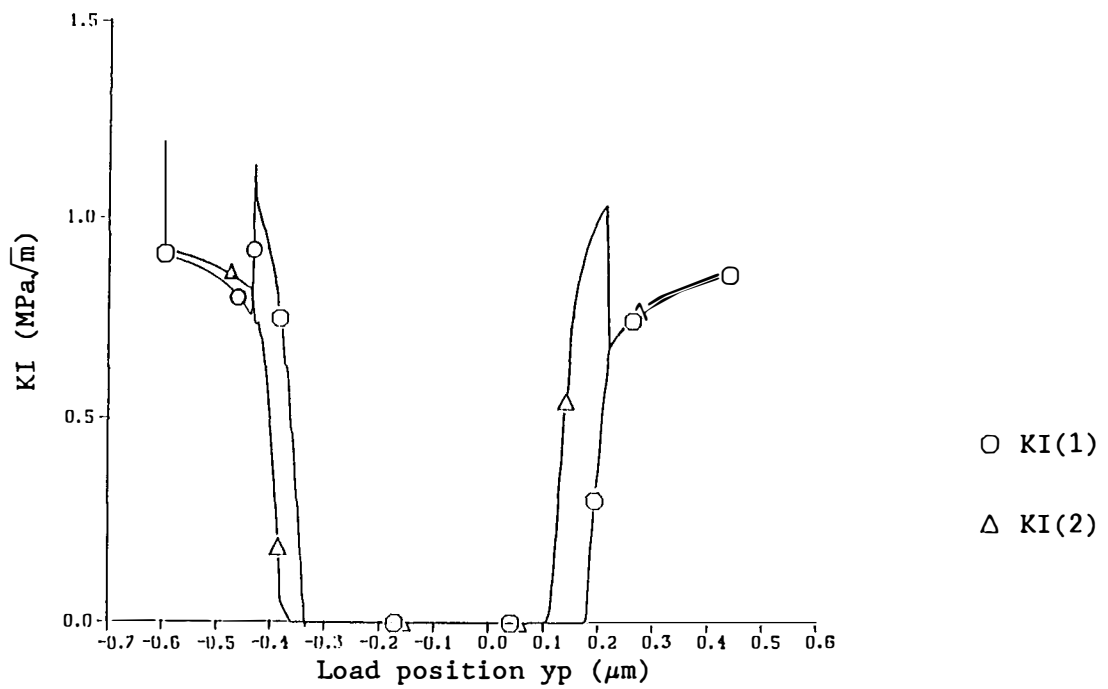


Fig. 7. Variations of  $K_{II}$  for cracks 1 and 2 versus  $y_p$

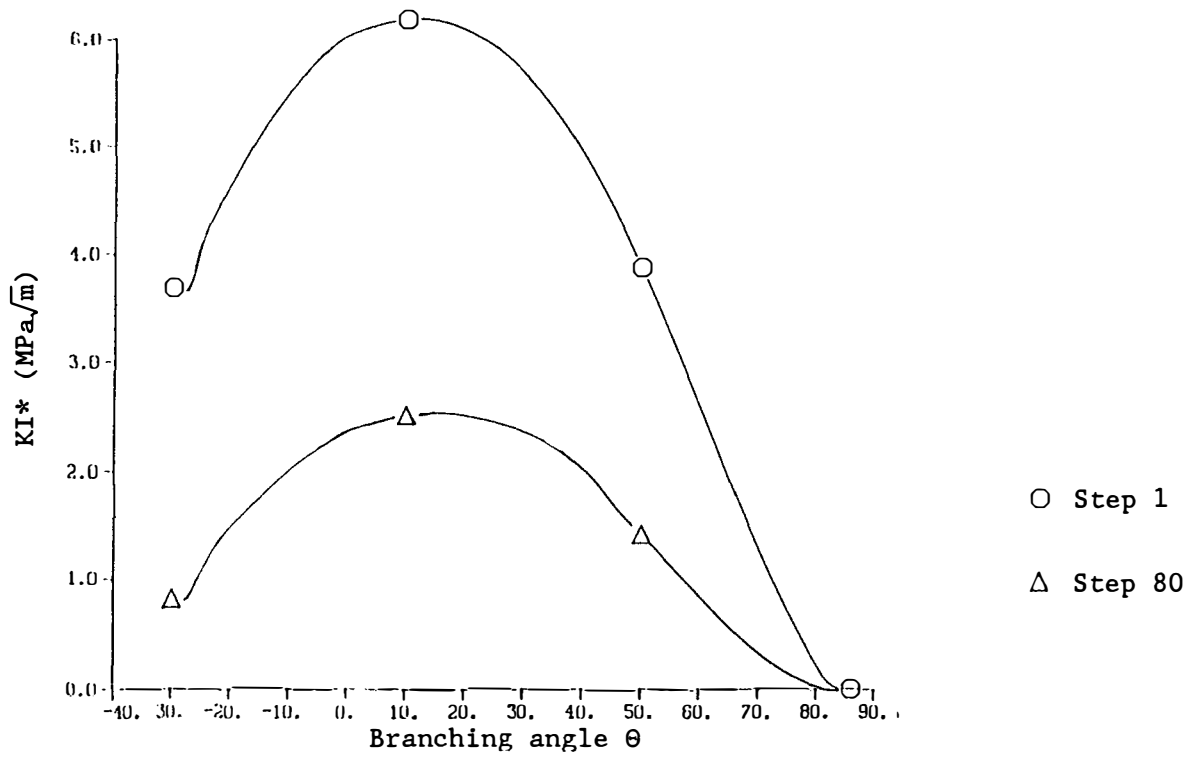


Fig. 8 Variations of  $K_I^*$  versus the orientation of the branched segment for crack 1

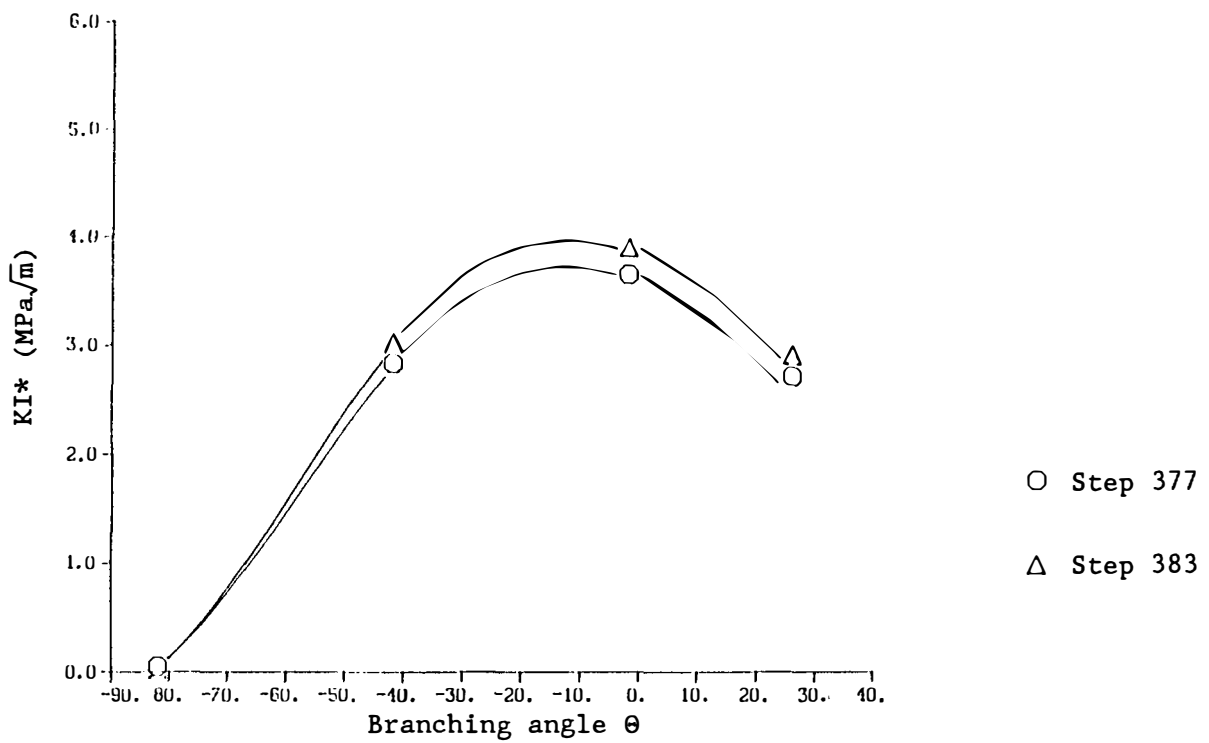


Fig. 9 Variations of  $K_I^*$  versus the orientation of the branched segment for crack 2

As the load approaches the cracks, compression occurs around crack 1. When the leading edge of the loading zone passes over crack 1 ( $y_p = -440\mu\text{m}$ ), crack 1 mouth closes abruptly but its tip opens further:  $K_I$  increases. Then crack 1 closes gradually from the mouth down until its tip closes for  $y_p = -340\mu\text{m}$  and  $K_I$  decreases continuously. Crack 1 opens again completely for  $y_p = 178\mu\text{m}$  when tension dominates again. Note that under contact loading alone,  $K_I$  is different from zero only from  $y_p = -430\mu\text{m}$  to  $-328\mu\text{m}$ .

Crack 2 behaviour is different from crack 1 as its orientation is different (fig.5). Compression starts at the tip of crack 2. Thus its faces close gradually from the tip when  $y_p = -360\mu\text{m}$ .  $K_I$  decreases continuously. Crack 2 faces open again from the tip upwards when  $y_p = 100\mu\text{m}$  ( $K_I$  increases), but are still closed at mouth. Crack 2 mouth only opens when  $y_p = 220\mu\text{m}$ , i.e. when the trailing edge of the loading zone passes over it:  $K_I$  decreases abruptly as tension between crack faces is relaxed. Note that under contact loading alone,  $K_I$  is different from zero from  $y_p = 138\mu\text{m}$  to  $y_p = 216\mu\text{m}$ .

During loading, condition for propagation in mode I is encountered: crack faces is relaxed.

- for steps 1, 80, 81, 82 and 83 for crack 1
- for steps 377 and 383 for crack 2.

Thus variations of  $K_I^*$  versus the branching angle  $\theta$  are presented in:

- figure 8 for crack 1 for steps 1 and 80 only, as loading conditions are quite similar for steps 80 to 83 and lead to similar variations of  $K_I^*$ ;
- figure 9 for crack 2 for steps 377 and 383.

Note that  $K_I^*$  values are much higher for both cracks than  $K_I$  values.

The propagation angle for both cracks derived from the  $K_{I\text{max}}$  or  $\Delta K_{I\text{max}}$  criterion is similar:  $\theta = 10^\circ$  for crack 1 and  $\theta = -10^\circ$  for crack 2. Cracks change direction, becoming nearly perpendicular to the uniform bulk tension, as observed experimentally by Waterhouse [17]. Cracks seem to be beyond the influence of the contact loading but its compressive action combined with the uniform tension causes high values of  $k_I$ , values higher than the ones obtained when tension dominates: for instance, for crack 1, when  $y_p > 178\mu\text{m}$ .

The compressive action of the loading contact has 2 different effects depending on crack depth:

- the crack depth is smaller than the compressive zone size: crack opening is prevented; the crack does not propagate,
- the crack depth is bigger than the compressive zone size: the crack is divided in 2 regions: compressive stresses exerted beneath the surface closes the crack at the mouth; tensile stresses exerted below open the crack at its tip. This configuration is similar to that observed when a fluid is trapped in a crack.  $K_I$  value obtained are higher than those obtained for a crack under tension alone. Here, propagation can occur, cracks bifurcate and grow in a direction nearly perpendicular to the uniform tension.

## 5. CONCLUSION

The identification of the conditions that lead to spall detachment are investigated through fracture mechanics. Conditions which lead to crack propagation and kinking are examined. The crack is assumed to propagate in local mode I when  $K_I$  is larger than the mode I threshold and in the direction where  $K_I^*$  or  $\Delta K_I^*$  is maximum. For the case studied here, 2 cracks under combined contact loading and uniform tension, results agree with practical observations:

cracks change direction and become nearly perpendicular to the uniform bulk tension.

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