

Bayesian Nonparametric Mixtures Why and How? Julyan Arbel

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Bayesian Nonparametric Mixtures Why and How?



Introduction

Bayesian nonparametric framework

• Massively many parameters

Inzia

- Inference on curves: pdf, cdf, hazard, link...
- Mixtures, exchangeable data $\mathbf{X}^n = (X_1, \dots, X_n)$

 $X_1, \dots, X_n \mid P \sim \begin{cases} P & \to (1) \\ \int_{\Theta} k(\cdot \mid \theta) P(d\theta) & \to (2) \end{cases}$

What prior for *P*?

- Learn about data through posterior dist.
- Discrete random probability measure prior
- Random weights $(p_i)_i$ and locations $(\theta_i)_i$

3 Survival Analysis

Bayesian hazard mixture (Arbel et al., 2016c) • Data are (remission) times possibly censored • Prior on hazard rate h(t) for every time t • Induces prior on survival function S(t)

Bocconi

- \rightarrow Availability of post. mean, median, mode
- → Smooth estimator VS Kaplan–Meyer

- © Natural uncertainty quantification
- © Flexibility, avoids over-fitting by regularization (prior)
- © Adapt to data complexity
- © Underlying clustering
- Sustify prior, expert
- © Efficient posterior sampling
- © Quantify truncation error



 \rightarrow Dirichlet process DP(α, G_0) (Ferguson, 1973) Predictive: Chinese Restaurant Process

$$\mathbb{P}(X_{n+1} \in \cdot | \mathbf{X}^n) = \frac{\alpha}{\alpha + n} G_0 + \frac{1}{\alpha + n} \sum_{j=1}^{k_n} n_j \delta_{X_j^*}$$

 \rightarrow Or for varying $\mathbb{P}(X_{n+1} \text{ new }) \dots \curvearrowright$





→ Proper uncertainty quantification



Open Questions

• How to best use underlying clustering? (Wade and Ghahramani, 2015)

(1) Species Modeling

Data can be species, microbes, words, genes...

Discovery probabilities (Arbel et al., 2016a)

- Estimation of *l*-discovery $D_{\ell} = \mathbb{P}(X_{n+1} \text{ is a species seen } \ell \text{ times})$
- Comparison with Good-Turing estimator
- \rightarrow Closed form posterior and estimators
- \rightarrow Uncertainty quantif., unavailable for GT
- \rightarrow 2nd order (fast) approximations

Diversity in ecology (Arbel et al., 2015, 2016d) • Assess impact of pollution on microbial community via study of diversity $\text{Div} = -\sum_i p_i \log p_i$

- \rightarrow Model detects an hormetic effect
- → Uncertainty quantification

Density Estimation (2)

Ecological risk assessment (Arbel et al., 2016b)

- Data are species critical effect concentrations (CEC), possibly censored
- Estimation of species sensitivity distribution (SSD), the density of CEC
- Safe concentration which protects most of the species: 5th percentile of the SSD (HC₅)
- Very moderate sample sizes, $\sim 10 50$
- \rightarrow BNP describes well variability of the data, without being prone to over-fitting
- \rightarrow Species clustering as an outcome



- Find consistent estimator of number of clusters: posterior inconsistent (Miller and Harrison, 2014), what about posterior mode?
- Devise efficient **posterior sampling**, truncation error (Arbel and Prünster, 2016)

References & Collaborators

- Arbel, J., Favaro, S., Nipoti, B., and Teh, Y. W. (2016a). Bayesian nonparametric inference for discovery probabilities: credible intervals and large sample asymptotics. Statistica Sinica.
- Arbel, J., Kon Kam King, G., and Prünster, I. (2016b). Bayesian nonparametric modelling of species sampling distributions. *In preparation*.
- Arbel, J., Lijoi, A., and Nipoti, B. (2016c). Full Bayesian inference with hazard mixture models. *Computational* Statistics & Data Analysis.
- Arbel, J., Mengersen, K., Raymond, B., Winsley, T., and King, C. (2015). Application of a Bayesian nonparametric model to derive toxicity estimates based on the response of Antarctic microbial communities to



fuel contaminated soil. *Ecology and Evolution*.

Arbel, J., Mengersen, K., and Rousseau, J. (2016d). Bayesian nonparametric dependent model for partially replicated data: the influence of fuel spills on species diversity. Annals of Applied Statistics.

Arbel, J. and Prünster, I. (2016). A moment-matching Ferguson & Klass algorithm. *Statistics and Computing*.

Ferguson, T. (1973). A Bayesian analysis of some nonparametric problems. *The Annals of Statistics*.

Miller, J. W. and Harrison, M. T. (2014). Inconsistency of Pitman-Yor process mixtures for the number of components. The Journal of Machine Learning Research.

Wade, S. and Ghahramani, Z. (2015). Bayesian cluster analysis: Point estimation and credible balls. *arXiv*.