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Bayesian Nonparametric Mixtures Why and How?

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Introduction

Bayesian nonparametric framework

- Massively many parameters
- Inference on curves: pdf, cdf, hazard, link...
- Mixtures, exchangeable data $\mathbf{X}^n = (X_1, \dots, X_n)$

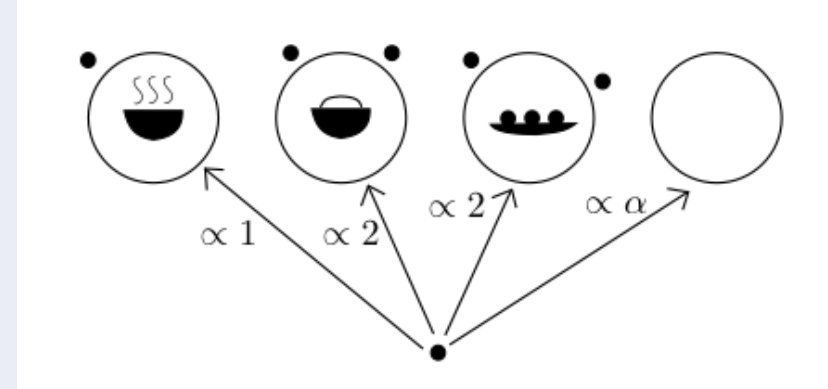
$$X_1, \dots, X_n | P \sim \begin{cases} P & \rightarrow \textcircled{1} \\ \int_{\Theta} k(\cdot | \theta) P(d\theta) & \rightarrow \textcircled{2} \textcircled{3} \end{cases}$$

- Natural uncertainty quantification
- Flexibility, avoids over-fitting by regularization (prior)
- Adapt to data complexity
- Underlying clustering
- Justify prior, expert
- Efficient posterior sampling
- Quantify truncation error

What prior for P ?

- Learn about data through **posterior dist.**
- Discrete **random probability measure** prior
- Random weights $(p_i)_i$ and locations $(\theta_i)_i$

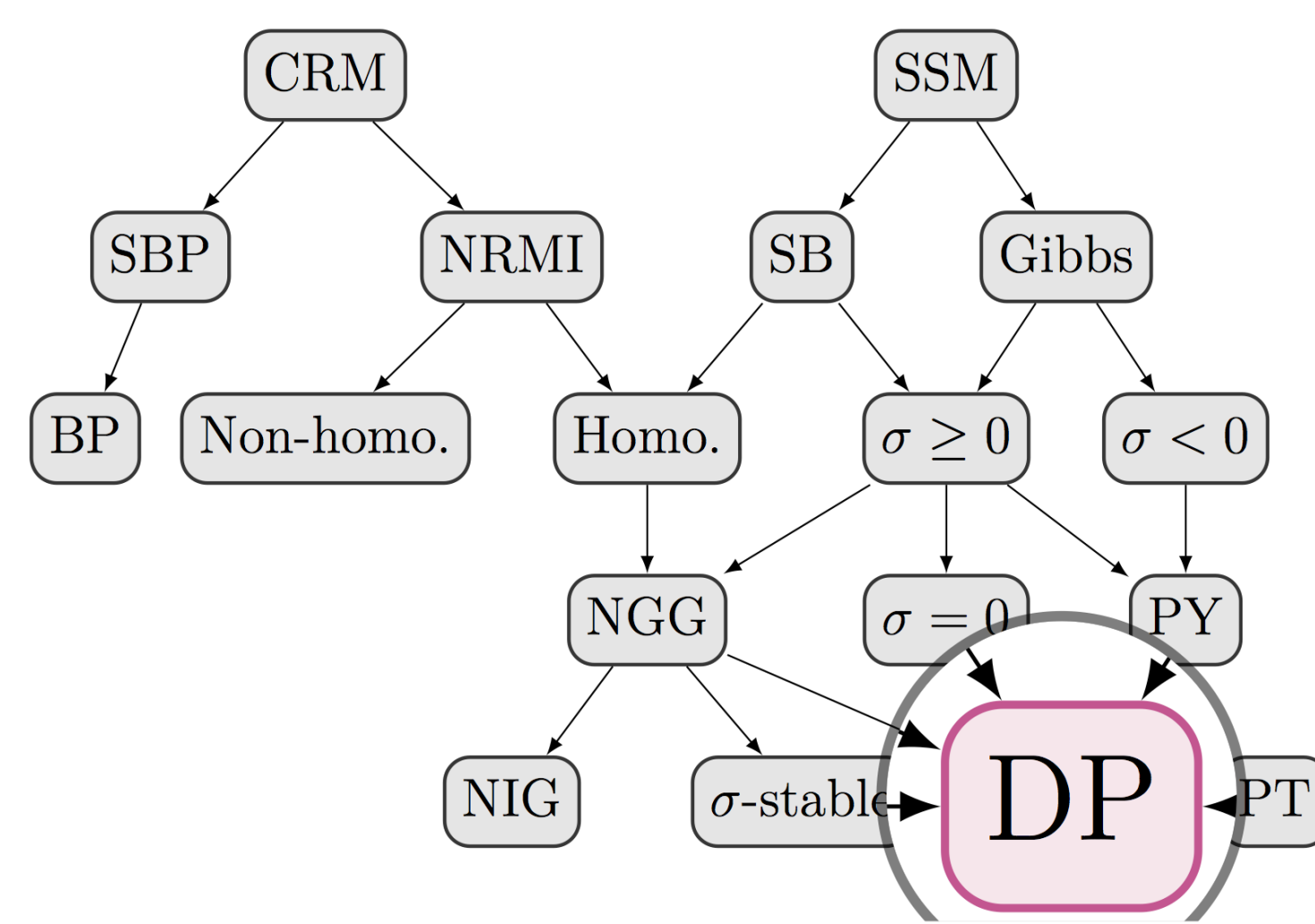
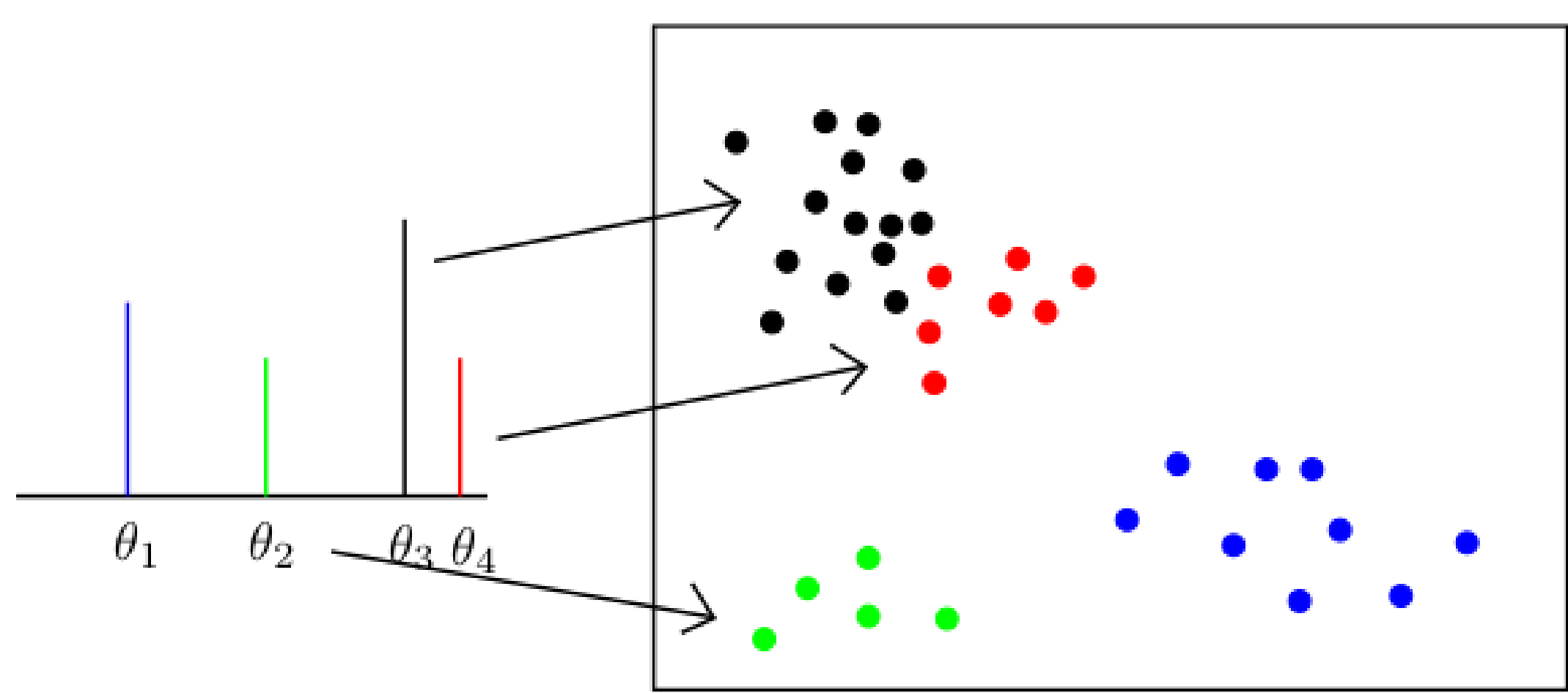
$$P = \sum_{i=1}^{\infty} p_i \delta_{\theta_i}$$



→ **Dirichlet process** $DP(\alpha, G_0)$ (Ferguson, 1973)
Predictive: Chinese Restaurant Process

$$\mathbb{P}(X_{n+1} \in \cdot | \mathbf{X}^n) = \frac{\alpha}{\alpha + n} G_0 + \frac{1}{\alpha + n} \sum_{j=1}^{k_n} n_j \delta_{X_j^*}$$

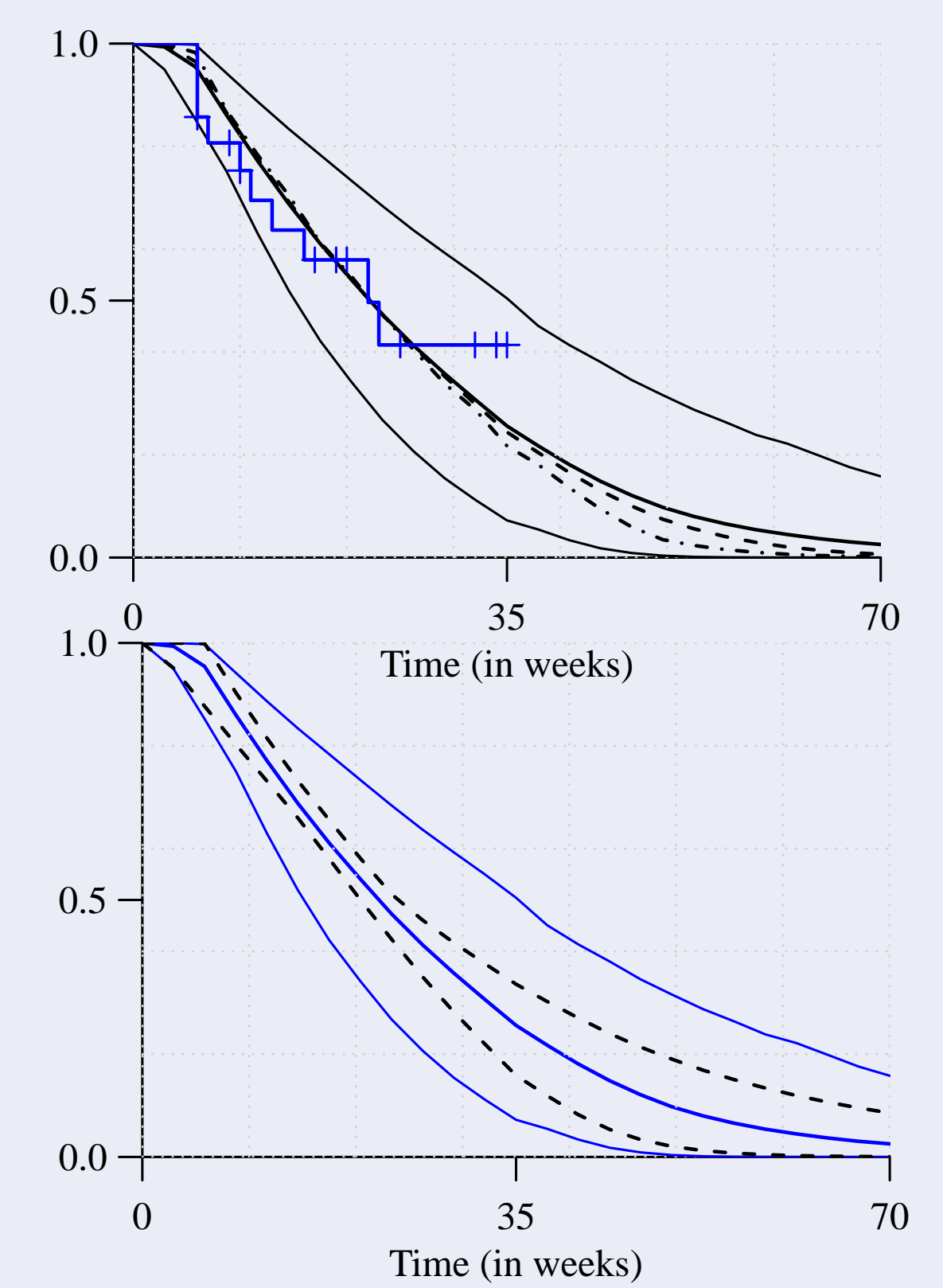
→ Or for varying $\mathbb{P}(X_{n+1} \text{ new}) \dots \curvearrowright$



③ Survival Analysis

Bayesian hazard mixture (Arbel et al., 2016c)

- Data are (remission) times possibly censored
- Prior on **hazard rate** $h(t)$ for every time t
- Induces prior on **survival function** $S(t)$
- Availability of **post. mean, median, mode**
- Smooth estimator VS Kaplan-Meier
- Proper uncertainty quantification



Open Questions

- How to best use underlying **clustering?** (Wade and Ghahramani, 2015)
- Find **consistent** estimator of **number of clusters**: posterior inconsistent (Miller and Harrison, 2014), what about posterior mode?
- Devise efficient **posterior sampling**, truncation error (Arbel and Prünster, 2016)

References & Collaborators

Arbel, J., Favaro, S., Nipoti, B., and Teh, Y. W. (2016a). Bayesian nonparametric inference for discovery probabilities: credible intervals and large sample asymptotics. *Statistica Sinica*.

Arbel, J., Kon Kam King, G., and Prünster, I. (2016b). Bayesian nonparametric modelling of species sampling distributions. *In preparation*.

Arbel, J., Lijoi, A., and Nipoti, B. (2016c). Full Bayesian inference with hazard mixture models. *Computational Statistics & Data Analysis*.

Arbel, J., Mengersen, K., Raymond, B., Winsley, T., and King, C. (2015). Application of a Bayesian nonparametric model to derive toxicity estimates based on the response of Antarctic microbial communities to fuel contaminated soil. *Ecology and Evolution*.

Arbel, J., Mengersen, K., and Rousseau, J. (2016d). Bayesian nonparametric dependent model for partially replicated data: the influence of fuel spills on species diversity. *Annals of Applied Statistics*.

Arbel, J. and Prünster, I. (2016). A moment-matching Ferguson & Klass algorithm. *Statistics and Computing*.

Ferguson, T. (1973). A Bayesian analysis of some nonparametric problems. *The Annals of Statistics*.

Miller, J. W. and Harrison, M. T. (2014). Inconsistency of Pitman-Yor process mixtures for the number of components. *The Journal of Machine Learning Research*.

Wade, S. and Ghahramani, Z. (2015). Bayesian cluster analysis: Point estimation and credible balls. *arXiv*.

① Species Modeling

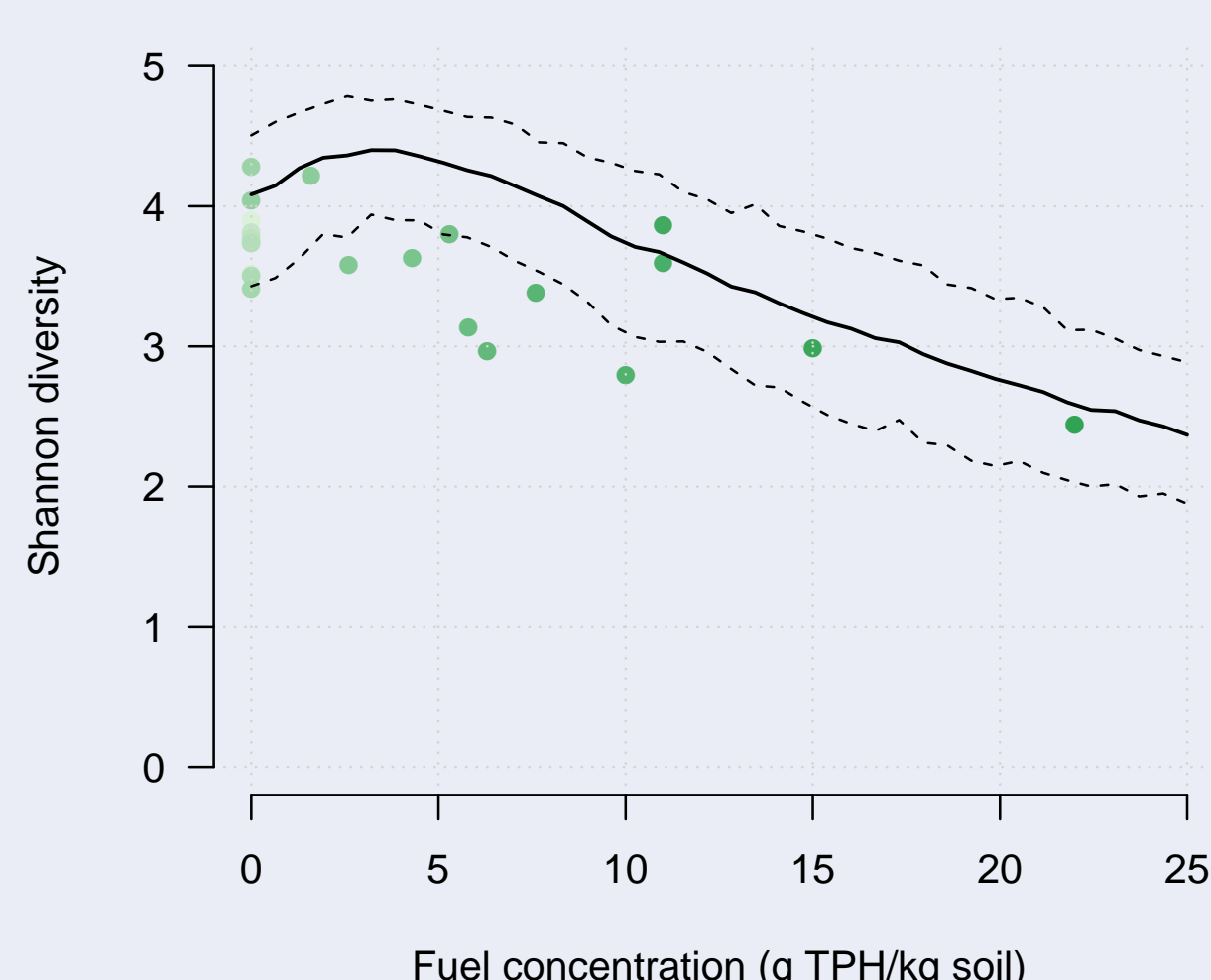
Data can be species, microbes, words, genes...

Discovery probabilities (Arbel et al., 2016a)

- Estimation of **ℓ -discovery**
- $D_\ell = \mathbb{P}(X_{n+1} \text{ is a species seen } \ell \text{ times})$
- Comparison with Good-Turing estimator
- Closed form posterior and estimators
- Uncertainty quantif., unavailable for GT
- 2nd order (fast) approximations

Diversity in ecology (Arbel et al., 2015, 2016d)

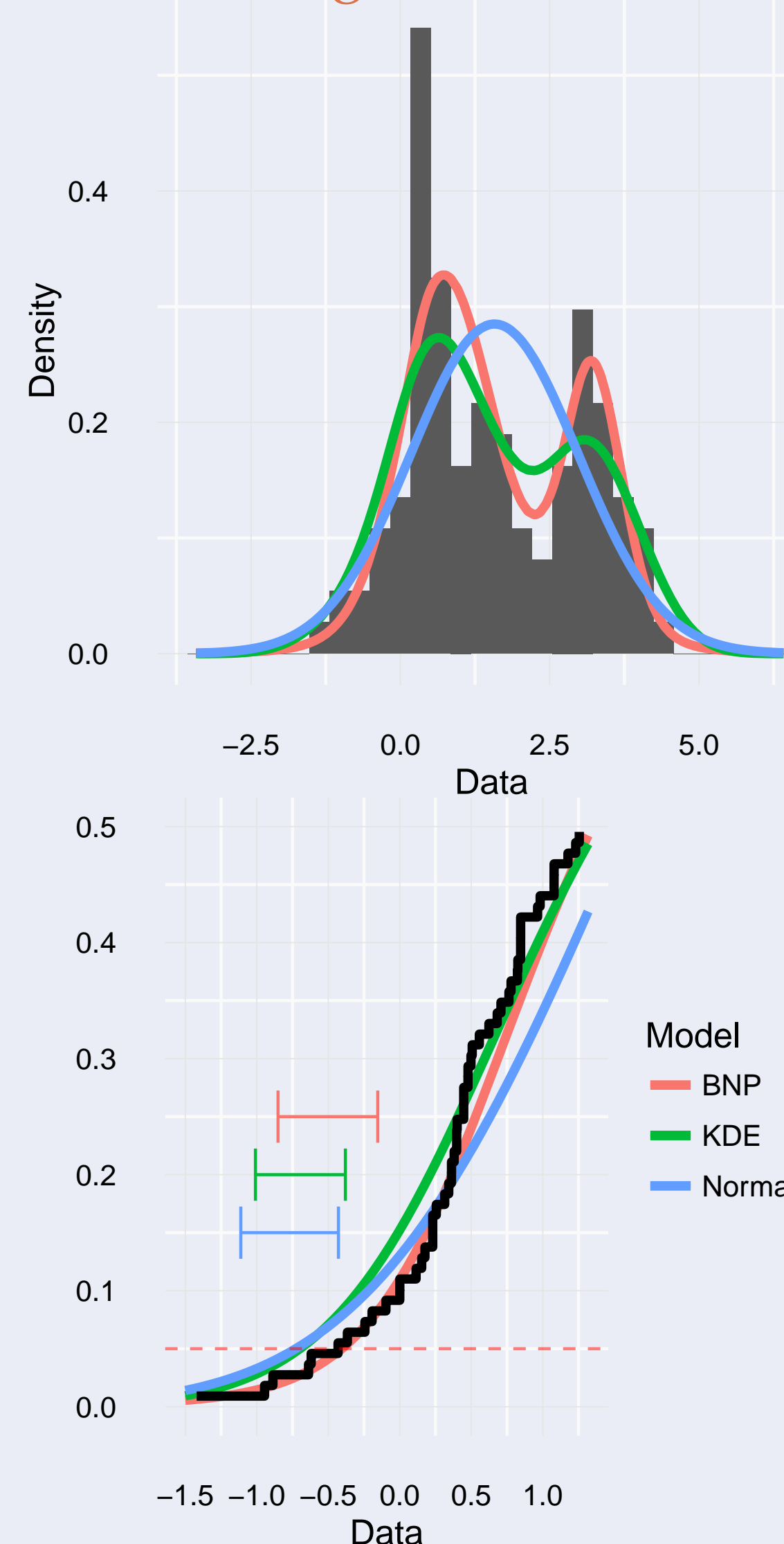
- Assess impact of pollution on microbial community via study of **diversity**
- $Div = -\sum_i p_i \log p_i$
- Model detects an **hormetic effect**
- Uncertainty quantification
- Prediction across full range of covariates



② Density Estimation

Ecological risk assessment (Arbel et al., 2016b)

- Data are species critical effect concentrations (CEC), possibly censored
- Estimation of **species sensitivity distribution (SSD)**, the density of CEC
- Safe concentration** which protects most of the species: **5th percentile of the SSD (HC₅)**
- Very moderate sample sizes, $\sim 10 - 50$
- BNP describes well **variability** of the data, without being prone to over-fitting
- Species **clustering** as an outcome



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