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Using a Models and Modeling Perspective (MMP) to frame and combine research, practice- and teachers' professional development

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This paper describes and discusses the framing of, and experiences from, a project that combine research, practice- and teachers' professional development based on the tenets of the Models and Modeling Perspective on teaching and learning (MMP). Besides providing a general description of the methodological considerations in the project design, the paper describes how the accumulated results and experiences in the research literature on so-called model eliciting activities are used to inform the design, implementation and evaluation of activities aiming at introducing functions to grade 8 students. The focus of the paper is on the implantation and aim to showcase how the teacher in question realized the offered perspective and tools in practice.

Keywords: Practice development, teachers' professional development, models and modeling perspective, model eliciting activities.

Introduction

Clarke, Keitel and Shimizu (2006) have shown that much of the teaching and learning of mathematics in many countries are centered around, and dominated by, a traditional use of textbooks. This practice seems to strengthen as the students progress through the educational system, as does the fact that many students lose their interest in, and motivation for, learning mathematics with increasing age. TIMSS 2007 for example shows that the attitudes towards mathematics in Sweden expressed by grade 4 students generally are much more positive compared to the attitudes among grade 8 students (Skolverket, 2008). This situation in combination with the declining results of Swedish students on the international assessments PISA and TIMSS, are reflected and frequent in the public debate as well as in many of the ongoing drives, projects and programmes trying to realize changes in schools. Learning mathematics is complex (Niss, 1999), and now more than ever is the role teachers play stressed for what mathematical understanding and knowledge the students develop in schools (Hattie, 2009; 2012).

However, the challenges that teachers meet in their everyday mathematics teaching are numerous and of various kind and nature. Students' lack of interest in combination with too monotonous (and "traditional") forms of teaching seems to be part of the reasons for the Swedish students' declining performances as well as interest in mathematics. Often, a proposed strategy to reverse these trends is to try to change the prevailing norms in the classroom (Yackel & Cobb, 1996) by increasing student interaction and the overall student activity. Teachers are encouraged to try to vary their teaching, increase students' activity levels and strive to make students 'talk more mathematics'. But how is this going to work in practice in everyday teaching? How do we get students to 'talk more math' and to be more engaged and (inter-)active in the mathematics classrooms?

The concerns mentioned above are part of the motivation for the installment for a joint collaborative initiative between two municipalities and a university, from which this paper will discuss some aspects. The overarching question for the initiative is: *How can we organize the mathematics*

teaching so that students are given the opportunity to develop their conceptual, procedural/methodology and reasoning abilities in, for the students, interesting and engaging ways?

A project combining research, practice- and teachers' professional development

As a response to the situation and challenges briefly outlined above, a collaboration between two municipalities and a university was initiated with the aim to establishing a long term and sustainable collaboration as well as to seek for ways to counterbalance the current trends. The initiative rest one three strands, namely to simultaneously (1) combine and produce research with (2) the development of teaching practices in schools and (3) to serve as professional development for the teachers in the municipalities. The project involves two researchers focusing on different grade levels: grades F to 6 (6- to 12-year-olds) and grades 7 to 12 (13- to 18-year-olds). The two researchers have autonomy in how they define, plan and conduct the work within the given boundaries defined by the university and the municipalities.

The project focusing on grades 7 to 12 runs a series of semi-parallel 1-year projects where the researcher in each project works together with 4-6 teachers from different schools and grades as *partners* (c.f. Jaworski, 1999) in, what ideally could be described as, a *co-learning agreement* (Wagner, 1997). Each project departs from the practices of the participating teachers and the possibilities and challenges they see in their everyday teaching. Based on the teachers' experiences a discussion leads to the formulation, planning and implementation of a 1-year long project with specified aims and goals. The research carried out in the projects is centered around the participating teachers own everyday teaching, and their engagement in research and developing their own practices constitute the professional development for the teachers. Within the context of the initiative two key questions then become: *How to coordinate the experiences and result from the individual projects?* and *How to communicate them?* Note that these questions also are at the heart of mathematics education more generally (aka the accumulation of research and the dissemination of knowledge; a main topic for CERME 10's TWG23). For the 1-year project discussed in this paper, the studied question was: *How can we create and work with joint classroom activities that challenge all students regardless of their levels of mathematical understanding and capabilities?*

The models and modeling perspective on teaching and learning

The *models and modeling perspective on teaching and learning* (Lesh & Doerr, 2003), MMP for short, sometimes given as an example of a so-called *contextual perspective* in the discussion on modeling (Kaiser & Sriraman, 2006), draws on and traces its lineages back to Vygotsky, Piaget and Dienes as well as influences from the American pragmatists' tradition represented by Mead, Peirce and Dewey (Kaiser & Sriraman, 2006; Mousoulides, Sriraman, & Christou, 2007). The central notion in this perspective is that of *models*, which are conceptual systems used to make sense of situations and phenomena. Models are considered to be human constructs which are fundamentally social in nature and can be described as systems consisting of elements, relationships, operations, and rules that can be used to predict, explain or describe the behavior of some other system. In the MMP learning is equated with model development, in which the role of modeling activities is to support this development by engaging the students in purposefully developing, understanding, modifying, and using their models to make sense of different situations and contexts (Lesh & Doerr, 2003).

The adaptation of the MMP at the macro level for all grade 7-12 projects establishes a common perspective and vocabulary that facilitate communication within as well as between different projects and levels of stakeholders in the initiative. The inherent recursive complexity of the MMP (researchers developing models of teachers' models for teaching and supporting students developing their models) connects the work and results from the different projects and levels. The inclusive and accessible notions models and model development (understood in a more mundane way) facilitates communication with teachers, high municipality officials and policymakers.

Model eliciting activities

Model eliciting activities (MEAs) are purposefully designed activities where students need to develop a model that can be used to describe, explain or predict the behavior of, for the students, meaningful contexts, phenomena and situations. Traditionally, much work within the MMP have been centered around so-called *model eliciting activities* (MEAs) developed by Lesh and colleagues (Lesh, Hoover, Hole, Kelly, & Post, 2000). Although originating in mathematics, MEAs have in the last 15 year been used to support and investigate the development of students' models (conceptual systems) in a range of disciplines and contexts (Diefes-Dux, Hjalmarson, Zawojewski, & Bowman, 2006; Iversen & Larson, 2006; Yildirim, Shuman, & Besterfield-Sacre, 2010; Yoon, Dreyfus, & Thomas, 2010).

The research involving MEAs have resulted in six design principles for MEAs, which also to some extent capture the essence of the MMP: (a) *the reality principle* – the MEA connects to students' previous experiences and is meaningful; (b) *the model construction principle* – the MEA induces a need for the students to develop a meaningful model; (c) *the self-evaluation principle* – the MEA permits the students to assess their work and models; (d) *the model documentation principle* – the situation and context in the MEA requires the students to externally express their thinking (models); (e) *the model generalization and sharable principle* – the elicited model in the MEA is sharable, generalizable and applicable to similar situations; and (f) *the simplicity principle* – the situation in, and formulation of, the MEA is as simple as possible (Lesh et al., 2000; Lesh & Doerr, 2003).

Teacher working with MEAs have proven to provide rich opportunities for professional change and development. Schorr and Lesh (2003) found that teachers working with MEAs in their classrooms

- (a) changed their perception regarding the most important behaviors to observe when students engaged in problem activities; (b) changed their views on what they considered to be strengths and weaknesses of student responses; (c) changed their views on how to help students reflect on, and assess their own work; and (d) reconsidered their notions regarding the user of the assessment information gathers from these activities. (Schorr & Lesh, 2003, p. 157)

These experiences and results suggest that MEAs might provide a productive tool to address the question about how to create mathematics teaching that is challenging for all students. The teachers in the project found this a promising approach and especially expressed the following aspects of MEAs appealing: MEAs build on and respect what the students bring to the classroom in terms of prior knowledge in a fundamental way; MEAs focus on the students' sense making of meaningful situations, representations and connections between representations; working with MEAs naturally includes a range of classroom organizations (working one-by-one, in pair, group or different whole class interactions); MEAs have the students work on explicitly formulating and expressing their

thinking using mathematics. In addition, the six design principles for MEAs come to play a few different roles: tools for design; tools for analyzing tasks; evaluative tools for students work in class; and tools for thinking about one's own view of mathematics, teaching and learning.

The MEA and its' implementation

We now end the paper by showcasing the result of one teacher's implementation of an MEA given as an introduction to linear function in grade 8.

The context and the design of the MEA *Candy time!*

The teacher wanted to use an MEA to introduce linear functions in grade 8. Functions, and different representations of functions, are something that the students have been exposed to more or less consciously in different forms during the majority of their mathematical schooling. In other word, students already have ideas and models for what graphs and tables are and how and when to use them. So, rather than systematically treat these concepts in a traditional manner, the teacher wanted to challenge the students to use their previous experiences and to see and explore the connections between tables, graphs and diagrams by engaging in a more exploratory activity (aka an MEA).

The context of the problem was chosen by the teacher to be about buying candy. In Sweden, there is a often practiced tradition, that the children on Saturday do their weekly candy shopping, called *lördagsgodis* – “Saturday's Candy.” Not seldom, the candy is bought in candy stores where you pick 'n' mix candy after your own preferences and liking, and pay by the hectogram (100 grams) or the based on the actual number of pieces of candy you picked.

In the design of the MEA the teacher stressed four of the guiding principles as especially important for this particular purpose: (a) *the reality principle*: the choice of context and situations (the candy store) was made in order to be familiar to the students in that it should facilitate the students in making connections and interpretations between different representations; (b) *the model construction principle*: the intention with the activity *Candy time!* is for the students to build on their previous experiences and knowledge in order to connect and coordinate them further; (d) *the model documentation principle*: to facilitate for the students to document their work, a in that they make their models visible and objects for discussion, a worksheet was developed with easy-to-read instructions, questions and diagram as well as generously with space for writing answers and comments; (e) *the model generalization and sharable principle*: to promote that students share ideas as a means for furthering their models, the MEA was designed to have the students working along as well as in pairs or small groups, and engaged in whole class discussions. Note that the four principles not are independent, and that they contribute to make the students' previous experience and knowledge the basis for the activity, to make students' ideas and thoughts (models) visibility, and to facilitate that the students' models are confronted with other students' models so that they through discussion can refine and develop their ways of thinking.

Implementation

After the teacher started the lesson and introduced the first part of the activity, the students began to work individually on the first part of the task about the three stores A, B and C; see Figure 1 below. However, it only took seconds until the students spontaneously started discussing with each other about which store would given them the most candy for their money, and they spontaneous formed

pairs and small groups. The teacher observed the students' work and listen to the students' discussions while walking around in the classroom and making sure all students understood the task, but otherwise intentionally kept a low profile.

Candy time!

It's Saturday and you're thinking about which of the three stores A, B and C you'll go to and spend 2,50 € so that you'll get as much candy as possible. Compare all three stores and motivate your choice.

Store A

You'll pay 1 € for a bag of 32 pieces of candy.

Store B

Pieces candy (#)	Price (€)
5	1,5
10	
15	

Store C

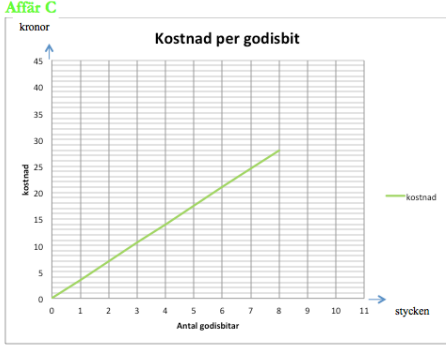


Figure 1: Part one of the activity *Candy time!*

The idea was that the students' should use an experimental approach and try different ways and strategies to approach the problem. If the teacher noticed that some of the student got stuck she approached the student with encouragements like *“Try to fill out the table for Store B!”*, *“What would it look like if one plotted the table-values for Store B in the same diagram displaying Store C's pricing?”*, or *“What would Store A 'look like' in the Store C diagram?”* When the majority of the students had decided in which Store to do their shopping of Saturday's candy, the teacher focused and pulled the class together by asking *“What would the graphs for Store A and Store B look like if you plotted them in the same diagram as the graph for Store C?”*.

When all the students had decided on which store gave them the most value for their money, the teacher, based on her observations in the classroom, chose a few of the students to orally present their solution for the whole class. The selected students showed, motivated and explained what method they used to approach and solve the problem. In the whole class discussion that followed the students' presentations, the teacher, based on continuous inputs of the students, showed what the graphs for the different stores would look like if they were plotted in the same diagram.

The discussion continued in smaller groups where the students were engaged in thinking about and explaining: *What use does one have of graphs and tables? What are the differences and similarities between the three stores? What factors other than the price can affect where one choose to buy one's candy?* Looking at the students' answers, there is a tendency to consider graphs as suitable tools for comparing things (*“when you want to compare something”*, *“you can see the differences in prices”*) or to illustrate how something develops over time (*“when you wanna show something along a timeline”*). Tables on the other hand the students put forward as good tools for presenting different kinds of compiled data or results (*“as for example results from sports”*, *“to present one's findings”*, *“sport results, lengths, weight, sizes, ages, sexes, opinions”*).

Regarding the differences and similarities between the stores the students mostly commented on directly observable features like “*all are selling candy*”, “*the price goes up with the number [of pieces of candy you buy]*”, “*all have different pricing*”. The selection of available candy in the different stores, both with respect to and quality and quantity, as well as to the geographical location of the store, were factors the students identified as things influencing where one buy one’s candy.

After the students had discussed and compared their answers for a couple of minutes, the teacher introduced part two of the *Candy time!* activity; see Figure 2 below:

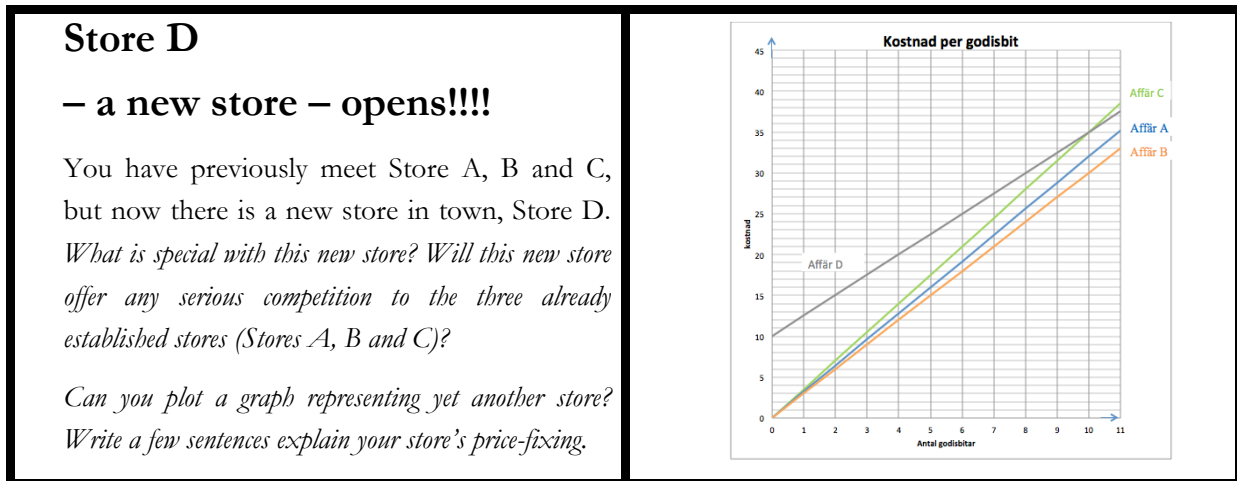


Figure 2: Part two of the activity *Candy time!*

While working on the second question in the second part of the activity, *Will this new store [the Store D] offer any serious competition to the three already established stores (Stores A, B and C)?*, the students concluded “*well, it depends on how much you buy!*”. Many of the students argued that Store D not would be any competition to the other stores if you as in the first part of the activity, only spent 2,50€. However, if you were spending a greater amount of money, then Store D should be the preferable choice. (“*No, this [Store D] is more expensive that the others [Stores A-C]. But this [Store D] becomes more affordable if you buy a larger and larger amount*”). The fact that the graph for Store D intersect the y-axis at $y=10$ some of the students interpreted as “*you have to pay 1€ to enter the store, like an entrance fee*” or that you pay for the box or bag you put the picked candy in: “*Surely it’s some kind of fancy candy store where you have to pay for the boxing. That’s is probably one of the reasoning people will come [and shop in the store] – that it’s a fancy shop that is*”.

The last task in the activity set lose the students’ creativity, drawing graphs describing other imaginary store’s pricing (see Figure 3). Most of the students draw in multiple stores and the most commonly pricing was a model giving the price proportional to the number of pieces of candy bought, as exemplified by Store H: “*Every single piece of candy costs 0,10 € each*”. One of the students wrote “*In this store the only sell giant pieces of candy*” (Store E) to explain the steepness of her graph. Store G was described by another student as “*I’ve made a cheaper store - one where you’ll get one piece of candy for free!*”, explaining the meaning of the graph intersecting the x-axis at $x=1$. Although the diagram only display the price for between zero and 11 pieces of candy, some of the students physically prolonged the lines representing the cost in Stores A – D and concluded that if you buy a large enough amount of candy, then Store D is the most price-worthy store. The

students also constructed stores that had price-fixing represented by a line with negative slope (“*The price decreases, and after 11 pieces the candy becomes free*”, Store I), and stores with a flat rate price-fixing (“*Take as much candy you want for 2,80 € ☺*”, Store F). After the lesson the teacher noticed and expressed her surprised over how much the students own examples of stores’ pricing showed and reveled about the students’ creativity and proficiency to interpret linear functions $y = kx + m$ with positive ($k > 0$), negative ($k < 0$) slope as well as zero slope ($k = 0$) in the given context of the activity.

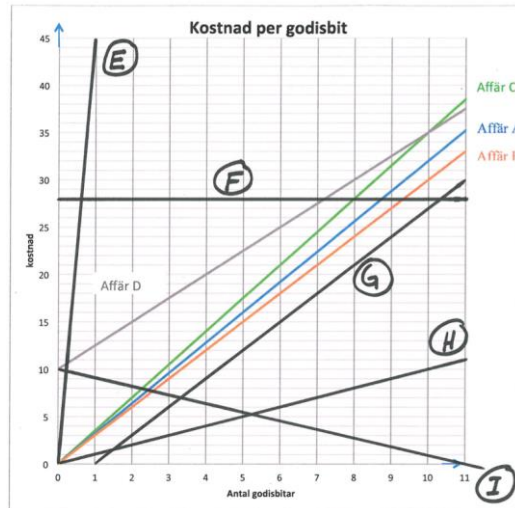


Figure 3: The students’ own stores (Stores E – I)

The students’ worked on the second part of the activity till the lesson ended. The teacher then collected all the students’ written work, and followed up the activity the following lesson, after having read and summarized the students’ explanations, with a whole class discussion about the students’ conclusions, interpretations and price-fixing of their own stores. The teacher was surprised over the interest and engagement the students showed when working on the activity as well as over the wide range of solutions and explanations the students offered. The fact that the activity allowed for a variety of solutions resulted in almost all students wanting to share their solution and thinking at the whiteboard in the whole class discussion. In a few instances the students asked for how to name certain concepts such as *origin* and *intersection point* to be able explain their thinking more precise and clear to their peers. In other word, the students wanted to express themselves mathematically correct.

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