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## NORMAL AND TANGENTIAL CONTACT BETWEEN ANISOTROPIC MATERIALS WITH AN ANISOTROPIC COATING

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### ABSTRACT

A contact model using semi analytical methods, relying on elementary analytical solutions, has been developed. It is based on numerical techniques adapted to contact mechanics, with strong potential for inelastic, inhomogeneous or anisotropic problems. Recent developments aim to quantify displacements and stresses of an anisotropic half space with an anisotropic coating which is in contact with a rigid sphere. The influence of symmetry axes on the contact problem solution will be more specifically analyzed.

### INTRODUCTION

Engineering problems are becoming more complicated when trying to reduce the gap between the model and the real application. It means that less restrictive assumptions should be made, or in other words more physics should be implemented in the model. Among the challenges to succeed in it, the material properties should be considered accurately. Supposing the material is isotropic is not enough. For most composite and mono-crystal materials their compositions or the elaboration and manufacturing processes imply that there exist one or two main directions or even a general anisotropy. Moreover, coatings are often used to prevent or control wear. Coatings do not have, generally, the same properties as the substrate and may have various thicknesses. The influence of the anisotropy orientations (in the coating and in the body) have to be taken into account to better predict the distribution of the contact pressure and the subsurface stress-field in order to optimize the service life of industrial components.

Several methods can be used for contact simulation of anisotropic materials. Semi analytical methods have proven their efficiency in contact mechanics and are developed here to account for anisotropy of materials. The main advantage is here the computing time compared to the Finite Element (FE) method that is however widely used for many contact problems. The contact model between an anisotropic material with an anisotropic coating and a rigid sphere will be presented.

### NOMENCLATURE

$E_1$  Young modulus in direction 1  
 $E_3$  Young modulus in direction 3  
 $R^T$  Transpose of the matrix R

$u_k$  Displacement in direction k  
 $u_{k,ij}$  Derivative with respect to i and j of  $u_k$   
 $\theta$  Rotation angle relative to direction 1

### SOLUTION OF THE CONTACT PROBLEM

The semi analytical method consists in the summation of elementary solutions known analytically. One of the difficulties is the derivation or the identification of these elementary analytical solutions, such as the well known Boussinesq and Cerruti solutions in isotropic elasticity [1]. The framework is simplified here by assuming the contact between one anisotropic elastic half space and a rigid body. The contact can be controlled by a prescribed load (which is used here) or by a prescribed displacement. Analytical solutions giving the contributions of normal and tangential loading assumed uniform over a single rectangular element will be used [2,3]. By summation the elastic deflection at each point within and near the contact area will be derived.

The elastic displacements are expressed by a double discrete convolution product between influence coefficients and the pressure or shear at the contact surface. The normal problem and the tangential problem in partial or gross slip are therefore solved.

The resolution is done by minimizing the complementary energy. An algorithm is developed with the conjugated gradient method. To accelerate the calculation, the Fast Fourier Transforms (FFT) are used to perform the double convolution product.

Once the contact problem solved, the strains in the half space are calculated.

### ANISOTROPY OF MATERIAL

Anisotropic materials are defined by the elastic stiffness tensor  $C_{ijkl}$ , which satisfies the full symmetry  $C_{ijkl} = C_{jikl} = C_{klij}$ . These materials can be divided into three parts: cubic, orthotropic and fully anisotropic. Depending on these families of materials, three, nine or twenty one parameters are necessary for defining completely the elastic tensor. Note that in this work the main directions or axes of symmetry are not necessarily the same than the contact ones. In such a situation a reference frame change has to be done. The elastic stiffness tensor links stresses to strains with this following relation  $\sigma_{ij} = C_{ijkl} \epsilon_{kl}$ .

## INFLUENCE COEFFICIENTS

The influence coefficients, which link the load to the displacements or to the strains, for an anisotropic material with an anisotropic coating, are obtained with the Green's functions [2,3]. These functions can be described explicitly in the Fourier domain, whereas it is more complicated to obtain their formulation in the physical domain due to the general anisotropy of the material [4]. The Fourier domain is therefore used.

A concentrated force (normal or tangential) is applied on the surface of an anisotropic elastic half space. In the absence of body forces, the equations of equilibrium in terms of displacements  $u_k$  are written as

$$C_{ijkl}u_{k,lj} = 0 \quad (1)$$

Three matrices  $3 \times 3$  are defined with the tensor  $C_{ijkl}$  and the vectors  $n$  and  $m$ , which form a right handed triad with the position vector  $x$ .  $Q$ ,  $R$  and  $T$  are a double projection of the elastic stiffness tensor.

$$Q_{ik} = C_{ijkl}n_j n_l, R_{ik} = C_{ijkl}n_j m_l, T_{ik} = C_{ijkl}m_j m_l \quad (2)$$

Six distinct eigenvalues  $p$  are obtained by calculating the roots of

$$\det(Q + p(R + R^T) + p^2 T) = 0 \quad (3)$$

The roots are three pairs of complex conjugates. The complex eigenvectors  $a$  of eq. (4) are not a trivial solution. The eigenvectors  $b$  are derived by eq. (5)

$$[Q + p(R + R^T) + p^2 T].a = 0 \quad (4)$$

$$b = (R^T + pT).a \quad (5)$$

The vectors  $a$  and  $b$  are the Stroh eigenvectors and  $p$  the associated Stroh eigenvalues [5]. The Green's functions are then expressed as series forms, by superposing the eigensolutions. Three or four terms are enough to solve this complicated problem (the remaining terms are ignored). The first order is obtained with the boundary conditions, then the successive orders are calculated by a recursive form.

## APPLICATION

The contact between an elastic anisotropic half space and a rigid indenter, with a spherical tip, is studied. The depth corresponds to direction 3, which means that the surface is defined by directions 1 and 2. In these examples, materials are orthotropic, with the same Poisson's ratio and the same Coulomb's modulus. The Hertz pressure is calculated with  $E_{\text{isotropic}}$ .

The influence of  $E_1$  and  $E_3$  on the contact pressure distribution (the depth  $z$  equals 0) is shown in Figs. 1 and 2. It is observed that the Young modulus in a direction parallel to the surface ( $E_1$  here) has a moderate effect (the maximum contact pressure is reduced by 3% only when  $E_1$  is multiplied by a factor 2), whereas the parameter  $E_3$  has a strong influence on the pressure distribution and the contact radius: increase of 32% of the maximum contact pressure and decrease of the

contact radius by 11% when the Young modulus is increased by a factor 2, in agreement with trends by other authors [6,7].

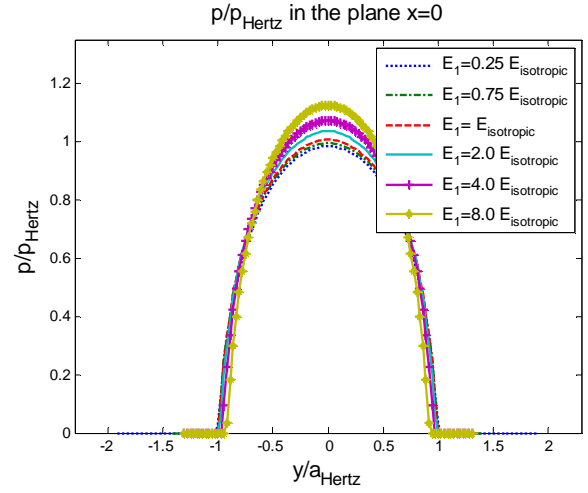


Figure 1: Influence of  $E_1$  on the contact pressure

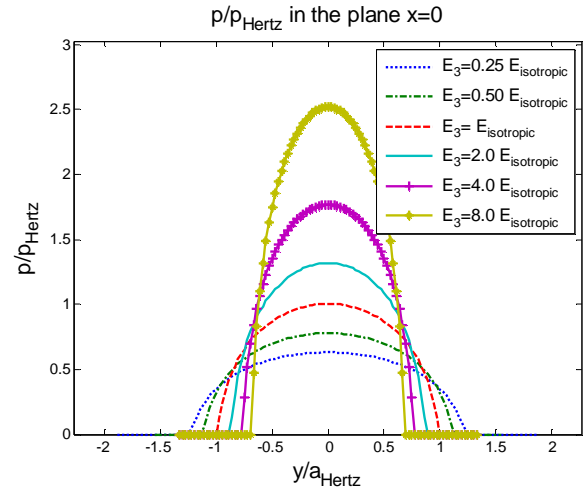
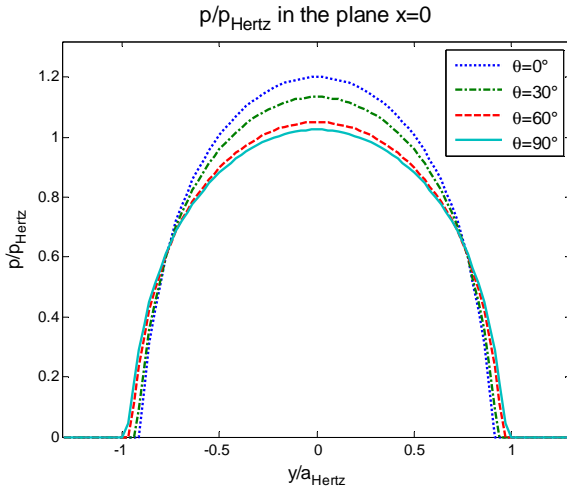


Figure 2: Influence of  $E_3$  on the contact pressure

The effect of the material's orientation relative to the contact is shown in Fig. 3. The material has the properties of an isotropic material except  $E_3 = 1.5 E_{\text{isotropic}}$ , but the material main direction is different than the contact normal. When the orientation angle  $\theta$  around the 1-axis increases up to 90 degrees, the numerical solution converges progressively to the solution where  $E_3 = E_{\text{isotropic}}$  and  $E_2 = 1.5 E_{\text{isotropic}}$  (as shown in Fig. 1).

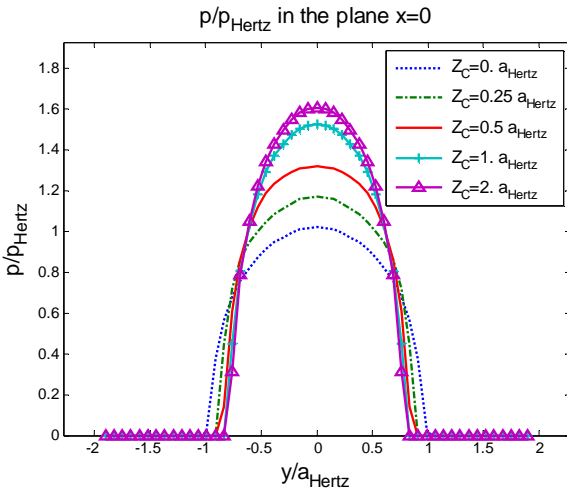


**Figure 3: Influence of the material's orientation on the contact pressure**

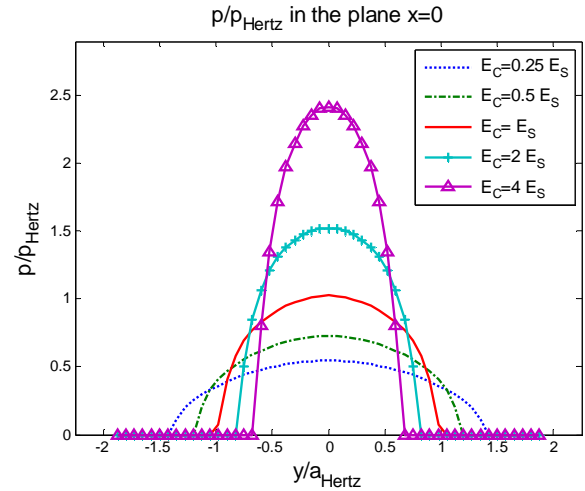
The contact between an elastic anisotropic half space with an anisotropic coating and a rigid indenter, with a spherical tip, is now studied. In these examples, materials are cubic.

The influence of  $Z_C$ , the coating thickness, on the contact pressure distribution is shown in Fig. 4. The Young modulus of the coating,  $E_C$ , is twice higher than the Young modulus of the substrate,  $E_S$ , and the Hertz pressure is obtained when  $Z_C=0$ . The pressure increases with the coating thickness, indeed the coating becomes predominant.

In Fig. 5, the influence of  $E_C$  on the contact pressure distribution is studied. The maximum contact pressure increases by 48% when  $E_C$  is twice higher than  $E_S$  and decreases by 29% when  $E_C$  is smaller than  $E_S$  by a factor 2.



**Figure 4: Influence of  $Z_C$  on the contact pressure**



**Figure 5: Influence of  $E_C$  on the contact pressure**

## CONCLUSION

A semi analytic method has been developed for the contact problem of anisotropic elastic materials. The model has been validated by comparison with the solution for isotropic materials. The advantage of this method is the low computing time compared to the finite element method.

It is found that the stiffness along the normal to the contact has a strong influence on the contact solution in terms of pressure distribution and contact; an increase of  $E_3$  leads to a higher maximum contact pressure and a smaller contact radius. Conversely a change of the Young modulus along a direction parallel to the surface (plane (1,2)) has a moderate effect on the contact pressure distribution (and contact area).

The performance of the method is highlighted by analyzing the effect of the orientation of the material main directions compared to the surface normal.

Moreover, the coating thickness and its Young modulus have a significant influence on the pressure distribution.

In order to optimize the engineering problems, the coating thickness and the properties of materials have to be studied accurately.

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