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► To cite this version:

Pessia Tsamir, Dina Tirosh, Ruthi Barkai, Esther Levenson. Noticing aspects of example use in the classroom: Analysis of a case. CERME 10, Feb 2017, Dublin, Ireland. hal-01949019

HAL Id: hal-01949019

<https://hal.science/hal-01949019>

Submitted on 9 Dec 2018

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Noticing aspects of example use in the classroom: Analysis of a case

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This paper investigates promoting knowledge of example use in mathematics education by way of analyzing a case using theoretical tools. Participants were both prospective and practicing teachers attending a university course. An event taken from a tenth grade geometry class was analyzed in terms of example use, and then discussed. Participants related to the type of example given, the timing of the example, agency, what the example was an example of, and the aim of giving the example.

Keywords: Examples, case analysis, teacher education, theoretical tools.

Introduction

Examples and non-examples are an integral part of learning and teaching mathematics. They are used in concept formation, when seeking relations between elements, abstraction, and generalization (e.g., Smith & Medin, 1981; Watson & Chick, 2011). Acknowledging the importance of example use in mathematics education, Shulman (1986) included knowledge of examples within the category of pedagogical content knowledge (PCK). This knowledge, he claimed, is essential for representing the subject, so that it will be comprehensible to others. Rowland, Huckstep, and Thwaites (2005) also included teachers' use of examples in their description of 'the knowledge quartet', a framework for thinking about the ways subject matter knowledge comes into play in the classroom. In their framework, the ways teachers use examples and the types of examples they use, are manifestations of teachers' own content knowledge, their meanings and descriptions, being transformed and presented in order for students to learn the mathematics. Ball, Thames, and Phelps (2008) also noted the importance of teachers' knowing how to sequence examples.

Although example use is a complex matter and promoting teachers' knowledge of example use could be quite challenging, few studies specifically investigated this aspect of teacher education. This study proposes fostering prospective and practicing teachers' knowledge and awareness of example use by applying research and theoretical tools to analyze cases, helping to bridge theory and practice for mathematics teachers. The material, which is the focus of this paper, is a case based on a classroom event, and used as an exemplar. The case material consisted of a classroom transcript, along with guiding questions to be answered by each participant. This was followed by a group discussion with the teacher educator. Specifically, we ask: What aspects of example use do individual participants notice when studying an authentic case? Can those aspects be traced back to theories learned during the course? What additional aspects arise during group discussion?

Example use in mathematics education

There are many aspects of example use which have been investigated. To begin with, there are different types of examples. Arcavi (2003) discussed visual representations, while Tabach et al.

(2010) focused on numerical examples. In addition to the form of the example, studies have categorized examples by how they are identified. For instance, there are intuitive and non-intuitive examples and non-examples (Tsamir, Tirosh, & Levenson, 2008). Intuitive examples are those examples which students immediately identify as such and are often derived from practical experience. Likewise, intuitive non-examples may encourage visual, rather than analytical thinking. On the other hand, it seems that non-intuitive examples and non-examples can encourage students to use reasoning based on critical attributes. Tsamir, Tirosh, and Levenson (2008) also discussed the sequencing of examples and non-examples and its effects on students' learning.

The difference between examples and non-examples is dependent on the mathematical lens one is looking through. When looking for polygons, a triangle is an example; when looking for quadrilaterals, a triangle is a non-example. In other words, another aspect of example use to consider is the way an example is being used. Zazkis and Chernoff (2008) introduced the idea of pivotal examples which can cause the learner to change his or her cognitive perception or way of approaching a problem. Rowland (2008) differentiated between two uses of examples in teaching. The first is when examples are used to motivate generality. In this case, the examples are examples *of* something where the aim is to teach a general procedure or to support abstraction and concept formation. The second type of example use is for students to practice what was taught. That is, students are given many examples to practice some procedure. This type of example use allows students to experience variation and can lead to additional awareness and understanding. Watson and Chick (2011) found that examples can be used as templates for dealing with other class members, indicating a relation between classes or to express equivalence.

If we take into account that the way an example (or non-example) is used by a learner is determined by the focus of that learner, then it becomes the teacher's task to choose examples and set them up in such a way that students will view those examples through the intended lens and focus on the intended pedagogical aim. Zodik and Zaslavsky (2008) identified six types of considerations employed by teachers when selecting or generating examples: starting with a simple or familiar case, attending to students' errors, drawing attention to relevant features, conveying generality, including uncommon cases, and keeping unnecessary work to a minimum. They found that the most frequent consideration was choosing to begin with the simple or familiar example. They also found that on the spot, teachers often choose an example that will attend to an error which arose in class, whereas pre-planned examples tend more to consider uncommon cases.

To notice something means to make a distinction, to stress some perceived feature and ignore others (Mason, 1991). As shown above, various aspects of example use can be considered when analyzing a classroom event. In this study we investigate which of these aspects, and perhaps other aspects of example use, practicing and prospective teachers notice when analyzing a case.

Methodology

The participants in this study were 13 practicing teachers (denoted as T) with between 1 and 11 years of experience (mean years of experience was four) and 10 prospective teachers (PTs) who had completed their first degree in mathematics or a mathematically rich field of study, such as engineering. The specific course which is the context of this study, aimed to promote participants' knowledge of explanations and examples, and the relationship between them in mathematics

education. The course consisted of a total of 13 lessons which took place once a week for a period of 90 minutes each. The first six lessons were devoted to explanations in mathematics education. Different types of explanations were reviewed, such as conceptual, procedural, mathematically-based and practically-based explanations (e.g., Levenson, Tsamir, & Tirosh, 2010), and theories concerning the roles of explanations in mathematics education were discussed (Levenson & Barkai, 2008). The last seven lessons were devoted to examples in mathematics education. Theoretical perspectives of examples, like those described in the background of this paper, were read and discussed.

During the fourth course lesson on examples (lesson #10), the teacher educator handed out a transcript of a geometry lesson which took place in a tenth grade classroom. The geometry lesson had taken place within a few weeks of the course lesson and was observed by three of the prospective teachers attending the course, lending the case authenticity and relatedness. The overall aim of the geometry lesson was to introduce students to Thales' theorem and show its connection to similar triangles. The students had previously learned about similar triangles in ninth grade. The case transcript began with the classroom teacher stating Thales' theorem. This was accompanied by a drawing on the board, made by the teacher, of two similar triangles, under the headline: Geometry – Thales' theorem. Next to the drawing it says that if the given is $BC \parallel DE$ then from Thales' theorem we conclude proportional line segments (see Figure 1).

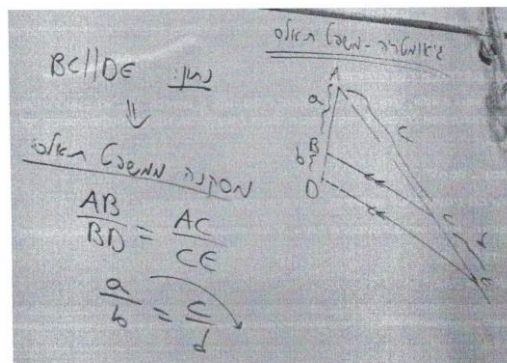


Figure 1: Presenting Thales' theorem on the board

The case transcript was handed out in the beginning of the course lesson with instructions to read it from beginning to end, without interruption, in order to understand the context and get a feel for the classroom. After reading through the transcript, participants were asked to reread the transcript and fill out a worksheet with the following questions: What did you learn about the use of examples during mathematics instruction from the examples given in the case presented? What would you do the same as the teacher did with regard to examples? What would you do differently from the teacher with regard to examples? After the participants wrote their answers and handed them in, a discussion followed. This discussion was audio-recorded and transcribed.

Data analysis

When analyzing participants' responses to the worksheet, our guiding question was, what aspects of using examples did the participants relate to when studying the case. We then used the literature background to help form a categorization scheme of those aspects. For example, we examined participant's responses for comments related to various types of examples that were discussed in class and were present in the case, such as intuitive examples, non-examples, familiar examples, and uncommon examples. Participants also commented on didactical aims of giving examples, such

as responding to a student's error. Because participants wrote freely, it happened that one sentence could encompass more than one aspect of example use. For instance, one teacher (T12) wrote "the teacher gave a numerical example, which in my opinion served the purpose of making it easier for the students to understand and generalize the idea." T12 refers to the type of example (numerical) as well as the purpose of the example (to help students generalize). Participants' comments that were thought not to be related to the giving and use of examples were categorized as 'unrelated'. Table 1 lists the categories along with examples from the data of each category.

Aspect of example use	Sample data
Type: What type of example is given?	The example uses simple numbers; it is a visual example of Thales' theorem.
Agency: Who is giving the example?	Only the teacher gives examples.
Aim: what seems to be the aim of giving the example?	The example is given to explain it again; the example shows the students how the theorem works.
Timing: When is the example given?	The example is good for the beginning of a lesson.
Example of: What is the example an example of?	The teacher gives examples of proportions.
Unrelated	The teacher presents a dry definition of Thales' theorem.

Table 1: Categorizing participants' comments

Findings

This section analyzes participants' comments from the worksheet on two segments of the case along with excerpts from the discussion which focused on those case segments. Thus, we review what participants noticed individually and what they discussed collectively. Analysis of participants' comments is carried out according to the aspects of examples listed in Table 1.

Segment one- Introducing Thales' theorem

The case transcript began a picture of the teacher at the board presenting to his students an example (see Figure 1). Accompanying the picture were the following lines from the case transcript:

- 1 T: In Thales' theorem, it is given that BC is parallel to DE. The conclusion is... wait, guess. If the length here (pointing to AB) is 6 and here (pointing to BD) is 2. And let's say that the length here (pointing to AC) is 12, what is the length of CE?
- 2 S: 4!
- 3 T: Right!

There are several ways to look at the examples in the above segment. One way is that there are two explicit examples. First, there is the drawing on the board (see Figure 1). Second, there is the numerical example given by the teacher in Line 1. However, the drawing on the board can be an example of Thales' theorem or an example of similar triangles, and indeed, that is what the teacher is trying to convey. In addition, depending on one's point of view, the example on the board and the

numerical example may be considered one complete example, with the example on the board written in a general matter, using parameters (a, b, c, and d), and the oral example, a specific example given with numbers.

The most frequent aspect of examples mentioned by participants was example *type*. Participants used such descriptions as: a simple example (PT2), an unfinished example (T8), a visual example (T12), a numerical example (T12, T13), and an intuitive example (PT 22). Although most participants wrote that the teacher gave the example, we do not take this as commenting on the *agency*, but rather as a description of what is going on. On the other hand, for a different segment, one participant wrote, “Only the teacher gives examples... but he should have requested examples from the students.” This participant is not merely describing the situation, but commenting on who is and who should be giving the examples. The *aim* of giving the examples was mentioned by T8 who wrote, “In Line 1, the teacher gives an example that students have to finish. He is checking to see if the students are listening and if they understand.” T13 wrote, “The teacher gave a numerical example so that the students could understand the example and draw on their previous knowledge of similarity and proportional triangle sides.” *Timing* of the example, that the example was appropriate for the beginning of the lesson, was noted by PT2. Two participants commented on what the example was *an example of* – T12 wrote that it was an example of the theorem, meaning that the example showed how the theorem could be applied. PT22, referring to the numerical example in Line 1, wrote that it was an example of equivalent ratios.

Segment two – Proportional segments

The following case segment is a direct continuation of the first one:

- 4-6. (The teacher and students review the concept of similar triangles.)
7. T: So, what is the ratio of their corresponding sides (referring to the example given in Line 1)?
- 8-10. Students: 1 to 3. 3 to 1. It depends on how you look at it.
11. T: The ratio is ... 3 to 4 because BD and CE are not sides of the triangle. So 6 is to 8 like 12 is to 16. Now, ..., what came first in Euclidian geometry? First, there was Thales’ theorem and only after that came the similar triangles theorems. So, let’s say we are in ancient Greece and we don’t know yet about similar triangles, but we do know Thales’ theorem. With that theorem, we can prove proportional sides in similar triangles.

In lines 4-11, no new examples are given. Instead, the teacher and students still refer to the first examples given on the board. Like the comments on the first segment, here too, most comments related to the *types* of examples being given: simple numbers (PT7), a non-intuitive example (T11), a general example with parameters (T17), and a numerical example with familiar numbers (T19). None of the participants mentioned aspects of *agency*, *aim*, or the *timing* of examples. T11 noted that the example was an *example of* ratios. Two PTs wrote remarks connected with the story of ancient Greece. PT23 wrote, “There is an example from real life that I like – in ancient Greece.” PT16 wrote, “In Line 11, the teacher made the material come alive when she told the story about Thales’ theorem from ancient Greece ... This example raises the question of why the students first learn about similarity and then about Thales.” T23 calls it an example from real life. Yet, it is not a

mathematical problem related to real life. The term ‘example’ when describing the ancient Greece story is not in line with the notion of examples discussed in the course.

Discussing the case

After the worksheets were handed in, the teacher educator (TE) opened up the discussion by asking who wished to comment on the case. After nine minutes of discussing general ideas, the discussion turned to the example given in Line 1 of the case transcript. Note that PT21 was present during the geometry lesson as part of their field work.

TE: Let’s look at Line 1.

PT21: But it’s not complete... He wanted to give an example of a ratio, but... Instead, he asked the students... He told the students to guess. And they did. He didn’t really give an example, in my opinion. But, in the next example...

TE: Where is the next example, in your opinion?

PT21: Line 11... In my opinion, it’s an explanation with a few numbers and that’s so you can see... I can’t decide. It’s, like, a numerical example. Here (in Line 11) is what was missing beforehand (in Line 1).

TE: Is it an explanation or an example? Can an example also be an explanation?

PT21: No. An explanation can be accompanied by an example. There can be an example and then the explanation generalizes it.

T8: I felt that way about Line 1. It feels like an explanation, and also like an example. On the one hand, there are numbers. On the other hand, it’s not complete.

The above excerpt gives us a glimpse into PT21’s and T8’s concept image of an example. Both infer that an example has numbers, but it must be complete, without any missing parts. There is also the question of the different roles an example may play in the classroom. Can an example be an explanation? Must all examples be specific and only the explanation can generalize it? These are some of the questions that the participants are grappling with. In the following excerpt, PT22, who was also present as an observer of the geometry lesson, tells what he observed.

PT22: He was trying to show how Thales’ theorem is really intuitive. That is, he gave an intuitive example.

TE: An intuitive example of what?

Many voices: Equivalent fractions.

PT22: And most answered correctly. The teacher was trying to show how easy it is.

TE: But what was it an intuitive example of?

T12: Of proportions. I know that it’s proportional because I know, I recognize it. It doesn’t really have anything to do with Thales’ theorem. Simply, 6 is to 2 as 12 is to 4. That’s it. That’s the example.

As stated in the background, what an example is an example of, depends on your focus and point of view. PT22 claims that the examples in Line 1 and Line 11 are examples of Thales’ theorem and

that the teacher used intuitive examples to simplify the concept for his students. However, other participants, those not necessarily present during the geometry lesson, see those examples as intuitive examples of equivalent fractions or of a proportion.

Discussion

During teacher preparation and professional development, participants are introduced to various theories. While field work is important, theories can help prospective and practicing teachers make the most of their field work by focusing their attention on different elements of practice. Findings showed that analyzing the case gave participants a chance to apply their knowledge of example theory when examining a classroom situation. Findings also showed that participants did not necessarily draw on the same theories when analyzing the same event. The example given in Line 1 of the case transcript was described alternatively as simple, numerical, visual, and intuitive. Each of these types can be traced back to different theories discussed in the course, but they focus on different issues. An intuitive example may also be numerical, but if the prospective teacher specifically comments on its intuitiveness, then that participant is remarking that a student will easily recognize it as an example (Tsamir, Tirosh, & Levenson, 2008). In other words, that participant is integrating knowledge of students (Ball, Thames, & Phelps, 2008) when analyzing example use and has appropriated a specific theory to accompany this integration. As teacher educators, we wish to encourage such integration of knowledge.

During the discussion, additional aspects of example use arose. Participants grappled with the nature of examples, and whether or not an example can be an explanation. This link between examples and explanations could have stemmed from the first half of the course which dealt with the topic of explanations in mathematics education. In any event, this question can help teachers and prospective teachers focus on the roles examples may play in the greater picture of teaching and learning mathematics. Finally, we also note how the integration of an authentic case, one that at least some participants actually observed, can help bridge theory and practice. In the discussion, participants held different views regarding what the example in Line 11 was an example of. Those who had actually observed the lesson had a chance to review the lesson again, focusing now on example use. They were also able to share with others some background of the lesson, perhaps adding to everyone's sense of 'being there'. This excerpt illustrates how fieldwork may be integrated into course work. It also reminds us, as teachers and as teacher educators, that it is not enough to offer examples. As Goldenberg and Mason (2008) said, exemplification is dependent on one's point of view. Analyzing a case using theories, and then discussing these analyses with participants, can raise awareness of how students may view examples and encourage planning example use in mathematics classrooms.

Acknowledgment

This research was supported by The Trump Foundation (grant No. 145).

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