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# Anticipating students' thinking through Lesson Study: A case study with three prospective middle school mathematics teachers

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*This study aimed to examine prospective teachers' anticipations of students' thinking on the measure of the arc and measure of the central angle, circumference, and area of the circle and to explore the changes in their anticipations as they conduct three lesson study. For this purpose, case study method was used. Three prospective middle school mathematics teachers participated in the study and completed three lesson study. The data were analyzed in terms of three components: the prospective teachers' anticipations of (1) how students' would think; (2) what difficulties they would have; and (3) what powerful ideas they would have. The results showed that lesson study cycles with real classroom experience provided opportunities for the prospective teachers to develop anticipations of students' thinking.*

*Keywords: Lesson Study, prospective teacher knowledge, anticipating student thinking, middle school mathematics.*

## Introduction

Recently, teachers' knowledge of mathematics has become an object of concern. Ball and colleagues have expanded teacher knowledge proposed by Shulman (1986) by defining "the mathematical knowledge for teaching" (Ball, Thames, & Phelps, 2008). Pedagogical content knowledge (PCK) is part of the mathematical knowledge for teaching, and it focuses not only on content knowledge and pedagogical knowledge, but also on integration and transformation of content and pedagogy (Ball et al., 2008). According to Ball and others (2008), the components of PCK are knowledge of content and teaching (KCT), knowledge of content and students (KCS), and knowledge of content and curriculum (KCC). This study focuses on knowledge of content and students (KCS). Hill and others (2008) defined "KCS as content knowledge intertwined with knowledge of how students think about, know, or learn this particular content" (p. 375). Ni Shuilleabhain (2015) added that KCS includes students' understanding of content, student developmental sequences, typical student errors, anticipation of what students are likely to think or find confusing and common student computational strategies. Teachers should consider students' needs and interests when they plan for lessons. They must anticipate students' typical thoughts, ideas, and difficulties when selecting teaching materials and making decisions about the implementation of the lessons. Teachers must also attend to and interpret students' emerging and incomplete ideas during the instruction. Each of these tasks requires knowledge and skills developed through the interaction between mathematical understanding and knowledge of student thinking (Ball et al., 2008). Prospective teachers who can recognize and appreciate students' thinking and cognitive development could design and implement learning activities to meet students' needs and interest (Ball et Al., 2008; Llinares, Fernandez, & Sanchez-Matamoros, 2016).

However, their experiences and knowledge of students' thinking are very limited (Peterson & Leatham, 2009). Hence, teacher education programs should be designed to help prospective teachers improve students' mathematical knowledge and skills related to KCS by anticipating student thinking, among other abilities.

Lesson study is one of the models that helps teacher candidates develop their knowledge related to student thinking. It is a professional development program in which teachers collaboratively work on teaching. In this program, teachers first determine learning goals of the lesson and plan the lesson. Subsequently, one of the teachers in the group teaches a lesson and other group members observe the teaching process. Finally, they evaluate and revise the lesson plan so that it can be implemented second time (Lewis, Perry, & Hurd, 2009; Murata, Boffering, Pothen, Taylor, & Wischnia, 2012). In planning a lesson, the teachers are usually guided to focus on expected learning activities, expected student reactions or answers, teacher's responses to student reactions, and possible evaluation activities. This professional development model may provide prospective teachers with opportunities to plan, implement, criticize, and reflect on lessons collaboratively (Carrier, 2011).

Although research studies emphasize the importance of teachers' knowledge of student thinking, we know little about the development of prospective teachers' knowledge and skills in anticipating student thinking (Webb, 2006). Lesson study practices are found helpful in developing teachers' competence to anticipate students' mathematical thinking (Lewis et al., 2009; Tepylo & Moss, 2011). In this regard, the present study aimed to investigate prospective teachers' knowledge of student thinking of the concept of circle as a two-dimensional figure (disk) and as a one dimensional curve and to explore any changes in their anticipations as they work on three lesson study cycle.

### **Conceptual framework**

The theoretical basis of the lesson study model acknowledges that cognition is social and that learning takes place in enriched learning environments in a cooperative way (Fernandez, 2005). In this regard, lesson study is a professional development program that encompasses constructive learning and creates learning opportunities (Lieberman, 2009). As lesson study is based on planning before the practice, observations during the practice, and cooperation and reflection throughout the practice, it contributes to making 'learning' a cultural activity, which ultimately makes this model more significant (Dudley, 2013). According to Davies and Dunnill (2008), this model differs from other cooperative models in that the cooperation adopted in lesson study continues before, during, and after the practice. All these characteristics of lesson study model give teachers different perspectives and show them how an effective mathematics education should be offered (Erarslan, 2008). Researchers emphasize that integrating lesson study model into teacher education programs can help prospective teachers develop knowledge and competence by learning from practice (Rasmussen 2016; Sims & Walsh 2009). Lesson study enables prospective teachers to work on lesson plans collaboratively and to conduct careful observations of learning and teaching activities. The model also allows prospective teachers to discuss and reflect on their own practice (Fernandez 2010). It mainly helps prospective teachers gain curriculum knowledge and pedagogical content knowledge, including knowledge of common student mistakes. Observing lessons gives prospective teachers the opportunity to notice things about the classroom environment, and most importantly, to see the situation from the perspective of students (Lewis, 2002). Thus, they can become familiar

with what students actually know, how they think, and what they can do as well as with areas in which they may have difficulty (National Research Council, 2001, Takahashi, 2005).

## Methodology

This study focuses on investigating the prospective teachers' anticipations of students' thinking of the measure of the arc and angle and circumference and area of the circle. Furthermore, it explores how such anticipations change during the lesson study. For this purpose, case study design was adopted. This design provides an in-depth analysis of single or multiple cases by means of various data collection tools (Creswell, Hanson, Clark, & Morales, 2007).

## Participants and context

The participants of this study were three prospective middle school mathematics teachers enrolled in a state university in Ankara. They were in their last year of the program and were willing and motivated to participate in the study. They were 22 years old, and their GPAs were between 3.29-3.47 out of 4.00. The teacher education program generally focuses on content knowledge (i.e., mathematics) in the first and second years. On the other hand, during the third and fourth years, most of the courses are related to mathematics education. Prospective teachers who graduate from the program can work as mathematics teachers in middle schools (grades 5 to 8).

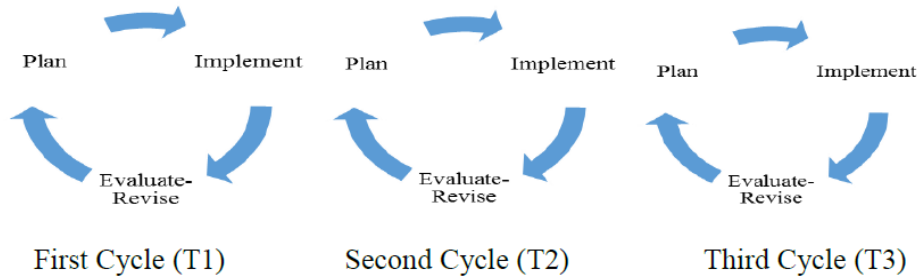
## Data collection

Within the scope of the three-week study, the prospective teachers worked as a group and planned, implemented, and revised three lessons (each of which lasted about 80-120 mins) for each learning objective on the topic of circle that is a part of the 7<sup>th</sup> grade math curriculum (MoNE, 2013). The prospective teachers used a lesson plan template to design the lessons. It consisted of four columns listing the (1) learning activities and key questions (and time allocation), (2) expected student reactions or responses, (3) teachers' responses to student reactions, and (4) goals and method(s) of evaluation (see column 2 of Table I).

Steps of the lesson: learning activities and key questions (and time allocation)	Student activities/ expected student reactions or responses	Teacher's response to student reactions / Things to remember	Goals and Method(s) of evaluation
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**Table 1: Lesson plan template**

Afterwards, one of the group members implemented each lesson plan in real classrooms. The implementation process was video recorded. Regarding the last stage, the first author and mentor teacher evaluated and provided feedback to group members. Subsequently, the group revised the lesson plan (see Figure 1). Moreover, prospective teachers were expected to write a diary about this process.



**Figure 1: Lesson study cycles**

The same cycle was repeated for each lesson plan. Prospective teacher 1 (T1) focused on the following learning objective: “Identifying central angles, their intercepted arcs, and the relationship between the measure of the arc and measure of the angle.” T2 focused on “Calculating the circumference of a circle and a segment of a circle.” T3 focused on “Calculating the area of a circle and a segment of a circle” (MoNE, 2013). The initial and revised lesson plans designed by the prospective teachers, the video recordings of the lesson study meetings, the video recordings of the lessons conducted, debriefing meetings after each lesson, and the observation notes and diaries taken by the prospective teachers, researchers, and training teacher during the implementation of the lesson provided the data for this study.

### Data analysis

The data were coded based on the themes from the relevant literature (Ball, et. al., 2008; Fernandez & Chokshi, 2002; Hill, et. al, 2008; Schoenfeld, 1994) as well as from the participants’ responses. For this purpose, researchers examined the prospective teachers’ articulations and behaviors during study lessons and searched for the incidents showing their anticipations of (a) student thinking, (b) difficulties that students would have, and (c) students’ powerful ideas. Anticipations of how students would think involve predicting how students think in general, what deductions they can make, and what types of connections they can make. Anticipations of students’ difficulties involve predicting the challenges, mistakes, and misconceptions of students regarding the concept of circle. Lastly, anticipations of the powerful ideas that students might have involve expecting the key ideas about the relevant concepts. The data were examined based on this analytical framework for each lesson study cycle, and the findings were compared across three lesson study to examine any changes observed throughout the study.

### Findings

This section presents the prospective teachers’ anticipations of (1) how students would think, (2) the difficulties that students would have, and (3) the powerful ideas that students would have, as teachers design and implement three study lessons on the concepts of the measure of the arc, measure of the angle, and circumference and area of the circle. In Table 2, frequencies represent prospective teachers’ anticipations related to each component of student thinking observed for each study lesson.

Table 2 shows that the prospective teachers pointed out that 26 different thoughts could be considered as typical for students (i.e., component 1) in the study lesson. For instance, they expected

that students could explain central angles as the midpoint angle of the circle, or they anticipated that students would know how to draw a circle using different tools, such as a coin or a compass.

	Types of plan	How students' would think typically	What difficulties they would have	What powerful ideas they would have
First	Lesson plan	10	2	3
Cycle	Revised lesson plan	10 (0 new anticipation)	3 (2+ 1 new anticipation)	3 (3+ 0 new anticipation)
Second	Lesson plan	8	-	5
Cycle	Revised lesson plan	9(8+ 1 new anticipation)	1 (0+ 1 new anticipation)	6 (5+ 1 new anticipation)
Third	Lesson plan	7	5	6
Cycle	Revised lesson plan	7 (7+0 new anticipation)	5 (5+ 0 new anticipation)	8 (6 + 2 new anticipation)

**Table 2: Frequency of prospective teachers' thoughts related to the components of anticipating student thinking**

Among these 26 different anticipations, only one new anticipation was included in the revised plan after its implementation in a real classroom. More specifically, in the second study lesson, they thought that the students would quickly give the correct answer when they were asked to spot the circumference of the circle. However, since the students pointed the region inside the circle as the circumference of the circle, the prospective teachers included such typical student thinking in the revised lesson plan and made some revisions, as illustrated in the following dialogue:

T2: Some students conceived the circumference of the circle as inside of the circle. We had never considered this.

T1: Yes, I didn't know what to say when they gave this answer.

T3: So, we need to include some questions in the lesson plan. For example, shall we say 'What do you think of when we say 'circumference of the school ground?' Then we can tell them to walk around the school ground.

T2: Let's decide a starting point and tell them to start walking from there and walk around the school ground until they reach the same point.

T1: Yes, then students would realize that circumference is not actually the same as inside [of the circle]. Then we can ask them to think about the circumference of a circle.

According to Table 2, the prospective teachers reported 9 different student thoughts could be considered difficult (i.e., component 2) in the study lesson. For example, they thought that students might confuse the concepts of circle and sphere. They also anticipated that students might consider meter as a unit of arc. Among these 9 different anticipations, two were included in the revised plans after its implementation in real classroom. In the first cycle, during the implementation of the lesson, the prospective teachers realized that students would confuse the central angle with inscribed angle. Thus, in the revised lesson, they decided to remind the students about the difference between these two types of angles. The following dialogue shows how they considered this idea as they were revising the lesson plan:

T1: Did you see that students confused central angle with inscribed angle?

T2: Yes, it seems that they didn't know the difference. We never thought about it. So, what shall we do? Let's draw a figure and ask them if it's a central angle. If they say 'yes', we'll use a material that shows an angle as a combination of two rays (half lines). Here, we can make a straight angle and ask the students what type of an angle it is.

T1: Then, students would say it's a straight angle based on what they learnt it in the previous lesson. Let's make it a 360-degree angle and draw a circle taking the origin of the angle as the center and ask what type of an angle it is.

T2: The students would say "It's a central angle because the origin is at the center of the circle."

T3: Yes, then we can show the example with an inscribed angle and ask if it is a central angle. I think students will say it's not because the origin does not cross the center.

T1: This way, they can differentiate between a central angle and an inscribed angle.

As seen in Table 2, the prospective teachers pointed out 17 different student thoughts could be considered as powerful ideas (i.e., component 3) in the study lessons. For instance, they expected that students could explain differences between circle and disk, and they anticipated that students would know what pi is. Among these 17 different anticipations, three new anticipations were included in the revised plans after their implementations in real classroom. In the third cycle, the prospective teachers asked students to find the area of a circle by using the area of a parallelogram. However, following the lesson, they realized that the instructions in the activity sheet were not clear enough to guide the students. This experience led them to contemplate about their expectations concerning students' powerful ideas (i.e., making connections between the area of a circle and of a parallelogram) based on which they made some changes to the instructions. The following dialogue illustrates how their expectations changed:

T1: The activity sheet was not very clear, so students did not understand the relationship between the area of a disk and the area of a parallelogram. We wanted to make it easier using what they learnt in quadrilaterals, but it didn't work. What shall we do?

T3: I think we should say that circle doesn't look like a quadrilateral. Then they can notice it. Later, we can ask what part of a disk and the height of a parallelogram look similar. They might say 'the radius of disk'. But, what's important here is whether they can tell, which part of a disk has a relationship with the base length of a parallelogram.

T2: Yes, so let's include this in the lesson plan.

To sum up, the data analysis showed that the prospective teachers identified various anticipations related to the three aspects of student thinking. They thought about how students develop ideas related to the fundamental elements of a circle as well as the circumference and area of a circle. They also recognized the importance of designing the lesson in the light of powerful ideas. In addition, they successfully anticipated different student difficulties and mistakes. The study lesson also help the prospective teachers produce new anticipations of student thinking after implementing and reflecting on the lessons. The prospective teachers might not have thought about these issues if they had not had a chance to implement and revise study lessons.

## Conclusion and results

The findings showed that prospective teachers' anticipations of student thinking for each study lesson varied. The study lesson provided the prospective teachers an opportunity to consider student thinking as an essential part of the planning. They were expected to work collaboratively and think deeply about students' thinking (typical student responses, misconception, powerful ideas, prior knowledge, and understandings, etc.) as they plan their lessons. The lesson plan template guided prospective teachers to document their ideas about student thinking, making them explicit and the object of discussion. In this way, study lessons created a context for discussion about student thinking.

Furthermore, the lesson study allowed them to develop knowledge and skills related to student thinking through powerful experiences in real classrooms. Such opportunities allowed them to get to know students well and analyze the learning process from students' perspective. Some previous studies reported similar results (e.g., Ni Shuilleabhain, 2015; Webb, 2006). In this regard, lesson study could be used as a model in undergraduate programs to help the prospective teachers develop knowledge and skills related to student learning.

Even though prospective teachers' anticipations of student thinking were observed for each study lesson, a clear development through the three cycle was not observed. In this study, each cycle focused on learning objectives involving different concepts and skills (e.g., identifying central angles and calculating circumference and area of a circle). Such differences might have influenced prospective teachers' anticipations. Another reason for not being able to detect a clear development through three cycles might be related to the analytical framework used in the study. To reveal different aspects of development of teachers' anticipations, a more structured and detailed analytical framework could be used.

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