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Self-scaffolding students' problem solving: Testing an orientation basis

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It is known not only that young students have difficulty solving mathematical problems but also that appropriate scaffolding can support them in the process. In this paper we describe the development and pilot implementation of a device for self-scaffolding grade 6 Catalan students' mathematical problem solving. Called an orientation basis (OB), the OB, which addresses cognitive, metacognitive and affective aspects of problem solving, drew on research describing the actions of an expert problem solver. The evidence indicated that the OB, while in need of refinement, had a positive impact on the problem solving behaviours of many participant students.

Keywords: Mathematical problem solving, scaffolding, orientation basis, Catalonia.

Introduction

Mathematical problem solving is difficult, both for students (Mason, Stacey & Burton, 1982; Pólya, 1945) and teachers trying to create appropriately conducive classroom environments (De Corte & Verschaffel, 2004; Schoenfeld, 2013). When children solve problems without being conscious of the relationship between their actions and their solutions their ability to transfer their solution processes to new situations will be limited (Coltman, Petyaeva & Anghileri, 2002). However, appropriate adult intervention can help children become aware of these processes (Coltman et al., 2002). Such interventions, known as scaffolding, build on what learners already know in order to close the gap between current learner competence and task objective (Bruner, 1985; Greenfield, 1984; Wood, Bruner & Ross, 1976). Moreover, given time, scaffolding can be provided by peers and, ultimately, students themselves (Holton & Clarke, 2006). In this paper we describe the development and use of a device, which we have called an orientation base (OB), for use in Catalan sixth grade classrooms. The OB's role is to support the transition from where teachers scaffold learner's problem solving to where students scaffold their own.

Problem solving

As a human activity, problem solving can be understood as an example of goal-directed behavior (Schoenfeld, 2007). It is a dynamic, but not necessarily linear, activity requiring the organization and activation of multiple skills and strategies (Mason et al., 1982; Pólya, 1945). At the heart of problem solving lies an appropriate mathematical knowledge, an awareness and experience of solution strategies, self-regulatory or metacognitive competence and a belief not only that the problem is worth solving but also that the solver can solve the problem (De Corte, Verschaffel & Op't Eynde, 2000; Schoenfeld, 2007, 2013).

A key aspect of the above, at least from the perspective of providing scaffolding for the learner, is the encouragement of students' metacognitive competence. For example, expert solvers spend more time understanding and analyzing the problem and solution process than calculating, and they continuously reflect on the state of the problem solving process (De Corte et al, 2000), behaviours that are typically absent with weak problem solvers (De Corte et al., 2004). Such students need

scaffolded support with respect to interpreting a task, identifying its sub-objectives and planning a strategy (De Corte et al., 2000; Mason et al., 1982). They need to learn how to reflect on their existing knowledge and thought processes; that is they need to learn how to evaluate and regulate their own thinking (Sanmartí, 2007). This regulative competence is not acquired automatically but emerges over time (De Corte et al., 2004). Thus, with support in understanding how things work, students can become more efficient and self-regulated problem solvers (Schoenfeld, 2013).

Scaffolding

Scaffolding in an educational context

Drawing on Bruner's (1975) initial observations with respect to the ways that parents scaffold their infants' learning, Wood, Bruner and Ross (1976) argued that knowledgeable adults can scaffold students' problem solving activity. Here, the adult seeks to reconcile implicit theories of the task components, the necessary steps to solution, and the child's capabilities (Stone, 1998). In this way, acknowledging a socially imitative process, six ways of assistance were differentiated; recruiting the child's interest, reducing the degrees of freedom, maintaining goal direction, highlighting critical task features, controlling frustration and modelling preferred solutions paths (Wood et al., 1976). Recent work has continued this theme, examining how teachers can best provide (temporary) support that enables learners to complete tasks they would otherwise not have been able to complete independently (Smit, van Eerde & Bakker, 2013; van de Pol, Volman & Beishuizen, 2010). In this process, whereby the learner becomes incrementally independently functional (Smit et al., 2013), both teacher and learner actively share and build common understanding (Stone, 1998; van de Pol et al., 2010).

Scaffolding strategies

Scaffolding is not a 'technique' that can be applied in every situation in the same way (van de Pol et al., 2010). As in the construction industry, where each scaffold is unique to a specific building, learning scaffolding can be provided at different ages and in a variety of ways, addressing learners' knowledge gaps as part of an ongoing progress (Wood et al., 1976). Significantly, effective scaffolding is thought to comprise three components, involving the six processes of feeding back, giving hints, instructing, explaining, modelling and questioning (van de Pol et al., 2010), which are

- Contingency: Support should be adapted to the student's current level of performance.
- Fading: Support is gradually withdrawn over time.
- Transfer of responsibility: Task completion is gradually transferred to the learner.

Moreover, effective scaffolding not only promotes learners' cognitive and metacognitive activities but also positive affect. Finally, acknowledging different agents in the process, whether they are informed adults, a group of learners or the individual student, scaffolding is progressively relocated to the learner, whereby the external dialogue of scaffolding becomes the inner dialogue of metacognition (Holton & Clarke, 2006).

Orientation basis for problem solving

One means of encouraging self-scaffolding of students' problem solving-related self-monitoring skills is to use an orientation basis (OB) (Sanmartí, 2007). Here we understand a problem solving-

related OB to be a necessary sequence of actions based on the problem solving behaviour of experts. An orientation basis leads the learner to a solution in ways that structure an emergent independence and problem solving autonomy. An OB is not a ‘one size fits all’ tool but tailored according to learners’ requirements and achievements. At every age and according to the learner’s needs, an OB can be presented through different statements.

Dimensions	Actions	Track
I understand the problem	A1. I have read the question twice, at least.	
	A2. I understand what the question wants.	
	A3. I have identified and understood the data.	
I devise a plan	A4. I have played with the data from the question.	
	A5. I have prepared a strategy.	
	A6. I have checked that my strategy fits the data.	
I apply my plan	A7. I have implemented my strategy.	
	A8. I have recorded all my actions in ways that I understand.	
	A9. I have recorded all my actions in ways others can understand.	
I review my task	A10. When I get stuck I go back to the beginning.	
	A11. When I have finished I have checked my answer(s).	
	A12. I have checked for other answers or better solutions.	

Table 2: The orientation basis (OB)

In this paper we discuss the development and implementation of an OB for grade 6 Catalan pupils. At this age, pupils are typically expected to have acquired a minimum background in problem solving. However, experience has shown that they lack regulative and problem solving competence, especially in understanding and analyzing the problem, and planning and implementing a solution process. Therefore, drawing on Pólya’s (1945) problem solving principles, the OB depicted in Table 2 was developed. Each of Pólya’s four dimensions comprised three particular actions, which can be tracked in the right hand column. The inclusion of each action was a consequence of earlier observations of the problem solving behaviours of grade 6 Catalan pupils and the problem solving strategies found in the literature (e.g. De Corte et al., 2004; Mason et al., 1982). The OB shown in Table 2, translated from the original Catalan, was designed to be a contingent, hint-giving, feedback tool focused on facilitating both fading and transfer of responsibility (van de Pol et al., 2010). As indicated above, the aim of this paper is to present an initial evaluation of the efficacy of the OB shown in Table 2 for scaffolding grade 6 students’ mathematical problem solving.

The study

The participants were students in a 6th-grade class of a Barcelona primary school. Their teacher was an experienced generalist primary school teacher. Such teachers, who receive relatively little subject knowledge instruction during their pre-service education, typically acquire their mathematical knowledge in practice, a situation much criticised (Egido, 2011; MECED, 2012). Data were collected during a regular, 50 minute, lesson at the end of the second quarter of the academic year 2015-2016. They derived from 22 students’ initial use of the OB as they tried to solve the mathematical problem posed in Figure 1, which was originally posed in Catalan.

Problem: My grandmother's blanket

My grandmother is a crochet expert. She loves doing crochet blankets. I haven't seen yet the last one she made, but she told me how she did it: "I joined squared pieces. Their styles are different but all of them have the same size."

She explained me that the boundary of the blanket is made with a dozen of green squares with a white rhombus, and with a dozen of white pieces with green dots. She set them up alternately: one green piece with rhombus, one white piece with green dots,... and she managed so that in the four corners there is always a green dotted blank piece.

She also told me that the interior part of the blanket is really different. It is made with different coloured pieces, but all of them have the same size of the external ones.

So,

1. How could my grandma distribute the pieces to make the boundary of the blanket?
Explain how you know it.
2. I wonder how many pieces she used to end the interior part of the blanket. How can I know it?

Figure 1: The Problem translated from Catalan

Before solving the problem, the teacher explained the purpose of the OB carefully and together with the class discussed and clarified the meaning and purpose of each element. This ensured, as far as is practicable, that students understood its vocabulary and overall purpose. Students were each given a copy of the OB's rubric, which included a grid in which they recorded their engagement with the OB as well as a paper copy of the problem on which their solution was to be written. Students were instructed to solve the problem, using the OB to guide their activity, and then record the OB actions they addressed. They were also told that their teacher would not intervene in the problem solving process but check, as they worked, that they completed their OB tracking.

Results

Table 3 shows the data from all 22 students' use of the OB as they worked on the problem. It can be seen that only one student, Student 21, failed to engage with the OB, while all others used it in varying degrees. Nine students obtained correct solutions for both parts of the problem, a further five managed just one part and eight failed to complete either, including the one who failed to complete any OB actions. Four students indicated some difficulty with respect to understanding some OB actions. In this respect, all four found A3, 'I have identified and understood the data', difficult to understand. The only other action that caused uncertainty was A6, 'I have checked that my strategy fits the data'. Thus, in the light of an OB being necessarily adaptive (Sanmarti, 2007), these issues would be addressed in the next iteration of its development. Importantly, even when faced with uncertainty, each of these students was able to continue the problem solving process to at least the next step. Student 12, the only student who found two statements difficult, completed 11 of the OB's stages but failed to complete either part of the problem. Importantly, from the perspective of the OB's development, Student 9 completed all the OB's actions but failed to solve either part of the problem, pointing, perhaps, to the need of cognitive interviews to determine in depth the nature of the difficulties encountered in completing the task.

	Student																					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A1	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		X
A2	X	X	X	X	X	X	X	X	X	X	X	X		X	X	X	X	X		X		X
A3	X	X	X	*	X	X	X	*	X	X	X	*		X	*	X	X	X		X		X
A4	X		X	X	X	X	X	X	X	X*	X	X		X	X	X	X			X		X
A5	X	X	X	X	X	X	X		X	X	X	X	X	X		X	X			X		X
A6		X	X	X	X	X			X	X	X	*		X		X	X			X		X
A7		X	X	X	X	X			X		X	X		X		X	X			X		X
A8		X	X	X	X	X			X	X	X	X		X		X	X			X		X
A9			X			X			X	X	X			X		X	X			X		X
A10			X	X	X	X			X		X	X		X		X	X			X		X
A11			X	X		X			X	X	X	X		X		X	X			X		X
A12			X	X		X			X	X	X	X	X	X		X	X			X		X
Total	5	7	12	11	9	12	5	4	12	10	12	11	3	12	4	12	12	3	1	12	0	12
Correct	0	2	2	1	1	2	0	1	0	0	2	0	0	2	1	2	2	1	0	2	0	2

Table 3: OB-related data for each student. An asterisk shows a difficult but completed OB action

Was the OB effective and did students take it seriously?

As is typical of classroom interventions, eliciting evidence of their efficacy and their being taken seriously is not straightforward. With respect to its efficacy, it is interesting to compare the number of completed OB actions with the number of completed problems, as shown in Table 4. A Fisher exact probability, $p = 0.008$, indicated not only that the figures of Table 4 were unlikely to have been due to chance but, importantly, that students who failed to complete the OB tended not to complete the tasks. Indeed, Table 4 shows that a necessary but not sufficient condition for the completion of both tasks was that students completed seven or more OB actions.

		Number of correct solutions		
		0	1	2
Completed OB activities	0-6	5	3	0
	7-12	3	2	9

Table 4: Number of completed OB actions and number of successfully completed problems

When viewed as four dimensions rather than as individual actions the data offer further interesting insights. For example, while probably not surprising, the data of Table 3 show that as they move down the OB, the number of students completing each dimension gets smaller. With respect to the first dimension, ‘I understand the problem’, 21 students began its three actions, of which 19 (90.5%) completed all three. With respect to the second dimension, 17 began with the first action, of whom 12 (70.6%) completed all three. The third dimension, ‘I apply my action plan’, was begun by 13 students (one more than completed the second dimension), of whom 9 (69%) completed the dimension. Finally, of the 12 students who began the final dimension, ‘I review my task’, 11 completed (91.7%) it. These figures tell two stories. The first is that students who begin working on a dimension can typically be expected to complete it. The second is that once they reach the final dimension students seem almost guaranteed to complete it. In other words, an in-depth examination of the four dimensions can also inform future developments of the OB; the first and last dimensions seem less problematic with high completion rates in comparison with the middle two.

Looking at the data qualitatively, it can be seen that students' solution attempts tended to show that they took the OB seriously. Students were able to connect OB actions to their own activity, and did not confirm those actions until after they had been completed.

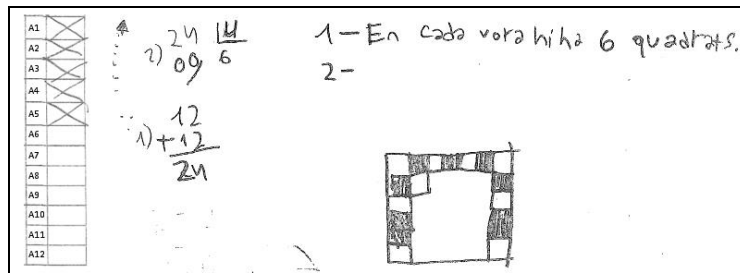


Figure 2: A solution of the problem and related OB tracking

For example, Figure 2 shows a student solution and his OB tracking. The picture confirms that he had read the problem and understood what the first question required. For example, his arithmetical operations and note, 'on each side there are 6 squares', indicate not only that he had identified and understood the required data but that he had also played with the data which let him to prepare a strategy for the first part of the problem. In short, the solution the student presented corresponded with the OB actions he claimed to have completed.

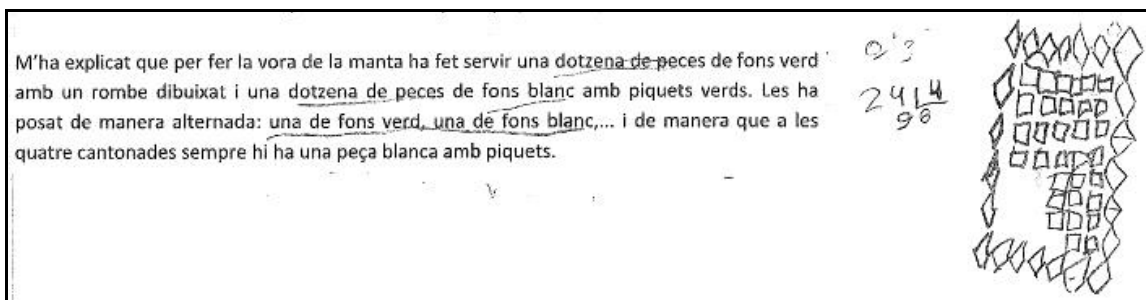


Figure 3: Solution of the pupil who failed to complete explicitly any OB action

Even when students failed to complete any OB action, there was evidence of its having influenced their solution attempts. For example, Figure 3 shows how the single pupil who failed to complete any OB action attempted to address the OB's first action.

Student response to the OB

Several students attempted to communicate with the OB, particularly when uncertain as to its intentions. Figure 4 shows, in the underlining of the word *quefer*, uncertainty as to its meaning and, essentially, an invitation for someone to explain. In similar vein, students annotated their OB in ways indicative of doubt or just a desire to comment on their response, both cognitively and affectively. Figure 5 shows comments inserted alongside the ticks indicating the student's completion of the various actions. The top two comments are the same and translate as, 'yes, but it takes me a great effort'. The lower comment, while similar in its intention, translates as, 'regular, because it takes me a great effort'.

A4. He jugat amb les dades del quefer?

Figure 4: A student's doubts with respect to the meaning of the fourth action

	S08	S12
A1.	✓	✓
A2.	✓	✓
A3.	Si p... m'ha costat.	Si p... m'ha costat.
A4.	✓	✓
A5.		✓
A6.		Regular per que m'ha costat
A7.		✓
A8.		✓

Figure 5: Two annotated record sheets

Discussion

In this paper we have outlined the development and trial of an orientation basis, designed to support 6th grade-students' problem solving-related self-scaffolding. Derived from the literature the four dimensions, and their respective actions, provided evidence suggesting that the OB has a role to play. The four dimensions and the means of their operationalisation make real for students the actions that guide problem solving (Holton & Clarke, 2006). The evidence supports earlier findings that appropriate scaffolding may have a beneficial impact on cognition, metacognition and affect (van de Pol et al., 2010). However, with respect to the extent to which the OB for problem solving is contingent, exploits fading and encourages transfer of responsibility (van de Pol et al., 2010) is variable. With respect to contingency, our view is that students were able to connect OB actions to their own activity and those who were affected by typically persisted until at least the next step. Also, students took the OB seriously, indicating initial support for both fading and transfer of responsibility, although a longitudinal study would allow these to be better examined. The dimensional structure and the ways in which students use the actions embedded within it point towards a productive cycle of refinement. Despite its linearity, based on the behaviours of an expert problem solver, students' engagement with the final dimension confirmed not only the cyclic nature of problem solving but also the role of the OB in supporting students' awareness of it. Finally, the OB comprised short statements written in the first person. Our view is that it helps learners' not only understand what problem solving expects of them but also anticipate possible actions.

Finally, this paper has reported on the first iteration of an emergent study. Since the completion of this first task students have solved two further problems using the OB. Their teacher has commented, anecdotally, that students are becoming more familiar with and confident in their use of the OB. Therefore, a longitudinal analysis of students' OB-related problem solving would seem an appropriate next step. As found with previous studies, the impact of scaffolding is difficult to evaluate (van de Pol et al., 2010) and this will remain a key objective of future work.

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