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Continuity of the students' experience and systems of representation, an example of teaching and learning in mathematics first grade students

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Our study is based on a national research project called Arithmetic and Cooperation at Elementary school (ACE). The main objective of this research is the designing of a curriculum for first grade students. This communication focuses on the analysis of an extract from a lesson proposed by this curriculum. During this lesson, the students work on the notion of difference, which is introduced with the help of several systems of representation. These systems are already known by the students. The analysis of the extract shows that the past and future situations can be related to each other. We point out that the students' continuity of experience can produce knowledge growth. This growth specifically occurs when the teacher's uttering activity directs the students' actions while they use the systems of representation. This uttering activity allows a reshaping / remodeling /modification of former knowledge through the systems of representation.

Keywords: Elementary school mathematics, continuity of education, semiotic representation .

This paper focuses on the continuity of epistemic experience in mathematics. We argue that systems of representation constitute a prominent way of achieving such continuity. We sketch the role of a specific system of representation (the number line) to build of the number sense at first grade. This research is based a larger French national research, arithmetic and cooperation at Elementary School (ACE). It offers a complete arithmetic program to 6-7 year-old-students (First grade). The conception of this curriculum relies on available scientific knowledge in different areas (Cognitive Neuroscience, Science of Education, Developmental Psychology and Didactics of Mathematics). A specific part of the conception of the curriculum is built within what we call a cooperative engineering (Sensevy, Forest, Quilio, Morales, 2011; Morales, Sensevy, & Forest, in press). This engineering consists of two spheres. The sphere 1 gathers a multi categorical team (PhD, teachers of study classes, researchers, teacher's trainers, pedagogical advisors) and the sphere 2 is constituted by 120 experimental classes. In the first year of the experiment (2011-2012), the sphere 1 designed the mathematical situations of the curriculum. These situations were implemented in the « study classes », and redesigned on line in the course of the implementation process. The first year of the experiment, the sphere 1 designed eleven modules corresponding to forty-five sessions. The second year of this experiment, this curriculum has been implemented in 60 experimental classes (*versus* 60 control classes) and in 120 experimental classes (*versus* 120 control classes) the third year (2013-2014). The experimental classes' involvement in this curriculum and their feedbacks allowed numerous improvements of the initial design proposed by the research team situations.

This cooperation between researchers and teachers showed a willingness to create a didactic continuity in student's experience through the use of representation of the number systems that are present in the progression throughout the year. We assume that in the joint action between students

and teacher, the systems of representation may authorize the continuity of the student's experience. But, how precisely this can be built in joint action between teacher and students? Here we can rely on Dewey's conception of continuity. "The principle of continuity of experience means that every experience both takes up something from those which have gone before and modifies in some way the quality of those which come after" (Dewey, 1998, p.27).

The Joint Action Theory in Didactics

Our analyses will build on the Joint Action Theory in Didactics (JATD) originated in comparative approach in didactics (Sensevy, 2011; Sensevy, Gruson, & Forest, 2015; Ligozat, 2009; Tiberghien & Malkoun, 2009; Venturini & Amade-Escot, 2013). Among the theoretical tools provided by the "JATD", we use mainly the contract/milieu dialectics and the reticence/ expression dialectics. When facing a new problem, students are confronted to what we call to a *milieu* (Brousseau, 1997; Sensevy & Tiberghien, 2015), as the epistemic structure. This milieu can be seen as the state of problem, what "has to be known" (Sensevy et al, 2015). Students have to face a rather enigmatic set of elements that they have to relate in order to build a system of meanings, in the knowing of what has to be known » (Sensevy, Gruson, & Forest, 2015). For example, for someone who has to do something with a representational system, the milieu is a specific symbolic organization of the system of representations itself. The milieu offered opportunities of enquiry, in which students have to connect elements of knowledge. They deal with this milieu by relying on the knowledge built in the preceding didactic joint action, *the didactic contract*, what "is already known". This relationship between contract and milieu is a dialectal one, because the understanding of a given milieu depends on the nature of the contract that guides the student's efforts.

In order to enable the student to learn, teacher enacts strategies to engage student's action. Interactions between teacher and students are determined by the didactic contract (Brousseau, 1997). In fact, the teacher knows the knowledge that students will have to learn. But he must not tell directly all what he knows. Thus, he has to make choices, in his teaching, about the equilibrium between saying/showing (expression) and remaining tacit/hiding (reticence). This is the reticence-expression dialectics. The two dialectics (contract-milieu and reticence-expression) are entangled, in that expression or reticence can be oriented to "contract " ("what is already-there"), or "milieu " ("what has to be known").

The research on the using of manipulatives and representations focus on the necessity of enabling the students to rely first on manipulative and concrete "objects", then to study iconic (analogical) representations of numbers (Bass, 2015; Schmittau, 2005, Davydov, 1975) then to write down equations in canonical form. This process seems very close to the tradition in Chinese textbooks (Bartolini Bussi et al., 2011; Sun, 2011; Ding & Li, 2014) and can be thought of as "concreteness fading" (McNeil et al., 2012; Fyfe et al., 2014). In this communication, we will try to show how in the new situation in which the notion of the difference (subtraction) is introduced, the systems of representation of number can guarantee a kind of continuity of experience. In fact, the "translational principle for representations systems" in a representational game (Morales, Sensevy & Forest, 2016) can allow students to understand the concept of the difference between two numbers.

Methodology

To discuss these questions, we focus on a specific moment of teacher's practice in a study class, with an experimented teacher, who belongs to the research team (sphere1). The data were collected in december 2013, in a first grade classroom of a French primary school. The twenty five students of this classroom were aged 6-7. This study follows a qualitative approach.

In this extract, the students collectively search the difference between two additive writings with two terms. In this communication, we focus on the introduction of a new piece of knowledge, the notion of difference between two numbers. In the preceding sessions, the students orally compared the production of two hands ads (students showed a number on the two hands, the statement) and a launch of two six-sided dice. The statement wins if it was bigger (in some cases smaller) than the two-dice throw. Then, students compared the two additions with two terms in reference to the situation of the "Statements" (fingers and dice). The result of this comparison was written in the form of a quality or an inequality with the mathematical signs « $=$, \neq , $>$, $<$ ». These two additions were represented in two number lines to solve or prove the comparison, as we can see below (figure 1).

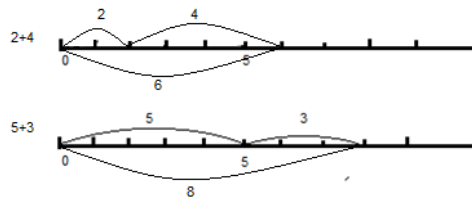


Figure 1: An example of comparison between $2 + 4$ and $5 + 3$ on the number line

The choice of this extract is motivated by the following reason: this extract shows how the continuity of student's experience could be developed through the use the systems of representation. This extract can be considered as a mesoscopic level of the description, the pivotal level (Sensevy et al., 2015), which allows relationship between what preceded and what is going to follow. So, we can analyze the didactic transactions *hic et nunc*. We can characterize and describe the motives and the forms that directed teacher and students 'action. This description can show the teacher's strategies to make the didactic time forward.

Analyze

Presentation

This part of the curriculum "ACE" is organized around a connected series of situations. The initial situation of this curriculum is the situation of the "Statements game" (fingers and dice). One die (marked with standard dot patterns for 1-6) is about to be thrown. Beforehand, the students use their fingers to make a "statement" (for example, a student shows two fingers on her right hand, and three fingers on her left hand). The die is thrown. The students compare their statement with what is indicated by the die. If the sums are equal, the pupils have won. After this oral comparison, students compare an addition (two terms ≤ 5 with a number ≤ 6). The progressive complexification of the situation guides students to increasingly rich comparisons: the number of hands (students) is increased, so the number and the nature of dice (1 to 10 dice are played with), the rules of the game are changed (for example a pupil no longer wins because he has the same number as in the

statement, but because he has a lower or higher number). These connected situations should allow the students to build a real mathematical experience, particularly in the handling of representation and symbolic writing systems, as we will show in what follows. A number line is also introduced on which students represent the numbers. Indeed, the students manipulated a concrete object (the fingers) and they translate fingers by an iconic (analogical) representation of number (the number line) and wrote down equation in canonical form ($2 + 4 = 6$). For example, a student shows two fingers on his left hand and four fingers on his right hand, then she draws these numbers on a number line:



And she writes down this addition: $2 + 4 = 6$

To understand various properties of numbers, students had to compare different representations of the same mathematical reality to become progressively able to recognize the differences and the similarities between these representations.

Since the beginning of the year, students acquired knowledge related to compare numbers. This comparison is performed first orally with the production of «two or three hands ads" and a launch of six-sided die. Then, students compared an addition in two or three terms with a throw of dice. They used the mathematical signs « $=$, \neq , $<$, $>$ » (for example, to compare $3 + 1$ et 5 , students write $3 + 1 < 5$). This situation become more complex when students have to compare two additions in two terms. Finally, students deal with the question of the subtraction on the basis of the comparison between two additions, in the continuity of the previous situations. The study is accompanied by the use of the number line.

The students have built a semiotic knowledge to represent the comparison between two numbers. The number is seen as a measurement. It refers to "the quantity of fingers" in two hands. The number line shows the number like a length measurement. This ancient knowledge is the *didactic contract*, the habits of action with which teacher and students are going to approach the new knowledge, the difference between two numbers. The extract of the session that we chose introduces the difference between two numbers from the comparison of two additions and the terms "larger than and smaller than". The difference is a gap between two numbers, two length measurements. Four episodes will be analyzed. Here is a synoptic view of this analyzer.

Episodes	Content
Episode 1	Presentation of the instructions by the teacher
Episode 2	a) Comparison, looking for the difference between " $1 + 3$ " and " $1 + 1$ ". b) Proposal of two students: (tdp 15) " $1 + 3$ " is larger than " $1 + 1$ ". c) Proposal of another student: the difference between " $1 + 3$ " and " $1 + 1$ " is 3.
Episode 3	a) The difference (tdp 53). b) Introduction of the two hands by the teacher to confirm the difference 3.
Episode 4	Introduction of the number line by the teacher to search the difference between " $1 + 3$ " and " $1 + 1$ " (tdp 68).

Table 1: Extract division

We abstract the four episodes and provide a short analysis.

Search for the difference between two numbers (episode 1, 2 3)

The teacher asks the students to look for the difference between “ $1 + 3$ ” and “ $1 + 1$ ”. He presents the instructions like this:

Teacher: Today, we are going to begin a new game. It is always a game with statements. But today what we are going to make, it is to compare ours statements. We look for which is the larger statement, the smaller statement but that I would know how much more and how much less (...) we are going to find, this calls in fact the difference.

Actually, the students meet difficulties to find the difference between “ $1 + 3$ ” and “ $1 + 1$ ”. They compare the two numbers and look for the larger number or the smaller number with the term-by-term strategy or by computing. They do not focus on “difference”. Confronted to the difficulties of the students, the teacher suggests to illustrate « the two additions “ $1 + 3$ ” and “ $1 + 1$ ” » by a statement with both hands. The following picture (figure two) shows such a statement comparison.



Figure 2: Translation of “ $1 + 3$ ” and “ $1 + 1$ ” by two statements

Unfortunately, this *translation* between the mathematical symbolic writing and the hands statements in a game of representation does not bring the students to produce an adequate answer. “The semiotic habit” of the contract, which considers fingers as instruments to compare numbers, impedes a new designation of numbers, the difference.

Using the number line

Therefore, the teacher introduces in a milieu two number lines on which students have to represent both additive writings. A student writes a first bridge above the first interval of the number line (hence representing the number “1”) and a second bridge above three intervals (between the second and a fourth graduation, hence representing the number “3”). He writes down above these bridges the numbers 1 and 3. Then, he draws below the number line a bridge of four intervals and writes down the sum number (4).

Teacher: It is good the statement makes four as Neil shows us. On the second line, what are you going to draw?

On the second number line, the same student draws two consecutive bridges and a bridge of two intervals, signifying the sum number (2)

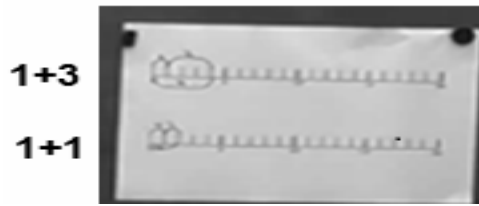
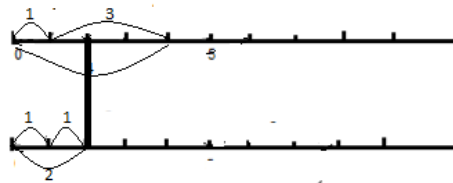


Figure 3: the representation of “1 + 3” and “1 + 1” on the number lines

Teacher: What do you see in the two number lines? Do you see if the 1+3 is largest than 1+1?

The students provide answers different answers: “three”, “four” and “two”. The teacher asks one student to show how he knows that his statement is largest.

Student: because here is the two [He slides his finger from the third graduation (number “2”) on the first number line to the third graduation (number “2”) of the second number line] It’s a part of four...



The student draws a bridge between the second and the fourth graduation on the first number line.

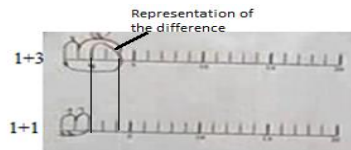


Figure 4: Representation of the difference on the number line

The teacher's expression encourages a translation between the symbolic writing and the number line. But, in the same time, she's reticent because she doesn't say how to draw these additions on the number lines. The teacher thus refers to the preceding contract. She just says: “What do you see in the two number lines? Do you see if the 1+3 is largest than 1+1?” This question can be seen as a “milieu-oriented situation”. The action of the student is moved toward the effective representation of the difference on the two number lines. But, though this expression, the teacher is reticent because she remains silent when the student searches the difference. She indicates where the student must look but the research stays under the responsibility of the student. The number line affords to show concretely the gap between the two numbers, the difference between the two length bounded by the bridges, the sum of the two additions. The students investigate an instrument, the number line, on which they know already how to play (in the situation of comparison). By using this instrument, they achieve not yet explored potentialities of this semiotic system. In particular, the number line shows a number included in another.

Results

The students investigate different systems of representation to find the difference between two numbers: symbolic writing, concrete representation with the hands, number line. All these

representations are known by the students. First, the symbolic writing is translated by statements on the two hands. Then these statements are translated into two number lines in what we may call a translation game, which is a particular representational game (Sensevy, 2011; Morales, Sensevy & Forest, in press). The preceding semiotic knowledge is "re-experienced" by the students. In this way, we can say that the systems of representation are instrumental (Dewey, 1938/1998) in that they allow investigating new knowledge. The semiotic habits are accommodated in a new situation, a new knowledge, allowing the continuity of experience. However, the re-experience of a semiotic system for introducing a new knowledge requires a subtle enunciative work for the teacher, given that the different strategic systems in teacher's action can be a contract oriented or a milieu-oriented transactional activity. In this communication, we have shown how the expression-reticence game of this teacher enable her to introduce in the milieu a system of representation (the number line) already known by the students (contract) in order to understand new properties of this system (then considered as a milieu). This teacher's strategy enables the students to investigate the difference between two numbers while leaving them the responsibility of this enquiry (Dewey, 1938/1998).

Discussion and conclusion

However, these results ask to be worked. It is necessary to explore on a long duration this continuity of the experience of the students in mathematics through the systems of representation (Joffredo-Le Brun, 2016). In particular, it is necessary to note that such continuity can be built only through a real epistemic continuity of the knowledges within the curriculum. The design of such a curriculum has to be performed through the effective experience of the teachers, within an iterative process, as it is the case in the ACE research.

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