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# Assessment in mathematics as a lever to promote students' learning and teachers' professional development

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*In this paper, we are presenting our analytical tools to characterize assessment activities as part of teachers' practice, on a specific mathematical content (algebra). We are also presenting the principles of our collaboration with high school teachers, inside a particular workgroup (LéA), to explain why we came to consider assessment as a potential lever to enhance both the students' learning in mathematics, and the teachers' development. We are presenting a few results on the effects of this collaborative work on teachers' practice when assessing students' learning, and on our means to analyze the students' results throughout the process.*

*Keywords: Assessment, algebra, teachers' practice, teachers' professional development, collaborative work.*

In this paper we are presenting the analytical framework that we are building to characterize the practice of high school teachers in mathematics regarding student assessment. We study assessment through one of its particular functions, promoting learning, with a didactical point of view, focussing on a particular content (algebra). We consider three inputs for assessment: assessment as a framework to characterize teachers' practice, assessment as a tool to enhance students' learning in mathematics, assessment as a lever for professional development. We will present some results on teachers' professional development, from the collaborative work we lead with high school teachers on assessment.

## Assessment as a framework to characterize teachers' practice

In this first part, we are presenting the framework that we have built to analyze teachers' assessment practice, leaning on previous studies about teachers' practice, and on the teaching of algebra in particular.

### Defining assessment practice

Assessment can be found in many aspects of the teachers' activity, and it would be easy to call assessment any interaction between the teacher and the students. To restrain our observations, we draw upon De Ketele's definition (1989), and call assessment any gathering of information by the teacher on the students' activity and knowledge, and the interpretation and use of this information.

By activity, we mean everything the students do, say, think, (or do not do). Of course, not everything is accessible to neither the researcher nor the teacher, but we consider that students' learning happens through their mathematical activities, at least partially (cf. Rogalski, 2013, about the use of Activity Theory as a framework for research in the Math Education field). These activities may consist on participating in a debate around a task or listening to a mathematical discourse in class. But in many occasion, they result from the tasks proposed by the teacher, and on the choices that the teacher makes

to manage the solving of the task, which are elements that we consider when analyzing the teachers' practice in class.

Assessment is easier to pinpoint for the researcher, when it is formal, for example a written summative test at the end of a teaching sequence, or short diagnostic tests happening at the beginning of every session. Informal assessment, on the other hand, is more difficult to identify, but can happen in many occasions during the class, through the interactions between the teacher and the students, giving information on the students state of knowledge, to the teacher or to the students themselves. To be able to characterize assessment practice in any case, we have drawn a list of criteria, whether the assessment is formal or not, and at any point in the teaching sequence.

### **Characterizing assessment practice**

One of the elements that we take into consideration to characterize assessment practice, is the distance between each assessment task and the similar tasks previously given by the teachers on the same mathematical content (Horoks, 2006). For example, when a teacher is assessing the students' knowledge on a mathematical content through a final test, we can question the choices of tasks made by this teacher and their link with the tasks that were actually worked on before the test. A certain gap between the test's and previous tasks (or between tasks from a diagnostic test and all the possible prerequisite tasks) can be interpreted in different manners: it could be explained by the function given to this test, (rewarding or challenging the students for example) or maybe by a lack of pedagogical content knowledge (Shulman, 1986) for the teacher. The epistemological and didactical analyses of the mathematical contents are crucial here to make the comparison between the two sets of tasks. In the case of informal assessment, each new task given by the teacher has to be considered among a set of previous tasks, depending on the "study moments" (Chevallard, 1999; Barbé & al., 2005), related to the distance from the first encounter with the mathematical notion. The distance between the tasks allows us to measure the complexity for the students compared to the tasks that they have already worked on. More globally, the whole range of tasks proposed by the teacher on a mathematical content, with the absence of particular tasks in the related assessments, can tell us some of the intentions of the teacher for assessment, in relation with teaching.

Another element that we take into account, is the "depth" of the information: indeed, the process of "taking / interpreting / exploiting information on students' activity" in mathematics, can take its roots in the solution that the students produce (the result of a task), the way they solve the task (procedure to achieve a result), or the knowledge that is put into action to complete this procedure. Linking the activity to the student's knowledge requires, from the teacher, an understanding of the conceptualization of the mathematical contents behind the procedure, which is usually specific to the particular content. Regarding algebra in particular, it leads us to consider some specific elements, such as the form of writing for calculations or the type of reasoning. Indeed, some forms of writing for the calculation for example (cf. figure 1) can inform the teacher on the meaning of the equal sign for each student (computation, equivalence), or on the student's structural / procedural view of numerical or algebraic expressions.

$$\begin{array}{l}
 - (3 + 4) \times 2 - 8 = 6 \\
 - 3 + 4 = 7 \times 2 = 14 - 8 = 6 \\
 \quad + 4 \quad \times 2 \quad - 8 \\
 - 3 \text{ -----} > 7 \text{ -----} > 14 \text{ -----} > 6 \\
 - 3 + 4 = 7 \\
 7 \times 2 = 14 \\
 14 - 8 = 6
 \end{array}$$

**Figure 1: Different writings for the calculation, giving information on the students' knowledge**

The interpretation of the information can also differ, depending on the reference taken for this interpretation: a comparison with what is expected by the institution (curricula, external assessment), or what could be expected by the teacher, considering all the previous work and the teacher's knowledge of the didactics of the mathematical notion at stake (errors, obstacles and breaches, steps in the conceptualization, etc). It can also be a comparison between the students' different procedures or a comparison in time for one student, to appraise his or her progress. These comparisons can be made explicit or not to the students. Here again, it can be linked to the various possible functions of the assessment.

Finally, the exploitation of the information differs depending on the moment when it occurs: whether it leads to immediate feedbacks related to students' result, procedure or knowledge or, in a more or less short term, when it influences the planning for the next activities.

Before giving an example of teachers' formal assessment practice, we will first describe our working context with a group of teachers.

### **Description of the collaborative work inside the *Léa***

A "Lieu d'Education Associé (*Léa*)" is an instance created by the French Institute for Education (IFE) to promote research with people who play an acting part in education. For 3 years, they are associated with a team of researchers to investigate questions about education and to build realistic resources for teachers or educators. Our *Léa* takes place, since May 2014, in a high school (students from 11 to 15 year old) in an Educational Priority Area, with 9 teachers (4 at the beginning) and 7 researchers, who meet every month to work together, to build teaching materials for algebra and to discuss assessment practice that could promote students' learning. The *Léa* can give us access to those teachers' evaluation practice in the long term.

### **An example of a comparison of teachers' formal assessment practice**

We asked *Léa* teachers to design a diagnostic test at the beginning of the year for their 7th-grade students, to assess their numerical and pre-algebraic knowledge before introducing algebra. The tasks of the tests that they individually proposed were not covering all the range of the required knowledge for the introduction of algebra (Carraher & Schliemann, 2007, Kieran, 2007). The teachers justified their choices by giving institutional or social reasons, rather than epistemological or didactical ones.

<b>Teachers</b>	<b>G</b>	<b>M</b>
Variety of tasks	Repetitive task	Different tasks
Complexity of test's tasks / previously given in class	Similar to the previous ones in class	More complex than the previous ones in class
Information (declarative)	On the result	On the procedure
Feedback to the students (declarative)	Marks on the paper	Marks on the paper
Function of the formal test (declarative)	<ul style="list-style-type: none"> <li>- To be able to give marks for the institution</li> <li>- To work on the basics</li> </ul>	<ul style="list-style-type: none"> <li>- To learn by adapting to a different situation</li> <li>- To adapt the teaching plan ahead</li> </ul>

**Table 1: Formal assessment (summative test)**

We also conducted interviews with these teachers to find out about their views about assessment, after they had proposed their first summative test of the year. They were asked questions about their choices of tasks and the feedbacks they gave to the students afterwards (cf. table 1 for two of the teachers). Their answers showed a great variety in the tasks they proposed, regarding the distance with previous tasks, and probably resulting from different views on the functions given to formal assessment, despite the fact that these teachers often worked together. What was common to all the teachers on the other hand, is that they did not usually give many feedbacks to their students. Indeed, those teachers gave a mark without informing the students with the necessary elements to understand their mistakes and the limitations of their reasoning. Another comparison, related to informal assessment, for one teacher at different moments, will be made in the last part of this paper.

## **Assessment as a tool to enhance students' learning in mathematics**

### **Definition of formative assessment**

For Black & Wiliam (1998), an assessment can be formative when a teacher uses the information on the students to help them engage in the work on a task, or to help each of them auto-evaluate their knowledge:

The term 'assessment refers to all those activities undertaken by teachers, and by their students in assessing themselves, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged. Such assessment becomes "formative assessment" when the evidence is actually used to adapt the teaching work to meet the needs. (page 2)

In terms of gathering/interpreting/exploiting information, formal assessment can play a more or less formative function, depending on the chosen tasks (if the tasks are way too complex or too simple; the students' productions might not reveal many useful information for the teacher). It depends also on the feedbacks made to the students. These facts can both be witnessed and analyzed by the researcher.

But when it comes to informal formative assessment in class, even if it is possible to see a teacher going around in the class when the students are working on a task, we can only witness the information actually gathered if the teacher is using it right away to guide the students' work. Deciding not to use the information right away, but reorganizing the plan of the next sessions, for the entire class or a particular student, could also be an exploitation of the information to promote the students' learning, but the researcher would then hardly acknowledge it. In any case, the research time allowed to the students to work on the task will probably have an influence on their mathematical

activity, and on the information that the teacher will be able to take on this activity, depending on the task.

### **A key moment: sharing the students' productions after letting them work on a task**

The moment of pooling of students' procedures, after letting them work on a task, alone or in groups, seems to us like a good opportunity for informal formative assessment, where students could compare their solution with others' and know if they are close to what was expected. It depends of course on how the teachers choose to manage this moment of the session, and on the use they will make of the students' productions. This is why we will look more closely at those moments in the classrooms to analyze the use that the teachers make of the students' productions: how is the students' work taken into consideration?

What kind of productions do the teachers choose to share with the entire class? Is there a variety in these productions, regarding the result of the task or the possible procedures? Are there errors, typical or not, showed to the students? These elements are indicative of the information probably gathered by the teacher on the students' work while they were working, but depends also on an *a priori* analysis of the task, strongly linked to the mathematical contents to be mobilized, in order for the teacher to anticipate the possible outcomes.

We also analyze the exploitation of these productions. However, the interpretation of the information by the teachers remains mostly invisible to the researcher, except when the teachers explicitly mention the reference they use to compare (with what is expected at the end of the year, with what the students already did before, between students...). We note if the teachers organize a comparison of the results or of the procedures. Do they rank them to show the relevance and limits of each solution? How is organized the (in)validation of the solutions? Who is (in)validating them? With which arguments? And which conclusion? These elements can inform us on the role given to the students in the validation and institutionalization process and in the assessment process in general. We will give an example of this type of analysis for one teacher, in the last part of this paper.

We have hypotheses on the conditions that we consider more favorable towards student's learning, for example by making use of various students' procedures and errors and by implicating them in the validation, using mathematical arguments. We will confront these hypotheses with the results in algebra of a hundred of high school students, whose teachers' assessment practice was analyzed in this study,. In order to do that, we will analyze each task given by each of the teachers participating in the *Léa*, as part of the formal assessment process in algebra during 3 years, in terms of kinds of tasks (Chevallard, 1999) and adaptations (cf. Robert 2003), to determine their variety and complexity. We will collect the students' productions, analyze their answers and characterize them according to the different degrees of algebraic competencies defined by Grugeon & al. (2012) and Chenevotot-Quentin & al. (2015) for the design of a diagnostic assessment in algebra. This analysis is now in progress, as we are beginning the third year of our work inside the *Léa*.

## **Assessment as a lever for professional development**

### **Our tools to analyze professional development**

For our research, we already have collected a wide range of data inside our *Léa*, that we still have to fully process: to document the teachers' assessment practice, we gathered their personal documents

for the class and asked them to film themselves regularly in their own class; to measure the possible effect of this practice on the students' learning in algebra, we collected many students' productions; and at last, to try to estimate the effects of our collaborative work on the teachers' practice, we recorded the discussions during our meetings, and kept the reports of these meetings, when written by the teachers.

The analysis of the teachers' assessment practice takes into account, as explained before, the list of tasks proposed to the students in algebra and the management of the resolution of these tasks in class (informal assessment) or after a test (formal assessment). Their point of view about assessment is also visible through the interviews we conducted at the beginning of the project, or through the discourse of the teachers during the meetings of the *Léa*. More specifically, we are interested in the ways they argument their choices for their class, through the type of reasons they give for their choices of tasks or management (institutional, social, didactical, mathematical, etc). The moments when the teachers disagree with the researchers, or try to convince new teachers who joined the collaborative workgroup, are particularly interesting for us for that matter; to gather information on a possible evolution on the teachers' point of view on assessment, and more globally, on the teaching of algebra.

### **Some results about the changes in the teachers practice and arguments**

To illustrate our analyses of the teachers' practice and their development, we are giving here an example of two sessions, for the same teacher, filmed one year apart. We analyzed the moment of sharing the students' productions, after working on similar tasks. These tasks both involve testing a calculation program with several numbers to notice an unchanging result or property and proving it with algebra. Our analyses are based on the indicators that we have already listed (the variety of the productions chosen to be displayed, the exploitation of students' errors, the initiatives in the validation, the arguments for justification).

Our analyses after the first year of collaboration (see table 2 "year 1") tend to find that teachers' assessment practice are very settled and stable. It seems that our didactical contributions about the teaching and the learning of algebra have helped teachers' practice to evolve (Horoks & Pilet, 2015): indeed they have better indicators to select the students' productions that they will use for the discussion after a task. But the exploitation that they make of these productions hasn't really changed after the first year: when sharing them with the class, the *Léa*'s teachers don't usually organize a comparison between the students' productions nor give a validation based on mathematical reasons. We also found that these teachers usually leave no initiative for students when working on the more complex algebraic tasks, which leads to the impossibility to rely on their production (see table 2, "year 1").

After the second year, where we decided to share some of our tools to analyze the sessions in class with the teachers, we can notice some evolution in the exploitation of the information (see table 2, "year 2"). Even though the second part of the task is more complex, this teacher still relies on the students' productions, even if they are not mathematically correct, to build, along with the students, the reasoning that will allow the class to invalidate the proposed solution.

However, even if the tasks are similar between year 1 and year 2, students have a higher grade in year 2 which may also explain the different choices made by the teacher. We should go on studying practice for a longer time, to identify its stability, and this is what we plan to do in the *Léa* project.

		M(year 1)	M(year 2)
Testing with numbers	Duration of individual work	6.00	this numerical step is not part of the second task
	Variety and comparison on results/procedures from the students' productions	3 procedures not compared	
	Presence of errors in the displayed productions	error in the procedure	
	Student's initiative in the validation	yes	
	Mathematical arguments of proof	no	
Proving with algebra	Duration of individual work	2.30	6.30
	Variety and comparison on results/procedures from the students' productions	this algebraic step is handled by the teacher without any support on the students' productions	4 procedures compared
	Presence of errors in the displayed productions		yes
	Student's initiative in the validation		yes
	Mathematical arguments of proof		counter-example

**Table 2: Evolution of informal assessment practice for teacher M**

Both the cognitive (contents and tasks) and mediative (organization of the sessions in class) elements of the teacher activity play a part in the assessment process that we are trying to analyze here. But, as emphasized by Robert and Rogalski (2005) there are other constraints of this professional occupation to be taken into account: the social (type of school), the institutional (curricula) and personal (career and education) components, playing a significant part when interpreting teachers' practice, including for us their choices in terms of assessment. We analyze teacher's practice through all these components, at different levels: locally in the classroom or globally within all the teaching plan, and we believe that it can give us access more deeply into the teachers' consistency and explain their stability. Yet, after the second year, we noticed some changes in the arguments that the teachers are giving to justify their choices, shifting a little from social and institutional reasons to mathematical or didactical ones, that we would hope to link to our work together, that is still going on..

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