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Subject matter knowledge and pedagogical content knowledge in the learning diaries of prospective mathematics teachers

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In this study, 26 learning diaries by prospective mathematics teachers were analysed in order to describe the characteristics of mathematical and pedagogical knowledge discussed therein and to evaluate the potential and limitations of the learning diary in mathematics teacher education. Conceptualisations of teacher knowledge are typically discussed in terms of subject matter knowledge, pedagogical knowledge and pedagogical content knowledge. A central goal of mathematics teacher education is to strengthen all of these areas of competency. The results of this study indicate that, although the learning diary is a potential learning tool, prospective mathematics teachers tend to emphasise pedagogical content knowledge, placing less stress on subject matter knowledge. Consequently, more structured learning diary tasks could be used to support all the components of mathematical knowledge for teaching.

Keywords: Teacher knowledge, mathematics teacher education, learning diaries, subject matter knowledge, pedagogical content knowledge.

Introduction

Finnish mathematics teacher education consists of three somewhat distinct parts: subject matter studies, educational studies and practical teacher training at schools. Subject matter studies at mathematics departments form a major part of the mathematics teacher studies. Finnish mathematics teachers, however, report that university-level subject studies in mathematics lack a clear connection to the mathematics taught at school (Koponen, Asikainen, Viholainen, & Hirvonen, 2016). The transition to university-level mathematics requires a major change in mathematical thinking (e.g., Tall, 1992). At university, mathematics courses typically emphasise formal reasoning, meaning reasoning based on axioms, definitions and proven theorems (Viholainen, 2008). Informal reasoning is based on visual or physical interpretations of mathematical concepts (Viholainen, 2008). Some empirical studies (e.g., Chin, 2013; Viholainen, 2008) have shown that prospective teachers may have difficulties connecting formal and informal reasoning.

On the other hand, prospective mathematics teachers may emphasise the importance of a teacher's personal characteristics and pedagogical knowledge, while diminishing the importance of subject matter knowledge (e.g., Hoffkamp & Warmuth, 2015). Subject matter knowledge nevertheless plays a significant role in a teacher's professional knowledge. Firstly, subject matter knowledge is typically seen as theoretically necessary for developing pedagogical content knowledge (Baumert et al., 2010). The quality of prospective teachers' subject matter knowledge also affects their pedagogical choices when participating in practical training (Even, Tirosh, & Markovits, 1996). In addition, subject matter knowledge along with pedagogical content knowledge can be seen as a foundation for effective teaching, as a teacher's professional knowledge affects student achievement (e.g., Baumert et al., 2010).

This study is a part of a design-based research and development project that has been carried out at the University of Helsinki. The aim of the research is to develop instructional practices in order for

prospective teachers to both strengthen their subject matter knowledge and build up their pedagogical content knowledge. The research also aims to give insight into prospective mathematics teachers' conceptions of the relationship between school and university mathematics.

In this study, the specific focus is on learning diaries written by prospective mathematics during a six-week seminar. The seminar focused on finding connections between the mathematics studied both at university and at school and on discussions of mathematical content from the teacher's point of view. That is, the aim of the seminar was for prospective teachers to, first, strengthen their (structural) knowledge of mathematical topics (such as derivative) and, second, enhance their pedagogical content knowledge with relation to these topics. The aim of this study was to examine the potential and limitations of learning diaries as a learning tool in this context and to conceptualise the kinds of knowledge these prospective mathematics teachers discussed in their diaries.

Theoretical background

Theories used in design-based research can be divided into grand theories, orienting frameworks, frameworks for action and domain-specific instructional theories (DiSessa & Cobb, 2004). In this study, the idea of constructive alignment (Biggs & Tang, 2011), which provided the instructional design of the research setting, is used as a framework for action. The data analysis for this study is based on domain-specific conceptualisation of teacher knowledge (Ball, Thames, & Phelps, 2008). In the next two subsections, these frameworks will be discussed in more detail.

Teacher Knowledge

The distinctions between content knowledge (or subject matter knowledge), pedagogical knowledge and pedagogical content knowledge (Shulman, 1987) are an established starting point for conceptualisations of teacher knowledge (Scheiner, 2015). Especially the distinction between subject matter knowledge and pedagogical content knowledge has gained significant attention and generated a great amount of research and further development of the conceptualisations of teacher knowledge. According to the Mathematical Knowledge for Teaching (MKT) model (Ball et al., 2008), a teacher's content knowledge consists of common content knowledge (CCK), specialised content knowledge (SCK) and horizon content knowledge (HCK). In the MKT model, pedagogical content knowledge, in contrast, is divided into knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of content and curriculum.

The components of MKT model have been shown as important for effective teaching (Jakobsen, Thames, & Ribeiro, 2013). Hence, the model is valuable for this study, which aims to offer insight into prospective mathematics teachers' discussions of their learning diaries and to use this information for further development of instructional practices in teacher education.

Constructive alignment and learning diaries

Present research and development of instructional practices in higher education is typically based on the constructivist view of learning and concepts, such as learner approaches to learning, self-regulation and reflection. Constructive alignment is based on the constructivist view of learning and suggests that the intended learning outcomes, implementation of teaching and assessment should be carefully aligned and support active learning. Biggs and Tang (2011) suggest that by using more

active ways of learning (such as problem-based learning) even ‘less academic’ students can achieve more advanced levels of learning, such as applying and theorising.

In this study, the seminar was designed in the spirit of constructive alignment (e.g., the students worked in groups and specified their own study/discussion topics). The learning diary task was one of the ways to promote active learning and reflection among students. Typically, learning diaries are seen as texts that include both the central arguments of a course or a seminar and the writer’s own interpretation of and reflection on these themes. That is, learning diaries are not supposed to promote *knowledge telling writing* (Bereiter & Scardamalia, 1987), which is understood as writing based on memorised facts. Instead, learning diaries promote *knowledge transforming writing* (Bereiter & Scardamalia, 1987), which is based on problem analysis and reflection.

Research questions

In Finnish higher education, learning diaries have been used successfully in subjects such as research methodology (Kyttälä, 2012). Journaling has also been found to be useful in studying university-level mathematics (Meel, 1999). There is, however, a lack of research evaluating the use of learning diaries in mathematics teacher education from the point of view of the teacher’s knowledge. Additionally, more insight into prospective mathematics teachers’ mathematical and pedagogical thinking is needed for further development of instructional practices in mathematics teacher education. Thus, the following research questions were formed.

1. Can learning diaries be used to promote knowledge transforming writing in mathematics teacher education?
2. What kinds of professional knowledge do the prospective mathematics teachers discuss in their learning diaries?

The first research question was posed in order to evaluate whether learning diaries have potential as a reflective learning method in mathematics teacher education. The second research question was posed in order to characterise the prospective mathematics teachers’ discussions on teacher knowledge. The question of whether some/certain aspects of teacher knowledge would be emphasised in the diaries was also considered, as prior research has shown that prospective teachers may emphasise pedagogical knowledge and diminish the importance of subject matter knowledge (e.g., Hoffkamp & Warmuth, 2015).

Method

The data was collected during a seminar held in autumn 2014. The students (prospective mathematics teachers) attending the seminar formed small groups of 4–5 members. All groups prepared an introduction to a topic (such as dot product), so that both mathematical and pedagogical ideas were covered. These introductions led to group discussion and, as homework, the students reflected on their ideas by writing a learning diary. In their diaries, the students were asked to discuss 1) *What was discussed and how do the topics of discussion relate to other contexts?*; 2) *What did I learn and what was its meaning for me?*; 3) *Was something missing or unclear?*

Participants were mainly mathematics students at the end of their studies. Three students were studying another subject (such as physics) with minor studies in mathematics. Also, six students were second- or third-year students and, thus, not yet at the end of their five-year studies. The participants

were studying in a subject teacher programme that qualifies them to work as a teacher in the last years of comprehensive school (with students aged 13 to 16 years) and upper secondary school (with students aged 16 to 19 years).

Student learning diaries (N=26) were analysed using content analysis (Elo & Kyngäs, 2008). The units of observation were first placed in categories from the MKT model using deductive content analysis. Subcategories were then formed using inductive content analysis. In addition, the individual diary entries were classified either as *knowledge telling writing* or *knowledge transforming writing* in order to classify the entire diary either as *knowledge telling* or *knowledge transforming*.

The author of the present article created the coding. As it was not possible to use two independent coders, during the process, the author reread the diaries and the coding to ensure that his thinking remained constant during the coding process. The components of the MKT model may be difficult to distinguish from one another and this boundary problem has been highlighted in the research literature. This poses a challenge for coding, as two researchers may create different categorisations. The most problematic category seems to be HCK. In this study, HCK was understood, as defined by Jacobsen et al. (2013), as ‘an orientation to and familiarity with the discipline (or disciplines) that contribute to the teaching of the school subject at hand, providing teachers with a sense for how the content being taught is situated in and connected to the broader disciplinary territory’.

The coding of *knowledge telling writing* and *knowledge transforming writing* was based on a prior study by Kytälä (2012). When coding each diary entry as either *knowledge telling* or *knowledge transforming*, the former was used if the entry included only repetition of the information discussed in the seminar and the latter code was used if the entry included personal reflection. *Knowledge telling writing* included excerpts such as ‘This week we discussed linear algebra. Firstly, we discussed vectors in \mathbf{R}^2 ’, whereas *knowledge transforming writing* included personal reflection such as ‘I soon realised that I didn’t remember much about dot product. I remembered that it had something to do with lengths and the perpendicularity of vectors.’

If at least half of the entries were labelled as *knowledge transforming* the entire diary was labelled accordingly. This methodology was chosen to ease the comparison of the results of this study to prior studies in the Finnish higher education context.

Results

The results of the study are presented in three parts. First, writing strategies (*knowledge telling* vs. *knowledge transforming*) are discussed. Then, in the following two subsections, the subject matter knowledge and pedagogical content knowledge observed in the diaries are discussed.

Knowledge telling writing vs. knowledge transforming writing

23 of the 26 diaries featured *knowledge transforming writing*. This seems to indicate that learning diaries can be used to promote reflective learning in mathematics teacher education as they have in other educational contexts, as Kytälä (2012) has suggested. However, while most of the diaries were categorised as *knowledge transforming*, the content discussed in the diaries varied significantly. In some of the diaries, both mathematical and pedagogical topics/issues were discussed comprehensively, whereas in others, the mathematical content was discussed only cursorily and the

pedagogical issues were discussed in depth. The coding of subject matter knowledge and pedagogical content knowledge aimed to highlight this variation in greater detail.

Subject matter knowledge

The distinguished subcategories of subject matter knowledge are presented in Table 1. The frequency of each category is indicated in brackets. In the main categories, percentages are also given. Discussing representations of mathematical content was labelled as SCK, as according to Ball et al. (2008), knowledge of 'how to choose, make, and use mathematical representations' belongs to SCK. The category 'nature of mathematics' included utterances such as 'In mathematics you don't prove absolute truths. Instead, the proofs are based on chains where assumptions lead to something.' These can be also seen in connection to HCK, but as these comments were general, they were coded as CCK. Additionally, some discussion of the curriculum, such as 'Does knowing probability require knowing set theory? I suppose so. It would be good if there would be more of that in secondary school', were categorised as SCK instead of KCC. These comments also seemed connected to HCK, but were more focused on rethinking school mathematics and were consequently categorised as SCK.

Common content knowledge (66; 25 %)	Specialised content knowledge (158; 59 %)	Horizon content knowledge (46; 17 %)
<ul style="list-style-type: none"> • Giving a list of concepts (20) • Giving a definition (11) • Giving a theorem (9) • Explaining a property of a mathematical entity (8) • Giving a solution strategy (8) • Giving alternative definitions (7) • Discussing the nature of mathematical knowledge (6) • Giving a mathematical example (2) 	<ul style="list-style-type: none"> • Discussing representations of mathematical content given in textbooks (72) • Discussing alternative representations of mathematical content (65) • Discussing relationship between mathematical knowledge and curriculum (11) • Going through some history of mathematics (6) • Giving and discussing matriculation examination tasks (5) • Reflecting on a mathematical example (2) • Modifying an example (1) 	<ul style="list-style-type: none"> • Discussing the hierarchical relationship of mathematical concepts (28) • Giving an application of a mathematical entity or method (18)

Table 1: Subcategories of subject matter knowledge distinguished in the diaries

Common content knowledge was mainly discussed in terms of giving a list of concepts (related to the subject), giving a definition of a concept (such as limit) or giving a theorem (such as 'If function f is derivative, then function f is continuous'). This discussion was typically limited to telling the facts and no explanations or proofs were given. The specialised content knowledge typically focused on discussing the representations of mathematical content. Only one student adapted an example so that different versions of a problem were considered. The least discussed aspect of subject matter knowledge was horizon content knowledge; only 46 units of observation included discussion of the

hierarchical relationship of mathematical concepts or an application of a mathematical entity or method. Overall, discussion of subject matter knowledge was somewhat focused on SCK. More specifically, discussing the different representations of mathematical content was common in many diaries.

Pedagogical content knowledge

The distinguished subcategories of pedagogical content knowledge are given in Table 2.

Knowledge of content and curriculum (106; 27 %)	Knowledge of content and students (133; 34 %)	Knowledge of content and teaching (148; 38 %)
<ul style="list-style-type: none"> • Upper secondary school curriculum (57) • University curriculum (51) • Comprehensive school curriculum (17) • University of applied sciences curriculum (6) • Vocational school curriculum (2) 	<ul style="list-style-type: none"> • Difficult content for students (45) • Student competence (28) • Student knowledge (23) • Learning process (18) • Misconceptions (10) • Affect (7) • Solving strategies (3) 	<ul style="list-style-type: none"> • Ways to approach mathematical content in teaching (87) • Teaching methods (54) • Encouraging students (5) • Differentiation (2) • Answering student questions (1) • Correcting misconceptions (1) • Mathematical language and notation (1)

Table 2: Subcategories of pedagogical content knowledge distinguished in the diaries

In many diaries, the secondary school and university curricula were compared. Students discussed, for instance, the content of secondary school calculus courses and university analysis courses. The *knowledge of content and students* sections mainly focused on difficulties or misconceptions that school students may have. For example, affect and learning (e.g., emotions) were little discussed and the cognitive studies in mathematics education were mainly used as references. The *knowledge of content and teaching* focused on discussing different means of approaching mathematical content in teaching and different teaching methods. For example, no imaginary learning situations were introduced and only one student pondered the answering of school students' questions.

Overall, PCK was discussed more than subject matter knowledge. However, the PCK typically discussed in the diaries can be described as content-driven, as it was mainly placed in subcategories such as 'Ways to approach the mathematical content in teaching' or 'Difficult content for students'.

Discussion and conclusion

It is worth noticing that the results of this study cannot be generalised to whole student populations or other contexts. This study contributes only case-specific information, which can, however, be used in further development of the specific learning environment. In addition, the reliability of this study could be enhanced by using two independent researchers in the data analysis phase. Nevertheless, this study found that in this specific context, many prospective mathematics teachers adopted a *knowledge transforming writing* strategy in their learning diaries. The knowledge discussed in the diaries was somewhat focused on SCK and PCK. More specifically, the most discussed topics were representations of mathematical content, curricula, student knowledge and teaching methods.

Some of the learning diaries discussed both subject matter knowledge and pedagogical content knowledge. Some of the diaries, however, were more focused on pedagogical content knowledge. This seems to indicate that some of the prospective teachers emphasised pedagogical topics, while other prospective teachers discussed teacher knowledge more comprehensively. Further research would be needed to discuss this variation in detail and, especially, to compare students who are at different stages of their studies. In addition, it is notable that horizon content knowledge was rarely discussed in the diaries. This was somewhat surprising as the aim of the seminar was to connect the content of university-level mathematics and school mathematics. If HCK is understood as Jakobsen et al. (2013) have presented it, connecting mathematics as a discipline to school mathematics means discussing horizon content knowledge. In addition, some aspects of SMK and PCK (such as modifying tasks) also received little attention. This implies that learning diaries may lead to reflections that are not fully aligned with the intended learning outcomes. Further research is needed to determine whether more structured learning diary tasks would help students to better discuss desired sides of mathematical knowledge for teaching.

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