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## Modeling a Multi-Objective Location-Routing Problem for Hazardous Materials with CO<sub>2</sub> Emissions Consideration

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### Abstract

This paper addresses a multi-period, multi-vehicle, location routing problem (LRP) where capacitated vehicles distribute hazardous materials from multiple plants to multiple retailers to meet the given demand of hazardous materials. Also, the risk of location and routing, cost of transporting hazardous materials from plants to retailers is considered. A series of middle nodes is considered which if needed to reduce the risk and costs, use them. CO<sub>2</sub> emissions from transport activities in a supply chain are key players in global warming. Routing decisions can help in the reduction of CO<sub>2</sub> emissions if these emissions are taken into account by the decision makers. The proposed framework is a multi-objective deterministic programming model. The first objective function aims to minimize the expected value of the supply chain costs including CO<sub>2</sub> emissions produced by the vehicles. The second objective function aims to minimize the total risk of establishing plants and risk of transportation. The demands associated with retailers is assumed to be time varying and deterministic. The proposed model is a mixed-integer linear program that was coded and solved in GAMS. We provide a numerical study showing the applicability of the model.

*Keywords:* Location routing problem, Green supply chain, Mixed-integer linear programming, Hazardous materials;

### 1. Introduction

With the rising of global warming concerns and environmental issues, many governments and researchers have begun to pay close attention to green logistics. Despite the fact that the concept of "green logistics" has recently attracted the attention of the researchers, few studies have addressed the environmental concerns in their LRP model [1]. There are few studies in this category [3] under which a set of plants serve a set of retailers. Some studies on supply chain have considered CO<sub>2</sub> emission [2,6]. They considered CO<sub>2</sub> emissions in the inventory-routing problem (IRP).

Due to the increasing environmental concerns, the problems of locating the plants which produce hazardous materials and the transportation of these materials have become the focus of many researchers [5]. Although, some studies on LRP have been devoted to distribute hazardous materials [4,5] but none of these none-traditional approaches has not applied the CO<sub>2</sub> emissions to elaborate the inherent conflict between economic aspects and environmental concerns in LRP decisions. In hazardous materials location and routing problems, in addition to cost, the risk of locating a facility should also be considered [5]. Our problem is also classified as a many-to-many structure.

The aim of this study is to present a green model of multi-period location routing problem (LRP) where capacitated vehicles distribute hazardous materials from multiple plants to multiple retailers. The proposed model can help in the reduction of CO<sub>2</sub> emissions and risk level.

## 2. Model definition

The location routing problem (LRP) can be stated as follows: Given a set of retailers with known demand, a set of potential plants, determine the location of the plants and the vehicle routes from the plants to the retailers to minimize the sum of the costs associated with locating plants, distribution to the retailers, vehicles' fixed and CO2 emissions costs. On the other hand, determine the location of the plants and the vehicle routes taking into account the risk generated to minimize the sum of the risks associated with locating plants and distribution hazardous materials to the retailers. Each retailer is assigned to just one plant which will dispatch a vehicle to fulfil its demand. Plants have several types of vehicles, each one is characterized by its own capacity, fixed and variable transportation cost rate and its CO2 emission index. Hazardous materials distribute from plants to retailers and if the risk of the route between them exceeds the allowed level, they use a series of middle nodes to distribute hazardous materials. The notations and mathematical formulation are as follows:

### Sets:

$f = \{1, 2, \dots, F\}$  Set of all plants

$r = \{1, 2, \dots, R\}$  Set of all retailers

$n = \{1, 2, \dots, N\}$  Set of all middle nodes

$\omega = f \cup r \cup n$  Set of all nodes

$k = \{1, 2, \dots, K\}$  Set of all vehicles

$t = \{1, 2, \dots, T\}$  Set of time periods

### Parameters:

$CF_i$  = The cost of opening the plant  $i$

$RF_i$  = The risk of opening the plant  $i$

$RL_t$  = Allowed level of risk generated in each period

$RS_{ijt}$  = Risk amount between node  $i$  and  $j$  in each period ( $i, j \in \omega$ )

$GHL_t$  = Allowed level of CO2 emission in each period

$GHG_k$  = CO2 emission by vehicle type  $k$  per unit distance

$P$  = Price of CO2 per unit mass

$Q_k$  = Capacity of vehicle  $k$

$c_{ij}$  = Distance between node  $i$  and  $j$  ( $i, j \in \omega$ )

$v_k$  = Variable transportation cost per unit distance for vehicle type  $k$

$u_k$  = Fixed transportation cost for vehicle type  $k$  per trip

$D_{jt}$  = Demand of retailer  $j$  in each period

$E_i$  = Maximum throughput at plant  $i$

$MA$  = A big number

**Decision variables:**

$X_{ijk_t}$  = A binary variable that determines if arc (i, j) is visited by vehicle type k in period t ( $i, j \in \omega$ )

$Z_{ijt}$  = A binary variable that determines if retailer j is allocated to plant i in period t

$Y_i$  = A binary variable that determines if plant i is opened

$W_{ikt}$  = A binary variable that determines if node i is visited by vehicle type k in period t ( $i \in \omega$ )

$M_{it}$  = Auxiliary variable for sub-tour elimination constraints in period t

**Mathematical model:**

$$(1) \begin{cases} \min z_1 = \sum_{i=1}^F CF_i Y_i + \sum_{i=1}^{\omega} \sum_{j=1|i \neq j}^{\omega} \sum_{k=1}^K \sum_{t=1}^T v_k c_{ij} X_{ijk_t} + \sum_{i=1}^F \sum_{j=1|i \neq j}^{\omega} \sum_{k=1}^K \sum_{t=1}^T u_k X_{ijk_t} + P \sum_{i=1}^{\omega} \sum_{j=1|i \neq j}^{\omega} \sum_{k=1}^K \sum_{t=1}^T GHG_k c_{ij} X_{ijk_t} \\ \min z_2 = \sum_{i=1}^F RF_i Y_i + \sum_{i=1}^{\omega} \sum_{j=1|i \neq j}^{\omega} \sum_{k=1}^K \sum_{t=1}^T RS_{ijt} X_{ijk_t} \end{cases}$$

s.t:

- (2)  $\sum_{j=1|i \neq j}^{\omega} \sum_{k=1}^K X_{ijk_t} = Y_i \quad \forall i \in f, t$
- (3)  $\sum_{j=1|i \neq j}^{\omega} X_{ijk_t} = \sum_{j=1|i \neq j}^{\omega} X_{jik_t} = W_{ikt} \quad \forall i \in n, k, t$
- (4)  $\sum_{i=1}^{\omega} \sum_{j=1|i \neq j}^R D_{jt} X_{ijk_t} \leq Q_k \quad \forall k, t$
- (5)  $\sum_{j=1}^R D_{jt} Z_{ijt} \leq E_i Y_i \quad \forall i \in f, t$
- (6)  $\sum_{i=1|i \neq j}^{\omega} \sum_{k=1}^K X_{ijk_t} = 1 \quad \forall j \in r, t$
- (7)  $\sum_{i=1}^F Z_{ijt} = 1 \quad \forall j \in r, t$
- (8)  $M_{jt} - M_{it} \geq 1 - MA(1 - \sum_{k=1}^K X_{ijk_t}) \quad \forall i, j \in \omega, t, i \neq j$
- (9)  $\sum_{i=1}^{\omega} \sum_{j=1|i \neq j}^{\omega} \sum_{k=1}^K GHG_k c_{ij} X_{ijk_t} \leq GHL_t \quad \forall t$
- (10)  $\sum_{i=1}^{\omega} \sum_{j=1|i \neq j}^{\omega} \sum_{k=1}^K RS_{ijt} X_{ijk_t} \leq RL_t \quad \forall t$
- (11)  $\sum_{b=1|b \neq i, j}^{\omega} (X_{ibkt} + X_{bjkt}) \leq 1 + Z_{ijt} \quad \forall i \in f, j \in r, k, t$

$$(12) \quad X_{ijkt} = 0 \quad \forall i, j \in f, k, t$$

$$(13) \quad X_{ijkt} = 0 \quad \forall i, j \in r, k, t$$

$$(14) \quad \begin{aligned} Y_i, X_{ijkt}, Z_{ijt}, W_{ikt} &\in \{0,1\} \\ M_{it} &\geq 0 \end{aligned} \quad \forall (i, j) \in \omega, k, t$$

The first part of the objective function (1) represents the minimization of the sum of plant opening cost, transportation cost, vehicles' fixed cost and CO2 emissions cost. The second part of the objective function (1) represents the minimization of the sum of risk of plant opening risk and transportation risk. Constraint (2) ensure that if the plant is opened, be sure to move the vehicle. Flow conservation constraints are expressed in Constraints (3). Constraint (4) guarantees that the vehicle's capacity should not be exceeded. Constraint (5) guarantees that if plant opened, the plant's throughput should not be exceeded. Constraints (6) ensure that each retailer belongs to exactly one route. Constraints (7) ensure that each retailer is assigned to just one plant. Constraints (8) are sub-tour elimination constraints. Constraint (9) ensures that the total amount of CO2 emissions does not exceed a certain emission capacity. Constraint (10) ensures that the total amount of risk generated does not exceed a certain emission capacity. Constraints (11) specify that a retailer can be assigned to a plant only if there is a route from that plant going through that retailer. Constraints (12) and (13) determine the impossible arcs. Finally, Constraint (14) defines the variable types.

### 3. Experimental result

In this section we solve a small-scale example to help understand the model. Retailer's demand is deterministic and the planning time horizon is assumed to be 3 periods. Suppose 3 plants, 2 middle nodes, 2 retailers and 3 vehicles are present. First, we want to identify the open plants then we want to determine the best route for allocation to retailers and in the end we want to determine the used vehicles. The travel distances are provided in **Table 1**. The information on the capacity and cost rate for 3 vehicles are summarized in **Table 2**. **Table 3**, **Table 4** and **Table 5** show the risks amount between nodes per period. The average permitted risk generated levels among all periods is assumed to be 15. The average permitted CO2 emission levels among all periods is assumed to be 100. We also assume that the Price of CO2 per unit mass (equal to 2). **Table 6** shows the information on the risk, opening cost and throughput of each plant.

	$f_1$	$f_2$	$f_3$	$n_1$	$n_2$	$r_1$	$r_2$
$f_1$	0	2	1	4	6	5	3
$f_2$	2	0	2	3	5	4	2
$f_3$	1	2	0	1	3	4	1
$n_1$	4	3	1	0	2	1	4
$n_2$	6	5	3	2	0	2	2
$r_1$	5	4	4	1	2	0	2
$r_2$	3	2	1	4	2	2	0

Table 1 - Travel distances

Vehicle type (k)	$v_k$	$u_k$	$Q_k$	$GHG_k$
1	16	20	400	5
2	14	22	450	5
3	12	25	500	5

Table 2 - Vehicle characteristics

$t_1$	$f_1$	$f_2$	$f_3$	$n_1$	$n_2$	$r_1$	$r_2$
$f_1$	0	1	1	5	1	1	1
$f_2$	1	0	5	5	1	1	1
$f_3$	1	1	0	1	1	5	5
$n_1$	1	1	1	0	5	5	5
$n_2$	1	1	1	1	0	1	1
$r_1$	1	1	1	1	1	0	1
$r_2$	1	1	1	1	1	1	0

Table 3 – Risk amount between nodes in period 1

$t_2$	$f_1$	$f_2$	$f_3$	$n_1$	$n_2$	$r_1$	$r_2$
$f_1$	0	1	1	5	5	5	20
$f_2$	1	0	1	1	20	20	20
$f_3$	1	1	0	1	1	20	20
$n_1$	1	1	1	0	1	20	5
$n_2$	1	1	1	1	0	1	1
$r_1$	1	1	1	1	1	0	1
$r_2$	1	1	1	1	1	1	0

Table 4 – Risk amount between nodes in period 2

$t_3$	$f_1$	$f_2$	$f_3$	$n_1$	$n_2$	$r_1$	$r_2$
$f_1$	0	1	1	5	5	20	20
$f_2$	1	0	5	5	5	20	20
$f_3$	1	1	0	1	1	5	5
$n_1$	1	1	1	0	1	5	5
$n_2$	1	1	1	1	0	1	1
$r_1$	1	1	1	1	1	0	1
$r_2$	1	1	1	1	1	1	0

Table 5 – Risk amount between nodes in period 3

Plant type (f)	$CF_i$	$RF_i$	$E_i$
$f_1$	200	300	100
$f_2$	200	200	100
$f_3$	300	100	100

Table 6 - Plant characteristics

The proposed model is solved by GAMS 24.1.2. We used goal programming for solve this model. In **Table 7**, the decision variables are reported for the 3 periods. According to the results, it is clear that in some periods the route from the plant to the retailer is direct and, if necessary, middle nodes are used. Objective value for the first objective is 1119 and for the second objective is 327 also time of run is 0.23 second.

Time period	Opened Plant	Used Vehicles	The path is traversed
$t_1$	$f_2, f_3$	$k_2$	$f_3 \rightarrow n_1 \rightarrow r_1$
		$k_3$	$f_2 \rightarrow r_2$
$t_2$	$f_2, f_3$	$k_2$	$f_3 \rightarrow n_2 \rightarrow r_1$
		$k_3$	$f_2 \rightarrow n_1 \rightarrow n_2 \rightarrow r_2$
$t_3$	$f_2, f_3$	$k_1$	$f_3 \rightarrow r_2$
		$k_3$	$f_2 \rightarrow n_1 \rightarrow r_1$

Table 7 – Vehicles rout

#### 4. Conclusion

Routing of vehicles and location of plants are two crucial operational decisions in logistics distribution management. The efficient routing of vehicles in a company's distribution network results in economic and, in most cases, in environmental benefits. In this study, a mathematical model was presented to address a multi-period, multi-vehicle location routing problem in a many-to-many supply chain network for distribution hazardous materials. Also in this study we followed two goals, reducing amount of risk and CO2 emissions. To validate and verify the proposed model we solve a small-scale example.

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