Prospective teachers interpret student responses: Between assessment, educational design and research

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We present a study about the development of interpretative skills in prospective teachers. In particular we discuss a kind of tasks designed by us for teacher education, containing the request of interpreting students' answers. The task in this study was built on an item concerning the sum of powers of 10 and was proposed to a group of prospective secondary teachers who were attending a Math Education course. The task was first faced individually by them and then discussed in group. We present the interpretations proposed by two prospective teachers before and after the collective discussion, in order to reflect on the differences in terms both of mathematical knowledge put in play and of attitudes exhibited.

Keywords: Teacher education, prospective secondary teachers, interpretation of students' answers, arithmetic and algebra.

Mistakes are the portals of discovery (James Joyce)

Introduction

In their daily practice, teachers are required to continuously interpret students' responses and productions. This not only assists them in evaluating their difficulties and achievements, but also allows to plan the next steps of the teaching activities. Therefore, this "interpretation activity" is one of the most crucial (and often most difficult) tasks teachers perform. Empirical evidence suggests that the true quality of a mathematics teacher stems largely from his/her ability to interpret students' productions, along with a flexible attitude to redesign the teaching approach based on them. However, the ability to make sound and accountable interpretations is rarely recognized as a crucial goal in teacher training. Moreover, in a previous research study (Ribeiro, Mellone, & Jakobsen, 2013), we observed that teachers do not naturally develop this ability as they gain work experience. So, presently, our research questions are the following. Is it possible for a teacher to acquire these interpretation skills or is it a matter of innate talent? In the first case, how can a prospective teacher develop this ability? Should we engage prospective teachers in mathematical discussions concerning the interpretation of students' reasoning?

Guided by our conviction that it is possible for a teacher to develop this ability, as a part of a joint research project, we explored a particular type of tasks we have conceptualized. In one part of these tasks, we asked the prospective teachers to interpret some students' responses to a problem and reflect on possible feedback they could provide to each student. This exercise had a twofold aim: to support prospective teachers in developing the skills required for interpreting and commenting on student work, and to investigate to what extent the particular mathematical knowledge and skills possessed by the (prospective) teachers support or hinder them in their interpretations and

"constructive" reactions. Our analysis shows that the abilities to interpret and to design an educational activity based on students' productions are inhibited in prospective teachers with a poor mathematical knowledge, due to their limited understanding of the subject and lack of appreciation of various ways that problems can be solved (Ribeiro, Mellone, & Jakobsen, 2013). In particular, this is true for prospective primary teachers, many of whom have poor mathematical knowledge. However, we have also observed that prospective secondary mathematics teachers—who have studied more advanced mathematics during the three years of their Bachelor in Mathematics degree (Jakobsen, Mellone, Ribeiro, & Tortora, 2016)—also struggle with this kind of work. This led us to posit that the ability and knowledge to interpret student work depends not only on (prospective) teachers' mathematical knowledge and its components, but also on their attitudes and beliefs toward mathematics and its teaching.

The tasks described herein are presently used in our Mathematics Education courses in three different modalities. First, we ask prospective teachers to individually solve the problem, before interpreting and reflecting on some selected students' productions, and finally engaging in group discussions on the mathematical aspects involved in these students' productions. Given that the prospective teachers (both primary and secondary) have difficulties in interpreting and in giving meaning to some students' answers, it was necessary to first assess their ability to solve problems that these students are given. This was informative, as some prospective teachers struggle with providing constructive feedback to the students even when they do not encounter any difficulties in solving the given problems for themselves. The findings yielded by this first phase of our research were utilized in the subsequent mathematical discussions of students' solutions and corresponding teachers' interpretations. These group discussions were helpful to most prospective teachers, as they were able to gain new perspectives on students' work and strategies that can be employed in teaching.

In this paper, we present the interpretations, given by a group of Italian prospective secondary teachers, of students' responses concerning a problem where sums of powers of 10 are involved. (see Jakobsen et al. (2016) for details). Here, we present analysis of the interpretations teachers gave before and after the mathematical discussions, in order to document their progress, as most demonstrated more sophisticated attitudes and greater mathematical knowledge following group discussions.

Theoretical framework

In order to characterize and study the features of teachers' interpretations of students' productions, in some of our previous work (see for example Ribeiro, Mellone, & Jakobsen, 2016), we have introduced the notion of *interpretative knowledge*, framed within the general Mathematical Knowledge for Teaching (MKT) framework (Ball, Thames, & Phelps, 2008). We define interpretative knowledge as the knowledge that allows teachers to give sense to pupils' answers, in particular to "non-standard" ones, i.e., adequate answers that differ from those teachers would give or expect, or answers that contain errors. We posit that interpretative knowledge is closely related to the ability of teachers to support the development of pupils' mathematical knowledge, starting from their own reasoning, even if students' ideas are incomplete or non-standard. Some similar ideas are implied in the notion of *discipline of noticing* (Mason, 2002). In particular, our construct encompasses the idea of teachers working "on becoming more sensitive to notice opportunities in

the moment, to be methodical without being mechanical" (Mason, 2002, p. 61). The development of pupils' mathematical knowledge starting from their own reasoning is, in our view, only possible if the teacher activates a real process of interpretation, shifting from a simple evaluative listening to a more careful hermeneutic listening (Davis, 1997).

In this sense, the notion of interpretative knowledge incorporates into the MKT framework the idea that errors and non-standard reasoning are considered as learning opportunities (Borasi, 1996). Moreover, the content of interpretative knowledge shapes teachers' ability to make informed choices in contingency moments (as defined by Rowland, Huckstep, & Thwaites, 2005), in order to respond to and deal with non-planned situations. In that sense, we felt the need to incorporate the role of beliefs and attitudes pertinent to the use of mathematical knowledge (Carrillo, Climent, Contreras, & Muñoz-Catalán, 2013).

With the goal of better understanding (prospective) mathematics teachers' act of interpretation, we characterized their interpretations of students' productions and attitudes, using the following three categories: (i) Evaluative interpretation: a process through which the teacher determines congruence between pupils' productions and the mathematical scheme of correct answers he/she has; (ii) Interpretation for the educational design: the manner in which the teacher designs educational steps based on the work produced by the students; (ii) Interpretation as research: teacher's willingness and ability to revise his/her mathematical formalization in order to ensure that it is coherent with students' productions (even when these seem in conflict with the traditional mathematics taught in school).

In Webster dictionary, "interpretation" is defined as "The act of interpreting, explanation of what is obscure"; however, it is also defined as "An artist's way of expressing his thought or embodying his conception of nature." This last definition stresses the potential creative nature of the act of interpreting that is in our context perceived as the potential new mathematical knowledge that can be developed owing to the process of analyzing students' productions.

Context and method

For several years, we have been studying the nature of (prospective) teachers' interpretative knowledge (e.g., Ribeiro et al., 2013; Jakobsen et al., 2016) by exploring the manner in which prospective teachers respond to specific interpretation tasks. In this design study the tasks are developed after the typical cycles of redesign of the design study method (Cobb et al., 2003). In their present form, essentially consist of three steps: (i) the teachers are initially required to solve a mathematical problem by themselves; (ii) they are given several students' productions in response to the same problem, some containing errors and some mathematically valid but following less standard procedures, which they are asked to interpret; and (iii) teachers are prompted to provide what they deem would be appropriate feedback to these students based on their solutions. The teacher trainees are asked to address these requests individually and in paper format (they are usually given 90 minutes to complete all three steps). In the next phase of the study, the educator engages all prospective teacher participants in a collective mathematical discussion (which again typically lasts about 90 minutes). The framework of the mathematical discussion is based on that proposed by Bussi (1996), as the aim is to allow the group of prospective teachers create a polyphony of articulated voices on the mathematical object starting from the interpretation of a

student's production. Upon completion of the group discussion, the prospective teachers are asked to provide in writing a new individual interpretation of the students' productions, allowing the researchers to determine if any progress has been made.

The task utilized in the present study is depicted in Figure 1, and was adopted from the annual Italian national assessment (2010-2011) for grade 10 released by INVALSI (Istituto Nazionale per la VALutazione del Sistema educativo di Istruzione e di formazione). A group of 34 fourth-year master students of mathematics enrolled in a Mathematics Education course took part in this investigation. Since most of these students are going to become secondary school teachers, we consider them prospective secondary teachers.

In our previous study, we focused on the interpretation these prospective secondary teachers gave to their students' productions (Jakobsen et al., 2016). Our analysis revealed that they experienced problems in mobilizing their mathematical knowledge for interpreting students' work. Indeed, while they were able to "see" some of the mathematical aspects involved in the solutions to the problems their students proposed, they seemed unaware of many important aspects relevant to mathematics teaching and problem solving.

The expression $10^{37} + 10^{38}$ is also equal to:

A. 20^{75}

B. 10^{75}

C. $11 \cdot 10^{37}$

D. $10^{37 \cdot 38}$

Figure 1: Item given to students to solve

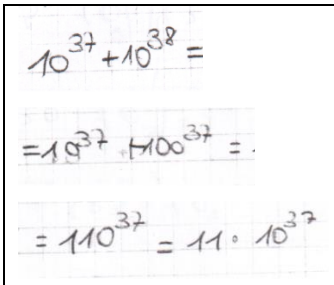
In the next section, we will present two out of seven students' productions that were included in the task given to the study participants (for their selection, see Mellone, Romano, and Tortora (2013)). This will be followed by the interpretations of these students' productions, provided by two prospective teachers—to whom we refer as Rossella and Gennaro (pseudonyms)—before (BF) and after (AF) the mathematical discussion.

Interpretation of two students' productions

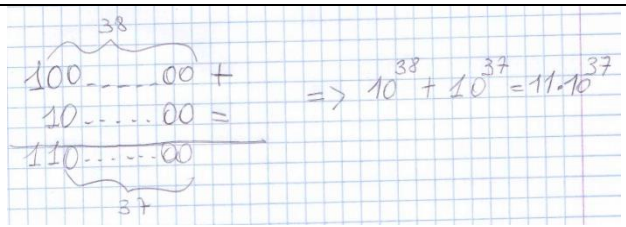
The following brief analysis of the students' productions included in the task aims to elucidate our reasoning behind the decision to deem these two students' productions effective for exploring prospective secondary teachers' ability to interpret the work of others.

Emanuela (Figure 2a) obtained the correct result, despite making three errors in her work: the first and the last can be described as lack of use of parenthesis and the second can be seen as a wrong application of linearity. Ciro (Figure 2b) arrived at the right answer using the arithmetical algorithm of the arrangement of the decimal representation of the numbers in column. In his responses, we can also recognize his perception of the algebraic structure connected with more general ideas implicit in calculus. Indeed, Ciro's use of the ellipses reveals the potentiality of generalization of his

response to other two consecutive powers of ten, and not only to the particular case given (as one of the mathematics students that took part in our study noted during the collective mathematical discussion).



(a) – From Emanuela's protocol



(b) – From Ciro's protocol

Figure 2: Two out of seven students' productions

Prospective teachers' comments on students' productions before and after mathematical discussion

Reflections on Emanuela's work

When individually interpreting Emanuela's response, prospective teachers seemed to experience difficulty in trying to understand the steps she used in arriving at the solution (see Jakobsen et al., 2016). Here, we focus on the interpretations given by a secondary prospective teacher —Rossella— before (BF) and after (AF) the discussion[1]:

The second prospective teacher, Rossella, shared the following:

Rossella (BF): There is no application of rules; it is pure invention. I don't know what I would say to the girl, but I would think that she had copied the solution and then tried to invent a justification.

Rossella (AF): Even if Emanuela's answer is correct, her arguments are far from being mathematically founded. Still, we can observe an interesting aspect in them, namely that if we repeat the steps with two powers having the base different from 10 and the exponents differing by one, we get the correct result. I followed this approach using different numbers, just to test this reasoning, which allowed me to assert that Emanuela's thought process appeared to work. More specifically, when you change the bases and use two consecutive numbers as exponents, or even not consecutive, her logic gives the correct result.

$$37^{25} + 37^{26} = 37^{25} + (1369)^{25} = 1406^{25} = (38 \cdot 37)^{25} = 38 \cdot 37^{25}$$

$$\left(\frac{1}{10}\right)^3+\left(\frac{1}{10}\right)^4=\left(\frac{1}{10}\right)^3+\left(\frac{1}{100}\right)^3=\left(\frac{9}{100}\right)^3=\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^3$$

$$2^{-3}+2^{-2} = 2^{-3}+4^{-3} = 6^{-3} = 3 \cdot 2^{-3}$$

$$x^n + x^{n+1} = x^n + (x^2)^n = (x+x^2)^n = ((x+1)x)^n = (x+1)x^n$$

[1] Of course, the students' words were translated from Italian into English. We tried our best to retain the exact expressions, including some errors, but some nuances are inevitably lost.

> In other words, the above three errors, made in sequence, yield a correct result that does not depend on the particular numbers used. This observation has prompted our reflection on how mathematics is usually managed. We are used to judge, without hesitation, arguments such as those mentioned above as wrong and completely invented. However, after our discussions, we were of view that errors should be seriously considered and, if possible, exploited as a stepping stone toward the construction of new knowledge. This is exactly what I tried to do in order to bring out something new from Emanuela's reasoning. Indeed I also tried to build a new system of rules for powers according with her reasoning. Several attempts have convinced me that this is not possible. Thus, my conclusion is that no new knowledge about the rules of powers can be derived from Emanuela's suggestions. However, her errors can be an invaluable tool to stimulate discussions and to highlight the need for a true comprehension of the rules of powers, which are often hard to grasp for students.

Reflecting on Rossella's BF and AF words, we can observe a change in the attitude when interpreting student's work. The first interpretation is an evaluative one—there is no effort to understand the rationality of Emanuela's steps. This results in Rossella's bias towards a solution like that Emanuela offered (referring to an her possible unfair behavior), which can be due to fear of moving toward an educational path she cannot (immediately) control.

The interpretation given by Rossella after the discussion is markedly different. She not only made an effort to derive a generalization from the errors in Emanuela's production, but further concluded that the three steps Emanuela used in solving the problem will give the right answer with other bases and exponents. Rossella thus went beyond the simple observation that Emanuela's steps are not mathematically sound, as she investigated the possibility to build "a new system of rules for powers according with her reasoning." For this reason, her second interpretation can be considered a form of interpretation as research.

Reflections about Ciro's work

> Gennaro (BF): Ciro reached the correct answer by a more practical method than those employed by his peers. In addition, the formalism seems original. He appears to have a strong expertise in the calculations with powers of 10, which highlights their significance and the importance of handling them correctly. Still, his method seems limited to powers of 10. It would be interesting to see how Ciro would proceed if presented with a different base. I think that Ciro's protocol could be used as an opportunity to explore differences between the properties of powers of 10 and those of other bases.

> Gennaro (AF): Ciro's argument is of an arithmetic character. Nonetheless, it allows us to appreciate some deep algebraic insights. Moreover, although it seems confined to powers of 10, it can actually be generalized to any base, if one represents the number in the base of the power. Hence, from Ciro's production going further, it would be possible to study the tables of operations in different bases, or even the divisibility rules in bases other than 10.

In his first interpretation of Ciro's protocol, Gennaro appreciates the originality of his method, while noting that it is limited to powers with base 10. Based on this observation, Gennaro proposed possible questions and issues that could be explored with Ciro and the rest of the students in the classroom, starting from his production. For this reason, Gennaro's first interpretation is aimed at educational design.

As with Rossella, whose interpretations we analyzed previously, the comments Gennaro gave on Ciro's protocol after the discussion shifted in focus. First, there is a subtle distinction between arithmetic and algebra that could be investigated and debated endlessly. Moreover, Gennaro's comments reveal his awareness that Ciro's method can be applied to other bases (indeed, 100…0 always represents the *n*-th power of the base). This fact was observed during the collective discussion by another prospective teacher, and for Gennaro, this discovery was so important that it became part of his new written interpretation. In other words, Gennaro's knowledge and interpretation benefitted from the mathematical discussion on Ciro's production. He reconceived the systems of representing numbers in different bases, which motivated him to explore the true meaning of digits, as well as of strings of digits. For this reason, Gennaro's second interpretation is perceived as interpretation as research.

Conclusive remarks

We started this paper by asking if mathematics teachers can develop the ability to interpret their students' productions in order to flexibly redraw the mathematical learning path, or if this should be considered as an innate talent. We are convinced that it is not only possible to develop this skill but is highly desirable. The observed difficulties these prospective teachers experienced when giving sense to student productions, along with the findings yielded by extant studies, indicate that the development of this ability requires a special attention in teacher education. These first results about our proposed method of working with prospective teachers appear to support its effectiveness. It stimulates prospective teachers' interpretive and critical skills and increases knowledge they must possess in order to teach effectively, taking into consideration the specificities of such knowledge. The value of our method stems from the nature of interpretive tasks involving student productions, as well as subsequent discussions among peers under the guidance of an expert on these interpretation tasks.

Our analysis of the interpretations given by two prospective teachers, before and after the collective discussion led by the educator, clearly demonstrates changes in terms of both their attitude and mathematical knowledge or awareness. We can hypothesize that the collective discussion mobilized mathematical knowledge that was previously present, but probably not put in play, and it also support the development of new mathematical knowledge, like for Gennaro. However, the improvements we witnessed were also due to the change in attitudes and beliefs supported by the discussion, and of course by the attitudes and beliefs incorporated in the educator's practice.

Still, our work leaves many questions to be answered in future research. It would be interesting to evaluate the sustainability of these changes, for example, by following the work of these prospective teachers in their future educational practices. Moreover, analysis of the mathematical discussion on interpretative task needs to be developed in order to clarify its features and dynamics.

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