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► **To cite this version:**

Lisnet Mwadzaangati. An exploration of challenges of engaging students in generative interaction with diagrams during geometric proving. CERME 10, Feb 2017, Dublin, Ireland. hal-01949145

**HAL Id: hal-01949145**

**<https://hal.science/hal-01949145v1>**

Submitted on 9 Dec 2018

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# An exploration of challenges of engaging students in generative interaction with diagrams during geometric proving

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*This paper reports from a single qualitative case study which investigated challenges that might arise in a geometric proof lesson involving generative interaction with diagrams. Data consists of video recordings from a 120 minute lesson, taught with a focus on geometric proof in a Malawian grade 11 secondary school classroom. A post-lesson interview was also conducted with the teacher soon after the lesson. The findings indicate that the teacher faced challenge of lack of time, hence he did not complete some planned activities. The students faced the challenge of lack of understanding of the problem. As a result, they were unable to devise a correct plan during their initial involvement in the generative activity. The findings suggest that both the teacher's and students' challenges would have been avoided if problem solving was conducted appropriately.*

*Keywords: Geometric proving, generative interaction with diagrams, empirical activity, geometric diagrams, problem solving.*

## Introduction

Malawi National Examinations Board (MANEB) chief examiners' reports show that students fail to construct geometric proofs because they do not interact with diagrams successfully (MANEB, 2013). The report attributes students' failure to construct geometric proofs to lack of teacher knowledge for engaging students in successful proving activities. The reason suggested by MANEB supports the argument that teachers are responsible for engaging students in activities that involve interaction with diagrams during proof construction (Herbst, 2004). In geometry, to prove means to construct a sequence of argumentation from X (hypothesis) to Y (conclusion) with supportive reasons (Cheng & Lin, 2009). Herbst (2002) describes the work of geometric proving as a didactical contract between a teacher and his/her students. In this contract, the teacher's responsibility is to provide the problem statement and a diagram which contains the givens and the unknown. The students' responsibility is to develop logically connected statements from diagrams by making appropriate geometric interpretations and relationships. Herbst (2004) proposed four distinctive ways of thinking about how students interact with geometric diagrams during proving. These are empirical, representational, descriptive, and generative modes of interaction. Empirical interaction supports hands on geometry in the sense that the student is free to make a variety of operations on the diagram (measuring, looking at, and drawing in the diagram). Representational interaction supports abstract geometry, in the sense that the student is restricted by prescribed rules when making operations on the diagram. In descriptive interaction, the diagram contains features like marks and labels. Students use the features to complete a proof. Herbst (2004) argues that the availability of features in a descriptive diagram reduces students' responsibility for producing the proof and portrays situations of doing proof as only learning of good logic rather than discovering of new mathematics. The author therefore proposes that teachers should engage students in generative activities by using generative mode of interaction with diagrams during geometric proving. In this mode of interaction with diagrams, students are authorised to anticipate operations and results on

the diagram. Depending on the given information and the anticipated result, students can add features like drawing in lines and labels into the diagram. This means that generative interaction with diagrams is supposed to involve exploratory teaching strategies. Despite acknowledging that it is not easy to involve students in generative activities, the studies by Herbst have not focused on clarifying the challenges that might arise during such activities. Likewise, to my knowledge, no study has been done in Malawi to examine challenges of involving students in different modes of interaction with diagrams during geometric proving. Therefore this study has both a local relevance to teaching of geometric proving in Malawi and general relevance to the field of teaching of geometric proofs. The study addresses the following research question: What are the challenges of engaging secondary school students in generative interaction with diagrams during geometric proving?

### **Theoretical framework**

Several investigations have been conducted with an aim of addressing challenges of teaching and learning geometric proof construction. Studies by Jones and his colleagues aimed at developing strategies for teaching proof construction with focus on helping students to understand the proof and appreciate its discovery function (Ding & Jones, 2009). The focus on value of geometric proofs arose because the authors found that there were some students who were able to construct proofs but could not appreciate its discovery function in mathematics (Jones et al., 2009). The authors therefore propose for a shift to exploratory pedagogical strategies. One of these teaching strategies is problem solving approach which involves four stages; understanding the problem, devising the plan, carrying out the plan, and looking back (Polya, 1945). For a geometric proof problem, understanding of the problem involves understanding of the hypothesis and the conclusion. Hypothesis is the given information and conclusion is the statement to be proved. Polya (1945) suggests that when a proof problem is connected to a figure, the stage of understanding the problem must involve helping the students to draw the figure, to introduce suitable notations, and to label in the diagram the hypothesis and conclusion. Devising the plan involves finding the connection between hypothesis and conclusion, deciding on the theorem to use, and making decisions whether to introduce auxiliary elements into the diagram to enable proving (Polya, 1945). Carrying out the plan involves writing of the proving statements logically, each statement accompanied by a valid reason. Looking back stage includes reviewing the solution and checking if the arguments can be derived using a different approach. This study agrees that problem solving strategy can provide opportunities for students to appreciate the value of geometric proving. In addition, this study argues that students' opportunities to involve in generative interaction with diagrams can be enhanced through exploratory activities. This means that teachers require knowledge of problem solving strategy to engage students successfully in generative interaction with diagrams. This study was therefore guided by Polya's (1945) problem solving framework in analysing the data.


### **Methodology**

The study was conducted using qualitative case study design because the goal was to expand understanding of social issues in their context (Yin, 2009). The study was conducted on one Malawian secondary school teacher and one lesson. The teacher, Kim (pseudonym) is regarded as one of the best teachers due to his long teaching experience and because his students perform well

in mathematics during national examinations. Kim was selected for the study on assumption that conducting research on an experienced teacher could offer an opportunity to study the issue in depth. The lesson episode analysed for this study is part of the video data that was collected for a larger project which aims at studying knowledge for teaching geometric proofs. The lesson episode was considered for analysis because it involved moments of generative mode of interaction with diagrams as well as problem solving teaching strategy. Post-lesson interview was conducted and audio recorded in the teacher's office soon after recording the lesson. The teacher was mainly asked to explain his views about the lesson in terms of what went well and what did not go well during the lesson. Although the data for the study is from one lesson, it is considered to be sufficient for illustrative purposes because it was generated from a real-life context (Yin, 2009). The empirical material, both from the video recording of the lesson and the audio recording of post-lesson interview were transcribed and further analysed separately by using thematic analysis. The aim of thematic analysis was to capture and interpret sense and substantive meanings in the data (Ritchie, Spencer & O'Connor, 2004). Polya's (1945) stages of problem solving were used as a priori themes for analysing the data. The transcribed data was read several times to understand it and to identify moments that were related to a particular stage of problem solving. The findings from the two types of data are discussed under each theme for purposes of comparison. Due to space limitations, this paper has mainly discussed findings related to the first two stages of problem solving; understanding the problem and devising the plan.

## Findings and discussions

Kim started the lesson by writing a geometric statement and making a drawing of a diagram on the chalkboard. He told his students that the aim of the lesson was to prove that an angle subtended by an arc at the centre is equal to two times an angle subtended by the same arc at the circumference. After explaining the lesson aim, Kim asked his students to go into their usual small groups to draw a similar diagram and discuss how to prove the theorem. There were six groups in the class and each group contained five to eight students. Table 1 shows the diagram that was drawn and the statement that written by Kim on the chalkboard.

	<p>Given: a circle with centre O, with arc AB subtending angle AOB at the centre and angle AMB at the circumference. Prove that the angle at the centre is twice the angle at the circumference.</p>
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**Table 1: Diagram and statement given to students for proof construction**

It can be argued that the problem statement in table 1 is ambiguous because the teacher has not specified the two angles whose relationship is to be proved. There is one angle at the circumference referred by the statement, but there are two angles at the centre (reflex AOB and obtuse AOB). Furthermore, the diagram in table 1 did not contain any features to indicate the required angles. This means that during group discussion, students were challenged to decide whether to relate the angle

at the circumference to the reflex angle or the obtuse angle at the centre. As such, the diagram required generative mode of interaction. Hence the discussion activity involves problem solving.

After about 10 minutes, Kim moved around to check what students were discussing and doing in the groups. The following lesson segment 1 presents a dialogue between Kim and students in group 6.

- Kim: Okay so what are you going to do, have you discussed?
- Student 1: Yes, we will join MO and prove that these two triangles (pointing at triangle AMO and BMO are congruent). Then relate the corresponding angles.
- Kim: Can you show me how you will relate the angles.
- Student 1: First,  $AO = BO$  (radii),  $OM$  is common, and  $AM = BM$  (third side)  $\triangle AOM$  is congruent to  $\triangle BOM$ . Then angle  $MAO = MBO$ , the two angle here are also equal (pointing at the reflex angle at  $AOB$ ) and the two angles here are equal (pointing at  $M$ ). (The student is silent).
- Kim: Go ahead.
- Student 2: Then we add angles here (pointing at the reflex angle at  $AOB$ ) and angles here (pointing at  $M$ ) uhhh.... (silence 4 seconds).
- Kim: Yes go ahead what about the other group members, how do you proceed from here to the theorem? (silence for 4seconds) how do you relate the two angles? (silence 6 seconds), how do you arrive at the question that you have been asked using that theorem? (silence 4 seconds). Do you know the angle at the centre referred in the theorem?
- Student 3: Yes this one (pointing at the reflex angle at  $AOB$ ).
- Student 4: No this one (pointing at the obtuse angle at  $AOB$ ).
- Kim: Can you try to measure the angles and see if it is the upper or lower angle which is twice the angle at  $M$ ? After that think of another way, this one might not work.

Then Kim went to check students in other groups and asked questions. When he noticed that most of the groups were not focusing on a correct angle at the centre, Kim interrupted the generative activity and asked all groups to measure the three angles and relate their values to find out the correct angles. The activity took about 15 minutes, students measured the angles and made comparisons. The following lesson segment (segment 2) is a continuation of a conversation between Kim and group 6.

- Student 3: This angle (pointing at angle  $AMB$ ) was  $52^\circ$  while this one (pointing at obtuse angle  $AOB$ ) was  $104^\circ$ . So this (pointing at obtuse angle  $AOB$ ) is twice this (pointing at angle  $AMB$ ).
- Kim: Okay so how are you going to prove the theorem?
- Student 5: We tried similarity but we found that it was going to be difficult as well because it was not saying anything about this angle (pointing at obtuse angle at  $O$ ) it was only saying about this one (pointing at the reflex angle at  $O$ ). So since this angle is outside these two triangles, we agreed to use the property of exterior angle of

triangle. So we extended MO to N to create exterior angles here (pointing at obtuse angle at the centre).

## Understanding the problem

Lesson segment 1 shows that the students agreed to add a feature (auxiliary line) into the diagram by joining MO to form two triangles (AOM and BOM). The students also agreed to construct the proof for the theorem by firstly constructing an in-between proof of congruency of triangles AOM and BOM. This means that in lesson segment 1, the students were devising a plan for the proof. Depending on geometry background at their level, there are two possible approaches that the students could use in developing their plan. The first approach involves joining of MO, then forming two equations using properties of isosceles triangles, sum of interior angles of a triangle and sum of angles at a point, and finally making substitutions to reach the conclusion. The second approach involves joining MO and extending it to some point within the circle to form exterior angles of triangles AOM and BOM, and then use properties of isosceles triangles and exterior angle of a triangle to form two equations, and finally make substitutions. This shows that congruency theorem is not appropriate for both approaches. As such, the students' decision to join MO and then use congruency theorem as an in-between proof was not correct. It can be argued that the students' decision was based on lack of understanding of the problem. This argument is based on several observations. Firstly, in segment 1 the students were unable to explain how they would use the congruency theorem to connect to the conclusion. But segment 2 indicates that after realising that the angle at the centre is the obtuse angle, the students changed their plan, and they were able to justify their decision to use property of exterior angle of a triangle. Secondly, the argument is based on students' disagreements regarding angle at the centre in segment 1. Some students pointed at the reflex angle while others pointed at the obtuse angle when Kim asked them to identify the angle at the centre referred by the theorem. This means that the students were not sure of the correct angle at the centre. The disagreement among the students regarding the angle at the centre provided an opportunity for Kim to shift students' focus from the stage of devising the plan to the stage of understanding the problem. The findings from the lesson segments are supported by the following extract in which Kim expresses the challenges that students faced when constructing the proof:

It was very difficult for the students to come up with relevant constructions when proving. Because I think the main trick in the proof was to know what type of construction and in-between theorem to use for the proof. But after doing the measurements, the students were able to make correct construction and to know the theorem to use. Their reasoning and their work showed that they now understood the statement they were asked to prove.

The extract shows that Kim realised the difference in the students' ability to come up with a correct plan before and after the empirical activity. Kim mentions two challenges that the students faced before the empirical activity. The first challenge involved failure to decide on features to be added to the diagram. The second challenge involved failure to use the diagram to generate a correct in-between theorem and proof. Both challenges involved generative interaction with diagrams as they required students to make explorations with the diagram (Herbst, 2004). The extract also indicates that the students were able to devise a correct plan upon understanding the problem. Cheng & Lin (2009) call the proving of an in-between theorem as construction of intermediary condition (IC).

These authors argue that students can only construct correct ICs if they understand the hypothesis and the conclusion. Polya (1945) advises that solving of any mathematical problem should not be started unless the problem is well understood. Probably, the challenge of understanding the problem could have been avoided if Kim carried out the phases of problem solving before going to the class (Polya, 1945). Through this preparation activity, Kim could have anticipated that students were more likely to regard the reflex angle as angle at the centre due to its location. Thus the reflex angle is close to and in the same quadrilateral (AMBO) with the angle at the circumference. In so doing, Kim could avoid the mistake by helping the students to understand the theorem (which also implies to understand the problem) before asking them to discuss how to develop its proof.

## Devising the plan

Both lesson segments show that students devised the plan for the proof in their groups through exploration. Lesson segment 1 shows that the first attempt to devise the plan was not successful due to lack of understanding of the problem. Lesson segment 2, shows that after the empirical activity, students came up with a proper construction and correct IC for linking angle at the centre and angle at the circumference. The students' suggestions to introduce a line into the diagram confirm that they were involved in generative interaction with the diagram. Lesson segment 2 also shows that the students were able to evaluate their ideas by considering the questions asked by Kim in lesson segment 1. The questions are, "how do you relate the two angles? How do you arrive at the question that you have been asked using that theorem?" The questions helped the students to focus on identifying a construction and theorem that could help them to connect the given information to the conclusion. This is observed in utterance by student 5 who explained that they tried to use similarity theorem but they realised that it was not appropriate because it could not help them to link angle at the circumference and the obtuse angle at the centre. The technique of asking questions that probe students' thinking is regarded as one of the strategies for helping students to devise a plan (Polya, 1945). The findings from analysis of students' explanations in lesson segment 2 show that the empirical activity which focused on understanding angle at the centre helped the students to devise a correct plan for the proof. This observation is also confirmed in Kim's explanation regarding what went well during the lesson. Kim explained that the lesson was generally successful because the students were able to construct the proof independently. Kim explained that his lesson objective was to help the students to understand the theorem and construct its proof on their own. He explained as follows:

If you just start proving without engaging students in an activity like measuring or discussions on how to prove, they just memorise the proof. So to avoid memorisation, I involved the students in discussions. When I found that they were referring to a wrong angle at the centre, I did not tell them the angle, I wanted them to find out on their own by measuring the angles.

The extract shows that during the first activity Kim regarded the students' inability to construct a correct proof as an opportunity for them to understand the theorem which is also the statement problem in this case. Kim's idea of giving students opportunity to do explorations on the problem to be proved is supported by Ding & Jones (2009). But Kim was not supposed to wait until students got stuck in order to suggest the empirical activity. According to Polya (1945) the teacher is

supposed to prevent students from answering a question that is not clear to them, and from working for an end that they do not desire. The author argues that students might be frustrated if they either get stuck or come up with undesirable solution to a problem. As such, teachers are supposed to avoid making students' frustrated by ensuring that they understand the problem before beginning to devise its plan. This suggests that Kim was supposed to first of all engage the students in an exploration activity that could help them to understand the problem before involving them in activity of discussing how to prove the theorem. By doing so, Kim could have avoided the challenge of lack of time that he pointed out when explaining what did not go well during the lesson. Kim explained that he had planned to discuss three examples of how to apply the theorem in solving different geometric problems, but he only managed to discuss one example because some time was spent on the unplanned activity of measuring angles. Apart from examples, Kim also explained that he had planned to give the students an exercise which he wanted to start marking during the lesson, but he turned it into homework due to lack of time. Kim seemed to have planned many activities for the lesson which involved problem solving. But Polya (1945) explains that problem solving might be time consuming because students explore different ways when devising a plan for finding solution of the problem. This means that a teacher is supposed to plan few activities for a problem solving lesson to ensure that students have ample time for explorations. The planning of many activities and the sequence of activities during the lesson indicates that although Kim used problem solving strategy, he was not aware of some of its skills. This agrees with Herbst's (2004) caution that engaging students in generative mode of interaction with diagrams is challenging.

## Conclusion

This study has found that both the teacher and students faced some challenges during the lesson of geometric proving which involved generative interaction with diagrams. The teacher faced the challenge of lack of time hence he did not complete some of the planned activities. The students faced a challenge of lack of understanding of the problem during their first attempt to devise a plan for the proof. Due to this challenge, the students came up with wrong construction and IC for the proof. As a result the students were unable to complete the plan for the proof during their first attempt to devise the plan. However the findings show that the students were able to devise correct plan for the proof after doing an empirical activity which focused on understanding the problem. The study implies that the challenges faced by both the teacher and the students could have been avoided if proper problem solving skills were followed. Thus the first activity could have focused on understanding the problem and the second activity could have focused on devising the plan for the proof. The findings suggest that successful involvement of students in generative interaction with diagrams require knowledge of several problem solving skills including proper planning and sequencing of activities. Lastly, the findings show that empirical interaction with diagrams enhanced students understanding of the problem. Further study is needed to explore whether successful involvement of students in generative interaction with diagrams require a combination with other modes of interaction with diagrams.

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### **Acknowledgement.**

This study was kindly funded by Norwegian Programme for Capacity Building in Higher Education and Research for Development (NORHED) under Improving Quality and Capacity of Mathematics Teacher Education in Malawi Project.