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# Teachers' collective documentation work: A case study on tolerance intervals

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*In this paper we use the documental approach to investigate teachers' collective work. We follow two teachers, preparing together a lesson on tolerance intervals for grade 11. We identify Mathematical Knowledge for Teaching (MKT) that influences the use of resources by the teachers. We evidence that their collective work fosters important documentation work; but we observe significant differences between the documents developed by the two teachers.*

*Keywords: Documentation work, resources, teachers' knowledge, tolerance intervals, variability.*

## Introduction

Teachers interact with curriculum resources in and out-of-class (TWG22 call for papers). In previous works we have identified the importance of these interactions in terms of teachers' professional development (Gueudet, Pepin & Trouche 2012), and we have evidenced that teachers often work collectively with resources. In this paper we study further this collective work of teachers with resources and its consequences.

The work presented here takes place within the French national project REVEA<sup>1</sup> (Living Resources for Teaching and Learning). We consider the case of two mathematics teachers at upper secondary school in France teaching sampling variability in statistics in grade 11. We firstly expose the theoretical perspective we use and our methods. Then we present the data we collected, and the context of teaching sampling variability in France. Finally we expose our analyses of the teachers' work and of its links with teachers' knowledge in particular.

## Investigating teachers' documental work: Theory and method

We use for our research the theoretical and methodological perspective of the documental approach (Gueudet et al. 2012). Mathematics teachers interact in their work with a large range of resources (Adler 2000). Resources designed for teaching purposes like textbooks or software, resources coming from the students, e-mails exchanged with colleagues etc. Teachers choose resources, transform them, use them in class; we call this work "teachers' documental work". In previous research, we have evidenced that this work is closely linked with teachers' professional knowledge. The choice of resources by teachers, the way teachers modify and use the resources is driven by their professional knowledge (and this is called an instrumentalisation process, drawing on Rabardel's instrumentation theory, Rabardel 1995). In a reverse way, the features of the resources used modify teachers' knowledge (in an instrumentation process). In the documental approach, we consider that from a set of resources teachers develop a document: transformed

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<sup>1</sup> <https://www.anr-revea.fr/>

resources associated with a scheme of use (Vergnaud 1998). A scheme of use comprises the aim of the activity, rules of action and professional knowledge. The development of a document is called a documentational genesis.

Teachers' Communities of Practice (CoP, Wenger 1998) have a shared repertoire that the documentational approach interprets as shared resources. In previous works (Gueudet, Pepin & Trouche 2016) we have investigated the documentation work of a CoP (Sésamath, an association of teachers in France) designing online resources and identified the development of shared documents. Here we study a more "ordinary" CoP, composed of two teachers working together for the preparation of their courses. We are interested in particular in the commonalities and differences in professional knowledge within the documents developed by these teachers. The research question we investigate can be formulated as:

How do professional knowledge and resources interact in the collective design and implementation of a lesson?

Concerning teachers' professional knowledge involved in the schemes, we are especially interested in the identification of Mathematical Knowledge for Teaching (MKT, Ball, Thames & Phelps 2008): professional knowledge linked with the mathematical content to be taught.

The documentational approach is associated with a specific method, called "the reflexive investigation method". Documentational geneses are long term processes; moreover documentational work can take place everywhere and at any time. Thus we follow teachers over long periods of time; and involve them actively in the collection of data. These data are interviews of the teacher; videos of the teacher's work in class and out-of-class (videos of collective work, if teachers work together); resources chosen and transformed by the teacher.

For analyzing the data, we start with the transcribed interviews of the teachers. We identify in them the aims of the teacher's activity. For each aim, we search in the data for the resources used, and the other components of the document (rules of action and professional knowledge). We submit these elements to the teacher who corrects and complements them if needed. We present in the next section the data collected for the case we study here.

## **Data collected and context**

We follow since 2014 two mathematics teachers in an upper secondary school of a middle-sized town in France: Valeria and Gwen. They are both very experienced: Valeria teaches for 34 years, Gwen for 36 years, they both regularly follow teacher education sessions and are trainers for new teachers in their school. They also regularly work together, we consider them as a CoP (Wenger 1998). In 2015-2016, they decided to take two grade 11 classes called "economics and science", a specialty they taught for the first time. We followed their work for these classes, in particular for a chapter entitled: "sampling variability" (that they both used to teach grade 11 "science", with a similar content). For this chapter, we video recorded their common preparation (one hour), their individual courses (four hours each), and for each of them an individual post-teaching interview. We collected all the resources they used and produced, and the students' productions for the final assessment of the chapter. For both teachers we identify the professional knowledge/beliefs, the

possible origin of these beliefs, the consequences in terms of the activities/resources produced and the resources used.

Sampling variability is taught in France since 2010. This teaching starts in grade 10 where the idea of sampling variability is introduced using material like coins and dice and simulations on the calculator and on the spreadsheet. In the curriculum in France, the concept of tolerance interval is central in the teaching of sampling variability. In grade 10 the students have to learn how to identify the population; the sample and its size  $n$ ; the probability  $p$  of a given feature in the population, and the frequency  $f$  of this feature in the sample. A first tolerance interval is introduced, without justification:  $[p-1/\sqrt{n}, p+1/\sqrt{n}]$ . If  $f$  does not belong to this interval, the students learn to reject the hypothesis “the sample follows the population’s law” with a 5% risk level. At grade 11 (scientific or economics and scientific) the binomial distribution is introduced; it provides another tolerance interval, which can be found using the table of the binomial distribution produced for example with a calculator or a spreadsheet. The chapter we followed concerns the introduction and use of this interval. At grade 12, the normal distribution is presented; this leads to a new interval (the asymptotical tolerance interval).

Many research works have investigated teaching variability; they emphasized the specific nature of reasoning in probability (Steinbring 1991) and the need for particular knowledge to teach this subject (González 2013). Eckert and Nilsson (2013) used the notion of Mathematical Knowledge for Teaching Probability (MKTP) for characterizing the specific knowledge needed by the teachers in probability and statistics, and for sampling variability in particular. Finding situations in relevant contexts; emphasizing the idea of variability, using the different kinds of possible representations all require specific knowledge from the teacher. Our aim here is to investigate how the interactions with resources are shaped by, and contribute to MKTP.

## Results

In this results section, we firstly consider the two teachers’ documentation work, during the common preparation session, the lessons taught by each teacher and finally during the design of a common assessment. Then we present our analyses of the most important aspects of this documentation work (in terms of MKT involved), focusing on the collective-individual articulation.

### A collective documentation work

During the common preparation, Valeria and Gwen used several textbooks (9 different textbooks). They did not actually chose resources together, but drew on exercises and problems in the textbook to illustrate their declarations. They also talked about resources they intended to use: exercises, software (GeoGebra, spreadsheet) and the calculator.

Their discussion during the common preparation started by stating a difference: Valeria intended to use the spreadsheet from the beginning and during the whole lesson. The students have learned, in the “binomial law” chapter to produce with the spreadsheet and read tables displaying the value of  $P(X=k)$  and  $P(X\leq k)$ , when  $X$  is a random variable following a binomial law of parameters  $N$  and  $p$ . Valeria wanted to recall this, then to introduce the binomial law tolerance interval and the method for finding it, using the  $P(X\leq k)$  table produced with the spreadsheet. Gwen, in contrast, intended to use only the calculator, and no other software. After introducing the binomial law tolerance interval,

she wanted to ask students to write and implement on their calculator a program producing the tolerance interval.

In the other aspects of the common preparation, Valeria and Gwen agreed on all the points they discussed. They mentioned in particular the need to recall the grade 10 tolerance interval and to compare it with the new interval introduced.

All these aspects discussed during the common preparation are present in the lessons actually taught by Valeria and Gwen. We analysed these lessons drawing on the observations and videos in class, the resources collected and the post-lesson interview.

Valeria started indeed by recalling how to produce and read the binomial law table with the spreadsheet. She also recalled the grade 10 interval with exercises chosen in a textbook's "revision section". Then she introduced the new interval through a problem concerning overweight in USA. This problem came from another textbook, and she modified it in particular by suppressing the table giving  $P(X \leq k)$ , because she wanted the students to produce it themselves with their calculator. She presented how to find the interval from the table  $P(X \leq k)$ . Valeria insisted on the need to formulate very precisely the decision rule. At the end of the chapter, she worked with her students on the algorithm: the students implemented it on their calculator, but this program was actually not used as a tool to find the interval in exercises.

Gwen started with a problem that she built herself, about red-haired people in Scotland (inspired by a textbook problem with a different context). The first part of the problem recalled the grade 10 interval, and more generally the idea of sampling variability. The second part of the problem introduced the new interval. Just after this session, Gwen worked with the students on the production of an algorithm and its implementation on the calculator to find the binomial law interval. Afterwards this program was always used to find the interval. Gwen said that she found the binomial law interval too technical, she did not want her students to learn how to find it. She preferred to use it as an opportunity to work on algorithms. She distributed a sheet to the students presenting the interval and a diagram. Then she proposed different exercises about decisions; in particular one exercise with samples of different sizes.

The final assessment of this chapter was also the final assessment of the year for the two Grade 11 ES classes. Valeria and Gwen wrote it together; we analyse their documentation work drawing on the resources they used and produced; the e-mail they exchanged and their interviews. The control text comprised one exercise on tolerance intervals. This exercise, with an introductory text (figure 1) and three questions, concerned the rate of twins in India; it came from a textbook. Valeria and Gwen modified the initial text which was, in their opinion, too long and complex.

“ India: Kodinji, the mysterious twins village

In the state of Kerala (south-west India), there is an amazing village. The rate of twins is much higher than the national average. 440 twins live indeed in this town for 14600 inhabitants. This average is outstanding, since the national average is 16 twins for 1000 births”. *Extract of an article* (Courrier International, 2009)

Let  $X$  be the random variable counting the number of twins in a 14600 Indians sample.

**Figure 1: Introductory text of the control exercise (our translation)**

They modified the first question: in the textbook the parameters of the binomial law followed by  $X$  were given; they wanted the students themselves to find the parameters. They also changed the second question, where the students are asked to produce the tolerance interval, to display the two different methods expected for each class: use of a table (given in the text) for Valeria's class, use of the calculator's program in Gwen's class. Question 3 was left unchanged.

42 students were present at the final assessment. In the first question, 35 students justified that  $X$  followed a binomial law of parameters  $n$  and  $p$ ; 29 students determined correctly the value of  $n$ , but only 15 the value of  $p$ . In the second question, 19 students determined correctly the endpoints of the tolerance interval. In the third question, 19 students justified correctly the rejection of the hypothesis ("the Kodinji village follows the national figures").

### **Valeria and Gwen documentation work and use of resources: General statements**

We can notice that like other research works using the documentational approach (Gueudet et al. 2012) this description evidences that Valeria and Gwen are designers of their own resources. They use various curriculum resources, but transform most of them. Only some exercises texts are left unchanged, and one textbook extract presenting the binomial law interval (for Gwen). Moreover, textbooks (on paper, they do not use digital textbooks) are central resources, used as a reference to discuss the lesson plan, and to choose exercises for practicing the new methods, for the assessment/test, or to find an introductory problem (for Valeria). More surprisingly, they did not search the Internet for resources – this can be a consequence of their common preparation: Valeria and Gwen sometimes search the Internet for preparing their lessons, but always at home. For the common preparation they were at school with no Internet access. Another result already well known in the documentational approach is that the observation of students (their written texts, or oral discussions in class) constitute a very important resource for the teachers, leading to a constant modification of the resources produced along the documentation work. Both Valeria and Gwen intend to modify this lesson on tolerance intervals next year, because they consider that the students made too many mistakes in the final assessment/test.

### **Documents developed by Valeria and Gwen**

In this section we analyse our data in terms of documents developed by Valeria and Gwen. Since our focus is on MKT, we do not give a complete description of each scheme of use but only mention the aim of the activity and the MKT involved. We have chosen three examples of documents, corresponding to different situations in terms of similarities or differences.

Valeria and Gwen had a shared aim that can be described as "Recalling previous knowledge needed for the binomial law tolerance interval". They both considered that this new chapter must start by recalling the grade 10 interval, because they knew from their observation of students during the year that "many students do not remember this interval" (Gwen even added that some students perhaps never saw it, since some colleagues keep this content for the end of the year and run out of time). This shared MKT lead however to two different documents for this aim, because of the teachers' different resources: Valeria used a lot the classroom textbook, and thus proposed revision exercises coming from this textbook, while Gwen wrote her own problem text.

Valeria and Gwen also shared a general aim that can be presented as: "Teaching how to find a tolerance interval with a binomial law". During the previous years, Valeria has developed for this

aim a document including various resources: the spreadsheet (as software or in the calculator), exercise texts, the illustrating diagram, and MKTP: “The students must learn to find the endpoints of the interval by reading the table”. This knowledge can have different sources; we claim that it comes in particular from institutional texts (the official curriculum) and from textbooks. For the same aim, Gwen has developed a different document, including: the calculator, an algorithm, the illustrating diagram, exercise texts as resources; and MKTP like: “The binomial law interval is too technical”; “there are no questions about the binomial law interval at the baccalaureate<sup>2</sup>”; and the MKT “it is important to work with students on algorithms”. This knowledge comes from reading the texts of the baccalaureate, and from a personal mathematical difficulty: Gwen declared that she “cannot remember [herself] how to find the endpoint of the interval”. Moreover she considered that this grade 11 curriculum is only a transition between the grade 10 interval and the grade 12 interval (with the normal distribution) while algorithms are always present in the baccalaureate texts. Valeria and Gwen discussed this difference during the common preparation. Valeria integrated in her lesson the programming of the algorithm on the calculator, but she did not want her students to use it, because she feared that the students do not really understand the algorithm and use their calculator as a “black box”.

For the aim: “Assessing the students’ ability to determine and interpret a tolerance interval”, Valeria and Gwen used shared resources: they wrote the assessment text together drawing on the same textbook exercise. The choice of this exercise was guided by MKT firstly expressed by Gwen, and adopted by Valeria: “the students must learn to find information in a text”. Nevertheless, the text produced was also transformed to incorporate the use of two possible methods, because of their different MKT concerning how to find the binomial law tolerance interval.

In Table 1 below, we synthesise these three documents, evidencing the common and different elements.

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<sup>2</sup> In France, the final secondary school assessment, at the end of Grade 12.

<b>Aim</b>	<b>Resources used</b>	<b>MKT/ MKTP</b>
<i>Recalling previous knowledge</i>	Valeria: Revision exercises in the classroom textbook	<i>“Many students do not remember the grade 10 tolerance interval”</i>
	Gwen: Her own problem text	
<i>Teaching how to find a tolerance interval with the binomial law</i>	Valeria: Problem and exercises texts from different textbooks, the spreadsheet, <i>algorithm on the calculator (coming from the collective work)</i>	Valeria: “The students must learn to find the endpoints of the interval by reading the table”; “they must not use the calculator as a black box”
	Gwen: Problem composed herself, exercises from different textbooks, <i>algorithm on the calculator</i>	Gwen: “it is important to work with students on algorithms” “the binomial law interval is too technical”
<i>Assessing the students’ ability to find and interpret a tolerance interval</i>	<i>Shared assessment text written together from a textbook exercise, but integrating two possible methods.</i>	<i>“The students must be able to identify information in a text” (shared)</i> + MKT/MKTP described in the above line

**Table 1: Synthetic presentation of documents developed by Valeria and Gwen. Shared elements are in italics.**

## Conclusion

In this paper we investigated the research question: “How do professional knowledge and resources interact in the collective design and implementation of a lesson?” in the case of a lesson on tolerance intervals for Grade 11 students in France. In the frame of the documentary approach, investigating how professional knowledge and resources interact means investigating the documents developed by teachers. For each of the two teachers we followed, we observed that they developed an important design work, choosing resources, associating and modifying them. This work was guided by their professional knowledge, in particular MKT and MKTP. We observe that this MKT is mostly of the type: “Knowledge of Content and Students”; deepening the analysis in terms of types of MKT is an interesting perspective for further work. In a reverse way, resources influenced the development of MKT, and this MKT can be different for each individual teacher. For example the official curriculum influenced Valeria and Gwen in different ways: while Valeria aligned with the curriculum about tolerance intervals, Gwen attached more importance to the algorithms. She developed a personal interpretation of the official curriculum, not focusing on the chapter she taught but taking into account the whole year.

In previous works (Gueudet et al. 2012) we evidenced that collective work is present in many aspects of teachers’ activity and that it is a stimulator of documentation work, especially when it takes place within CoPs. In (Gueudet et al. 2016) we analysed the common documentation work in a CoP: an association of teacher designing an e-textbook. We evidenced that they developed



common documents, drawing on their individual documents. In the present study we investigated a CoP composed by two teachers preparing their courses together. We evidenced that, in spite of the collective work the documents developed by the two teachers for the same aim are never completely identical. The consequence of the collective work is that these documents sometimes share common elements. The main reason for the differences seems to be the long experience of both teachers: they already developed in previous years documents for the same aims, and thus have MKT or MKTP associated with these aims and also specific resources. The collective work can bring new resources (the algorithm on the calculator for Valeria) or new knowledge (the students must be able to find information in a text, for Valeria again), but the previous knowledge developed during interactions with resources over many years is still present and produces differences in the documents.

These teachers will go on working together; with a longer common work, evolutions may take place, and we will try to analyse these evolutions. We also hypothesize that evolutions of practice are more likely to take place in teachers' CoPs when the members of the CoP are involved in a common design activity (for example in the Sésamath case, Gueudet et al. 2016, or in the context of professional development, Pepin & Miyakawa 20016). We intend to investigate this hypothesis in further research.

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