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Asymptotes and asymptotic behaviour in graphing functions and curves: An analysis of the Croatian upper secondary education within the anthropological theory of the didactics

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Introduction and methodology

Asymptotes and asymptotic behaviour play an important role in many traditional and modern mathematical disciplines and are supported by a rich and well-established abstract theory. Some basic aspects of these concepts can be utilized already in elementary mathematics as powerful tools in graphing and analyzing behaviour of elementary functions at infinity and near singularities and in graphing simple plane curves such as hyperbola. Therefore, this body of knowledge is a common part of upper secondary mathematics curricula worldwide. Here, we investigate its didactical transposition to the Croatian general upper secondary education. The survey is conducted within the theoretical framework of the Anthropological theory of the didactics (ATD), developed by Y. Chevallard (Chevallard, 1981, 2007). The main idea of the ATD is to determine a relation $R_I(p,O)$ between a body of knowledge O and a person that occupies a position p in an institution I . Accordingly, mathematical knowledge and activities are described in terms of a *praxeology* $[T, \tau, \theta, \Theta]$, where its practical component, *praxis*, is represented with a *task* T and a *technique* τ and its discursive or theoretical component, *logos*, with a *theory* Θ and a *technology* θ . Research within the ATD should include relevant data about the relation $R_I(p,O)$ for all institutions I involved in the educational process i.e. the process of didactic transposition (Bosch, Chevallard & Gascón, 2005). Questioning and understanding *conditions* and *constraints* on the relation $R_I(p,O)$ is necessary for setting up attainable, justified and significant educational interventions (Barbé, Bosch, Espinoza, & Gascón, 2005). Hence, as relevant for our setting, we analyzed and compared the relations $R_B(p,O)$ and $R_S(p,O)$, where O is a graphical representation of an elementary function and a hyperbola in a square coordinate system, together with corresponding techniques regarding asymptotes, while institutions considered are two actual Croatian mathematical gymnasium textbooks B and the cohort of 40 the final, fifth year mathematics education students S at the largest mathematical department in Croatia.

This is a part of a more comprehensive study regarding asymptotes and asymptotic behaviour in the Croatian pre-university education which also included the institution of academic mathematicians. The methodology included a praxeological analysis of the textbooks as representatives of the *knowledge to be taught* and three questionnaires with open-ended questions for the prospective mathematics teachers to provide an insight in related *knowledge available* to students as a potential *taught knowledge*. Based on this, a reference epistemological model (REM) is proposed and then verified and improved with scholarly knowledge gained from semi-structured interviews with two

academics. Here, we focus on textbook topics, tasks, techniques and discourses related to O and on three particular tasks from the questionnaires administered: graphing and describing a simple rational function and a shifted exponential function from a real world problem, given by formulas, and graphing a hyperbola and its asymptotes.

Results and conclusions

The results of the textbook analysis show that graphing a function f pointwise, that is, by plotting some corresponding points $(x, f(x))$ and connecting them by a smooth curve dominates all other available techniques for graphing functions given by formulae. A technique of graphing a function or a curve regarding its properties recognized from a corresponding algebraic expression (formula or equation) occurs only in relation to the tangent and cotangent function and to a hyperbola, while the tasks on graphing polynomials and rational functions appear only as common practical activities of utilizing derivatives (calculus). Finally, a technique of transforming a prototype graph of an elementary function to get a graph of its composition with a linear function (by translations and dilations) is rarely implemented. Although textbook praxeologies of graphing functions often elaborate function properties and flow, discursive accents are set on establishing a function's domain, monotonicity and symmetry of its graph, neglecting its asymptotic behaviour. Asymptotes are seen relevant only for tangent and cotangent functions. Results of the questionnaire analysis completely reflect those of the textbooks. The students applied techniques and provided discourse to the same extent as it is given in the praxeological organization of the textbooks. Namely, their dominant techniques are drawing curves through their points and drawing graphs regarding function properties determined by using calculus, their chosen techniques are not the most efficient for the task in question, and asymptotic behaviour is available to them but not fully utilized in praxeologies relevant to graphing functions or curves. Considering this, it is our suggestion for the teaching practice and for an ongoing curricular reform in Croatia that: all relevant function properties should be emphasized when describing its behaviour; common properties of a function should be more related to its algebraic representation and utilized for its graphing; functions should be graphed by graph transformations, whenever fitted; and asymptotic behaviour should be more emphasized, adequately graphically represented and described by formal and informal mathematical discourse.

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