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Measuring beliefs concerning the double discontinuity in secondary teacher education

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In this paper, we discuss a research project to measure prospective mathematics teachers' beliefs towards their perceived disagreements between mathematics at university level and school mathematics that is known as double discontinuity. Firstly, we introduce the double discontinuity problem. Secondly, we refer to the construct of beliefs as the main part of our theoretical framework. Afterwards, we outline our method including a brief discussion on our approach of bridging the double discontinuity problem with so-called teacher-oriented tasks that are appropriate to illustrate connections between university mathematics and school mathematics. Furthermore, our results of pilot studies aiming to measure prospective teachers' beliefs are provided.

Keywords: Beliefs, double discontinuity, prospective teachers, teacher education.

Introduction

Over 100 years ago, Felix Klein coined in the preface of his textbook “Elementary Mathematics from a Higher Standpoint” the term of a “double discontinuity” (Klein, 1908, p. 1). This notion embodies the challenges of transitions in the mathematical socialization of mathematics teachers. The first discontinuity, i.e. the transition from secondary to tertiary education, became a main topic in research on university mathematics education during the last decades (e.g. Gueudet, 2008; Thomas et al., 2015). A challenge that accompanies the second discontinuity is the transformation of academic mathematics gained at university into educational forms of school mathematics (Prediger, 2013; Winsløw & Grønbaek, 2014). As a consequence of both discontinuities, teachers may lose sight of academic mathematics after university studies and, thus, teach on the basis of experiences from their own schooldays (Bauer & Partheil, 2009; Hefendehl-Hebeker, 2013).

Although mathematics instruction in schools as well as teacher education in universities changed considerably since Klein's claim, the phenomenon of a double discontinuity still seems to exist and prospective teachers nowadays frequently believe that the topics of university mathematics do not meet the demands of their later profession in school (cf. Ableitinger, Kramer, & Prediger, 2013; Hefendehl-Hebeker, 2013). The prospective teachers' perception of the mentioned relationships between school mathematics and university mathematics could be regarded as part of teachers' beliefs (Eichler & Isaev, 2016). Considering this background, we primarily rely on teachers' beliefs as the main construct of our theoretical framework in the next paragraph. Subsequently, we outline the method for our research project with a specific focus on pilot studies. Finally, we use the results of these pilot studies to explain the development of our instrument aiming to measure prospective teachers' beliefs concerning the double discontinuity.

Theoretical framework

Prospective teachers' beliefs

On the construct of beliefs we refer to the definition of Philipp (2007, p. 259), who defines beliefs as “psychologically held understandings, premises, or propositions” affecting an individual’s view of the world or a special part of it, such as filters for receiving information. Beliefs can be considered as a component of teachers’ mathematics related affect (Hannula, 2012), which itself can be defined as “a disposition or tendency or an emotion or feeling attached to an idea or object” (Philipp, 2007, p. 259). Thus, in our research we regard prospective teachers’ beliefs as their propositions concerning the relationships between school mathematics and university mathematics. Following Calderhead (1996), teachers’ beliefs are understood not only to impact on teachers’ professional knowledge but also to potentially have an effect on teachers’ classroom practices (cf. also Skott, 2009). Therefore, it seems essential to tackle the issue early and to investigate, how beliefs concerning the double discontinuity develop in secondary teacher education.

In related literature, beliefs are partially seen and sometimes even defined as relatively stable dispositions. In contrast, several studies provided a change of beliefs due to interventions, especially concerning prospective teachers (e. g. Decker, Kunter, & Voss, 2015). In this paper, we aim to provide an instrument in order to investigate, whether and how prospective mathematics teachers change their beliefs concerning the double discontinuity phenomenon based on our intervention project. From this background, we address beliefs being changeable and consider change as “a natural part of the development of beliefs and the reaction of beliefs in the face of experiences” (Liljedahl, Oesterle, & Bernèche, 2012, p. 35).

Related Research

Although numerous projects and institutions across the world aimed to overcome the perceived gap between school mathematics and university mathematics in recent years, not much research has been done on the perception of prospective teachers’ concerning the double discontinuity problem. Winsløw and Grønbæk (2014) distinguished three dimensions of Klein’s double discontinuity, which are not independent but important to separate: the institutional context (i.e. school vs. university), the difference in the subject’s role within the institution (i.e. student at university or school vs. teacher of mathematics), and the difference in mathematical contents (i.e. elementary vs. advanced). In our research, we primarily refer to the content aiming to figure out possibilities of “building bridges” (Winsløw & Grønbæk, 2014, p. 64) and to investigate the effect of bridging activities on the prospective teachers’ beliefs referring to the double discontinuity.

Becher and Biehler (2016) used narratives in order to ask prospective secondary teachers in their third year of university studies about what benefits they see in learning university mathematics for their future career as a school teacher and which aspects are articulated by the prospective teachers in their evaluation of benefits of university mathematics. The results revealed a wide range of prospective teacher’s beliefs on benefits of learning university mathematics with regard to school mathematics. Most of the statements can be matched with one of four levels of mathematical content knowledge based on Krauss et al. (2013), i.e. “A deep understanding of the content of the secondary school mathematics curriculum (e.g., ‘elementary mathematics from a higher standpoint,’ as taught at university)” (Krauss et al., 2013, p. 155). Taking these studies into account, we developed a

questionnaire for measuring prospective mathematics teachers' beliefs towards their perceived disagreements between mathematics at university level and school mathematics.

Method

The institutional frame

Referring to this theoretical framework, the main target of our project is to investigate changes in prospective teachers' perception of a double discontinuity that could be explained by our approach of building bridges. Prospective secondary mathematics teachers in Germany are usually enrolled in the same mathematics courses as mathematics majors (e.g. analysis), particularly in the first semesters. A big challenge for all students in the initial phase of the studies is the task to complete a range of exercises every week as homework (Ableitinger & Herrmann, 2013). These tasks can be solved on the basis of the plenary lectures (by usual four hours per week) and are reviewed in additional small courses (by two hours per week) which are organized by student assistants. Our aim is to develop and establish the desired bridges in these introductory mathematics courses for prospective secondary teachers. Our focus is here to enrich the set of tasks for homework with so-called "teacher-oriented tasks" that are appropriate to illustrate connections between university mathematics and school mathematics to prospective secondary teachers.

Teacher-oriented tasks

We conceptualize specific tasks which potentially demonstrate bridges between school mathematics and university mathematics to a model of domains of teacher knowledge according to Ball, Thames, and Phelps (2008). More precise, we differentiate our teacher-oriented tasks referring to the subdomains of specialized content knowledge (SCK), knowledge of content and students (KCS), knowledge of content and teaching (KCT) as well as curriculum knowledge. One example that represents specialized content knowledge (SCK) within the notion of mathematical tasks for teaching is provided below. In this exercise, "Evaluating the plausibility of students' claims" and "Giving or evaluating mathematical explanations" (Ball et al., 2008, p. 400) are requirements which can be used well to describe the setting.

In the subsequent task from a mathematics contest for students ("Känguru der Mathematik 2009") the participants were asked to solve which of the following figures is the greatest one.

$$(A) \sqrt{2} - \sqrt{1} \quad (B) \sqrt{3} - \sqrt{2} \quad (C) \sqrt{4} - \sqrt{3} \quad (D) \sqrt{5} - \sqrt{4} \quad (E) \sqrt{6} - \sqrt{5}$$

A student in grade 12 chose answer (E) and stated:

" $\sqrt{6} - \sqrt{5}$ is the greatest figure, because roots are monotone. So, the greater is x, the greater is $f(x)$. Thus, their difference is the greatest, as well (by going more to the right)."

1. Analyze the student's answer. Where do you see problems with the argumentation?
2. Provide an own student-oriented answer to this topic.
3. Show in general: $\lim_{n \rightarrow \infty} \sqrt{n} - \sqrt{n-1} = 0$

Figure 1: Task "roots" for prospective secondary mathematics teachers

The design

In order to gain empirical evidence for the efficiency of our method, prospective teachers in the relevant mathematics courses are assigned at random to a treatment group and a control group. While the control group is taught traditionally, the treatment group gets an extra teacher-oriented task for homework every week that focuses on bridging mathematics at university level and school mathematics. Our main research question is whether and how prospective teachers change their beliefs about the double discontinuity phenomenon based on our intervention in the introductory mathematics courses. In this paper we only refer to pilot studies where prospective teachers got homework including an extra teacher-oriented task on a trial basis. The main aim of these pilot studies was to develop an instrument for measuring teachers' perception of a double discontinuity.

During the winter semester 2015/16, two basic mathematics courses at the University of Kassel were selected in which the mentioned teacher-oriented tasks bridging mathematics and mathematics education was administrated: "principles of mathematics" and "analysis". Prospective mathematics teachers attend these courses usually in the first or in the third semester of their university studies. Three prospective teachers from the third semester were interviewed at the end of the semester. The data was analyzed by qualitative content analysis. We also developed a questionnaire with 16 items for measuring prospective teachers' beliefs concerning their perceived disagreements between mathematics at university level and school mathematics. The questionnaire was piloted in a mathematics course for prospective secondary teachers ($N = 60$) and is outlined in the following paragraph.

Discussion of results

The analyses of the interviews revealed that all prospective teachers took the contents dimension into account when reflecting their university studies. For example, the first prospective teacher (PT_1), believes, that in university mathematics, there are too little relations to school with regard to the content.

PT_1: "There are not many connections - direct contents connections, so - as well as in the other lectures. And also in analysis I notice, that it - that the university stuff of the mathematics lecture - almost simply goes beyond school and it is more or less by chance, if there are contents, which fall together with school mathematics - I have the feeling."

Further, two of the prospective teachers also mentioned another aspect of the double discontinuity problem, i.e. their beliefs concerning the relevance of learning university mathematics with regard to school mathematics, such as can be found in the proposition of PT_2.

PT_2: „If I finished school, I would have the same status, which I want to teach the students. And if now deeper questions arise, I would not be able then to answer them, for instance, because I myself have never had this and then - such a teacher one also did not want formerly, who could only tell, what he has just done.“

Indeed, Klein (1908) addresses both aspects in the double discontinuity phenomenon since on the one hand the problems at university may not suggest the things at school, and on the other hand, university studies may remain only a memory with no relevance upon teaching.

The collected data from the interviews also supported the development of our current questionnaire. Our first version (cf. Eichler & Isaev, 2016) contained 9 items using a 6-point Likert scale to assess students' beliefs about the double discontinuity problem. The questionnaire was piloted in a mathematics course for prospective secondary teachers ($N = 60$) and seemed to provide good internal consistency (Cronbach's alpha 0.782). Interestingly, a higher reliability (Cronbach's alpha 0.831) was achieved when regarding only the prospective teachers in the course ($N = 35$) and not all university students in the mathematics course. As a possible reason for this phenomenon, we identified two theoretical discernable domains in our questionnaire. On the one hand, we asked items which contained a personal statement including the words “I” or “me” like “I think that I require a deep understanding of mathematics in order to teach mathematics in school.” On the other hand, we provided a few statements such as “University mathematics has mostly little relation to school mathematics”, which were rather matter-of-fact. These items might have led to a different extent of identification within the different groups of university students. To provide useful information, we applied both dimensions in our current questionnaire containing 16 items.

Taking into account the four levels of mathematical content knowledge based on Krauss et al. (2013), we derived further items from the interviews such as “By the use of university mathematics, gaps are filled in the mathematical knowledge that is required in school”. We grouped all items into three subscales which we identified through our development process: “contents relationship”, “relevance for profession” and “higher standpoint”. In a further pilot study with prospective secondary teachers in higher semesters ($N = 24$), we approved these subscales to be internal consistent¹ with a total reliability value of Cronbach's alpha .911. Moreover, we used our qualitative approach in order to validate our survey afterwards.

Contents relationship – beliefs concerning the connections between university mathematics and school mathematics on the contents dimension (4 items; Cronbach's alpha .821):	
4.	University mathematics offers many parallels to school mathematics with regard to contents.
9.	School mathematics and university mathematics are applied to each other in contents.
11.*	In university mathematics, there are too little relations in contents to school.
12.*	School mathematics and university mathematics are two different worlds with regard to contents.
Relevance for profession – beliefs concerning the relevance of university mathematics for the later profession as a school teacher (6 items; Cronbach's alpha .814):	

¹ 7 Personal items with Cronbach's alpha .845 and 12 non- personal items with Cronbach's alpha .858.

1.	University mathematics is very useful for the teaching profession.
3.*	I will hardly ever need university mathematics after studying.
5.	By the use of university mathematics, I am well prepared to the job profile of a mathematics teacher.
10.	Without university mathematics, I could hardly teach mathematics in school.
15.*	Learning mathematics at university is not so important for the teaching profession.
19.	The relevance of university mathematics for the teaching profession is ²
Higher standpoint – beliefs concerning the usefulness of university mathematics as a higher standpoint for elementary mathematics (6 items; Cronbach's alpha .818):	
2.	University mathematics helps me to get deeper into school mathematics.
6.	By the use of university mathematics, gaps are filled in the mathematical knowledge that is required in school.
7.	By the use of university mathematics, I gain a deeper understanding of concepts in school.
8.	By the use of university mathematics, I understand relationships within school mathematics much better.
14.	Learning mathematics at university promotes me to be in thinking “one step ahead” of the students.
16.	As a mathematics teacher, an in-depth mathematical content knowledge is required.

Table 1: questionnaire for measuring prospective teachers' beliefs concerning a double discontinuity

The possible range of scores for each component is between 1 and 6. Higher scores correspond to more positive beliefs (by reversing the responses to the negatively formulated items indicated with an asterisk*).

Concluding remarks

The main topic of this paper was to discuss our approach of measuring prospective mathematics teachers' beliefs towards their perceived disagreements between mathematics at university level and school mathematics that is known as double discontinuity. In order to be able to investigate changes in prospective teachers' beliefs referring to the double discontinuity problem, we chose a mixed methods design and developed a questionnaire including 16 items that actually seems to measure these beliefs. Grounded on our preliminary results, the following steps of our research will be a comparison between two groups of prospective teachers - one group in a traditional course and one group in a course using the mentioned teacher-oriented tasks to prove if the type of the course has an effect of the prospective teachers' beliefs. Since a variety of other factors may be related to our outcomes, we also collect among others data to study interest (Schiefele, Krapp, Wild, & Winteler,

² Whereas in the previous items the prospective teachers may choose an option in a scale from “strongly disagree” to “strongly agree”, the last item refers to a scale from “very low” to “very high”.

1993) and study satisfaction (Dargel, 2005) in a pretest and a posttest as well as additional items referring to relevant demographic and academic background information.

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