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# Future mathematics teachers noticing mathematics: Knowledge-based reasoning

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*The article explores how future mathematics teachers (n = 26) at the end of their master studies notice moments in a lesson deemed important by experts and looks into their knowledge-based reasoning. A reflective task was given to the students and their written observation about the videoed mathematics lesson was compared against the expert analysis of the lesson. While all students commented on at least one important moment (the median was 3.5, expert rate 6), a third of their comments about these moments was of a subjective evaluation nature. They were mostly unable to provide a theoretical justification for their opinions. On the other hand, most students were able to suggest an alternative action to what they observed in the important moments of the lesson.*

*Keywords: Preservice mathematics teachers, teacher noticing, knowledge-based reasoning.*

One of teachers' specific skills is the ability to notice aspects of a mathematics lesson which "matter". There is a body of literature on (student) teachers' noticing skills showing the pattern of their attention to various aspects of lessons and investigating how it can be developed. In our previous work, we showed that while two-year master studies do not influence the pattern of attention much, even a short video-course does (Simpson, Vondrová, & Žalská, in press). After this course, student teachers (here students) described events rather than evaluated, noticed more of pupils' actions rather than the teacher's and paid more attention to the mathematical aspect of teaching rather than pedagogy. In a different study, we found that students paid limited attention to what experts considered to be crucial in the observed lesson (Vondrová & Žalská, 2015).

## Theoretical framework

Professional vision means seeing phenomena in a situation from the area of expertise which are different from those arising from lay viewings of the same situation (Goodwin, 1994). According to Sherin (2007), professional vision consists of selective attention (what the teacher pays attention to, or marking (Mason, 2002)) and knowledge-based reasoning (how he/she reasons about it). Professional vision in education overlaps the idea of teacher noticing: "teachers' professional vision involves the ability to notice and interpret significant features of classroom interactions" (Sherin & van Es, 2009, p. 22).

Some studies focus on the selective attention only, others take into account knowledge-based reasoning. There is a difference in noticing if the event is only described or reasoned about. For example, Sherin and van Es (2009) introduce Stance and distinguish Describe, Evaluate (from a subjective point of view) and Interpret (from a theoretical point of view). Stockero (2008) introduces five levels of reflection: Describing, Explaining, Theorizing (references to research or

course readings to support the analysis, or substantial evidence from transcripts and/or pupil written work), Confronting (alternate explanations for events and/or considering others' points of view), Restructuring (theorising and confronting to consider alternative instructional decisions).

It is the reflection on observation which is considered to be crucial to developing professional vision. Much of the research is focused on “video club” interventions where participants are guided to reflect on videoed lessons (e.g., Friesen, Dreher, & Kuntze, 2015; Liston & Gill, 2011; Simpson, Vondrová, & Žalská, in press; Star, Lynch, & Perova, 2011; Stockero, 2008). It has been shown that even a short video-course has an impact on noticing skills such as more specific comments, more attention to mathematical thinking and less on pedagogy, more attention to pupils and less to the teacher.

Studies on noticing mostly do not distinguish between less and more important events in a lesson to be noticed. Star, Lynch and Perova (2011, p. 120) even write:

[...] some classroom events are certainly more important than others, and it is critical that preservice teachers be able to attend to and interpret these important events. However, we believe that teachers do not have the ability to notice important events [...] until after they have developed the ability to notice (even trivial) classroom features.

The authors do say that the ultimate goal is for teachers to be able to notice important classroom events and they admit that it is not clear yet “whether it is better to focus first on improving teachers' awareness of the full range of (trivial and important) events (as was done here [in their course]) or to focus explicitly on only important events from the outset” (p. 132).

If researchers do look for *important moments*, they mostly find them in the mathematics of the lesson. For example, Star, Lynch and Perova (2011) identified ‘important questions’ in all observation categories, with the fewest from classroom environment and most from pedagogical choices made by the teacher, mathematical content addressed in the lesson and teacher-initiated communication. They say that “it is always more important to observe mathematical content carefully than to observe classroom environment carefully” (p. 132). In our research (Vondrová & Žalská, 2015), important moments are those which have been shown to play the key role in pupils' learning of mathematics. Hiebert and Grouws (2007) take it for well documented that pupils learn best when they have the most opportunity to learn. This concept is defined as “circumstances that allow students to engage in and spend time on academic tasks such as working on problems, exploring situations and gathering data, listening to explanations, reading texts, or conjecturing and justifying” (Kilpatrick, Swafford, & Findell, eds., 2001, p. 333). The opportunities to learn are influenced, among others, by “the kinds of tasks [teachers] pose, the kinds of questions they ask and responses they accept, the nature of the discussion they lead” (Hiebert & Grouws, 2003, p. 379). Finally, two key features of teaching that promotes conceptual development are part of important moments for us (ibid, p. 383, 387): teachers and pupils attend explicitly to concepts; pupils struggle with important mathematics.

The above was the starting point of our *expert analysis* of lessons. This methodology is not unknown in research on noticing. In (Blomberg, Stürmer, & Seidel, 2011), experts prepared items for rating video clips and an expert norm value system. In (Star, Lynch, & Perova, 2011), important

features of the lesson were selected by two raters and then used as a measure for assessing the participants' ability to notice. To validate measures to be used for assessing the ability to notice, the authors identified features of the lesson to be noticed and compared them against the video analysis made by six experienced teachers. Mitchell and Marin (2015) calculated the percent alignment between participants' scores and the master rater scores. Stockero, Rupnow and Pascoe (2017) made an expert analysis to determine *pivotal teaching moments*. We used the same process in our previous research to determine so called *expert mathematics specific phenomena*, i.e., the events in the lesson which experts deemed important for the successful learning of mathematics (Vondrová & Žalská, 2015). We found out, among others, that future mathematics teachers paid limited attention to the expert phenomena both at the beginning and at the end of their two-year master studies.

The study has two *research questions*: How do future mathematics teachers notice moments, deemed important by experts, at the end of their master studies? What is their knowledge-based reasoning?

## Methodology

We asked students to observe a video of a whole Czech Grade 8 mathematics lesson from TIMSS Video Study 1999. The piloting stage showed that the video included teaching practices understandable for students which they could relate to. The video is self-contained (the lesson has a clear introduction and ending, so that the knowledge of the previous lessons is not necessary) and it is quite rich in generic and subject-specific content (Blomberg, Stürmer, & Seidel, 2011) which is reasonably observable. The lesson leads to the discovery and formulation of *Thales' theorem*: “For any triangle  $ABC$ , it holds: a) If  $ABC$  is a right-angled triangle with hypotenuse  $AB$ , vertex  $C$  lies on circle  $k$  with diameter  $AB$ . b) If vertex  $C$  lies on circle  $k$  with diameter  $AB$ ,  $ABC$  is a right-angled triangle with hypotenuse  $AB$ . Circle  $k$  is called *Thales' circle* with diameter  $AB$ .”

The *participants* are future mathematics teachers of pupils aged 12 – 19 studying a two-year master studies at our university. In their final semester in January 2016, the whole group ( $n = 32$ ) was asked to watch a video of the lesson and write observations. They were to write what they “considered important and noteworthy”. They were told that there “were no correct or wrong answers” and that they should “feel free to write their honest views”. We received 26 written observations. The majority of students were in their mid-twenties and had teaching experience only from two fortnights of field placement at the primary and secondary schools.

The *analysis of data* was twofold. First, we used an expert analysis of the lesson in order to capture attention to important moments. Two authors of the paper, both mathematics educators, and four educators of future teachers of other subjects independently watched the lesson, repeatedly met and finally agreed on six important moments (here *expert phenomena*, Tab. 1). Naturally, what is considered important is rather subjective. We based our identification of such moments on the considerations from research on effective teaching presented above. By including experts from other fields, we ensured that the phenomena were really observable and not only apparent to an expert in mathematics education. In order to be coded as an expert phenomenon, the unit had to have a connection to mathematics; e.g., “The game was motivating for pupils.” is not coded as M1 (Tab. 1).

Second, to capture the participants' knowledge-based reasoning, we modified Stockero's (2008) levels of reflection: *Description* – a description of what can be seen in the lesson; *Explanation* – a naïve explanation of what was seen in the lesson using one's experience as a pupil or as a teacher; *Theorising* – interpreting what was seen using a theory; *Evaluation* – evaluation from a subjective point of view; *Alteration* – suggestion of an alternative approach to what was seen; *Prediction* – elaborating possible consequences of the event seen. We did not distinguish the depth of alterations or the quality of the theory used and we did not follow if the students' reasoning was in line with that of experts (this is something which can be done in further research).

Expert phenomenon	Comments about the:
M1: Work with geometric concepts within a game activity.	Ways concepts are made more precise and mathematical terminology is developed in the game.
M2: Discovering Thales' theorem, direction a).	Implementation of the activity (experimenting with a triangle ruler and two pins to find points making Thales' circle) and the way it can influence pupils' understanding.
M3: Inter-subject links.	History of the theorem and connection with geography.
M4: Thales' theorem, direction b).	Implementation of the task to verify the properties of triangles whose vertex $C$ lies on circle $k$ with diameter $AB$ and the way the teacher connects directions a) and b).
M5: Formulation of Thales' theorem.	Implementation of the task to formulate the theorem.
M6: Problems "to think about".	Implementation of two, seemingly more difficult tasks.

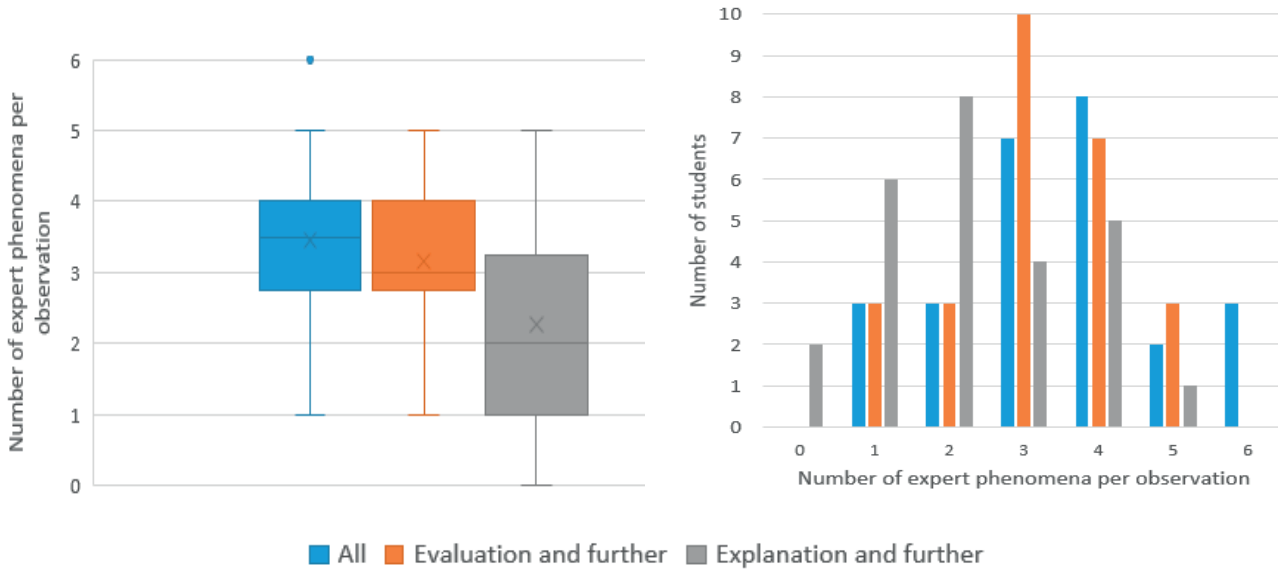
**Table 1: Results of the expert analysis of the lesson to be observed**

During the analysis, we first searched written observations for instances in which a student commented on M1 to M6. All parts commenting on the same phenomenon became one unit of analysis coded as M1, M2, etc. Thus, each written observation was assigned at most one code M1, M2, etc. We got 90 units from 26 students. Each was assigned one or more of the attributes capturing knowledge-based reasoning. An example is the unit: "The teacher accelerated the phase of the exact formulation of Thales' theorem very much and did not give pupils enough time to independently reach a conclusion. The teacher tried too hard so that Thales' theorem is formulated in a textbook way which, in my opinion, was for the worse." It was coded as Description, Evaluation, Explanation. Or the unit: "Pupils tried to formulate the theorem about the discovered property. I would first ask them to describe the property in their own words and only afterward to formulate the theorem mathematically. The pupils would not get tangled into it. The teacher puts down the theorem in an informative way, she emphasises that it is a right angled triangle. This can prevent some mistakes." It was coded as Description, Evaluation, Alteration, Prediction. All the units were coded by two authors independently and any discrepancy was discussed until a 100% agreement was reached.

## Findings

Fig. 1 shows box and whisker plots and a bar chart of the number of expert phenomena M1 to M6 per student/written observation. The box and whisker plot on the left concerns all 90 units of analysis. Bearing in mind that we focus on knowledge-based reasoning, we discounted the units

coded as Description only (the middle box and whisker plot, the number of units 82) and as Evaluation only, i.e., units coded at least Explanation (the box and whisker plot on the right, the number of units 59). The high median (3,5, the maximum being 6) of the number of comments on expert phenomena per student is not a surprise as the phenomena were rather visible in the lesson even to educators in other fields than mathematics. However, a third of the units (31) were of a subjective or descriptive nature only; students evaluated what they saw without any theoretical support: e.g., “I like that the teacher forces pupils to explain concepts (the game).” “The experiment leading to Thales’ theorem was nice.”



**Figure 1: The number of noticed expert phenomena M1 – M6 per student/observation**

The median for the number of *elaborated comments* on expert phenomena per student (Explanation and further) drops to nearly 2 (and there are even 2 students who have none). Tab. 2 shows the distribution of the 59 elaborated comments in terms of the phenomena noticed.

	Units ( <i>n</i> = 90)	Students ( <i>n</i> = 26)	Explanation	Theorising	Alteration	Prediction
<b>M2</b>	15	57.7 %	10	2	11	0
<b>M5</b>	15	57.7 %	10	1	7	2
<b>M4</b>	12	46.2 %	7	1	10	0
<b>M1</b>	9	34.6 %	6	4	4	1
<b>M6</b>	6	23.1 %	6	0	0	0
<b>M3</b>	2	7.7 %	1	1	0	0
<b>Total</b>	<b>59</b>		<b>40</b>	<b>9</b>	<b>32</b>	<b>3</b>

**Table 2: Distribution of elaborated comments among expert phenomena and the nature of elaboration**

The most commented on were M2 and M5. Both concern the core of the lesson – pupils experimenting to discover Thales’ circle and formulating Thales’ theorem. Fewer students (12) explicitly elaborated on the fact that the teacher also led pupils to see the inverse implication (M4). It is worth noticing that only about 58 % of students at the end of their master studies somehow

elaborated on the core activity of the lesson. Only 6 students (about 23 %) elaborated on the “problems to think about” the teacher used at the end. Note that the experts deemed M6 important because the first problem the teacher posed can be answered intuitively without knowing Thales’ theorem and thus there is really nothing for pupils to “think about”. On the other hand, the second problem is too difficult for the pupils as they have just learned about the theorem and moreover, they got no time to think about it.

Finally, we will look into the nature of comments. There were very few predictions in written observations. It appears that it does not come as natural to the students to comment on what might be the consequences of what they see in the lesson. There were 9 theorising units made by 6 students and 32 alterations made by 22 students. Tab. 3 depicts the types of theorising comments and suggestions for alternatives with examples (some units include two types and thus the total in Tab. 3 does not match the total in Tab. 2). Few units included the theory explicitly. Students mostly proposed alternatives concerning subject matter, for the implementation of the task to discover the theorem. They were suggestions of rather small changes. Students also wanted to modify the teaching methods used. Many proposed a constructivist way of teaching with a bigger involvement of pupils which is in view with the approach taken in their mathematics education courses.

Theorising	Diagnosis of knowledge	3	“The <i>diagnosis value</i> of the game activity was small as there was no explanation of problems which caused problems.”
	Concept development	5	“The pupil was to describe the concept in her own words which can in interaction with others <i>consolidate understanding of the concept</i> .”
	Inter-subject relationships	1	“By showing the picture of Thales and asking pupils to see where he came from, she made an <i>inter-subject link</i> in a nice way.”
Alterations	Subject matter	15	“The teacher could have asked what the radius of the circle is. It would also be appropriate to mention a right-angled triangle and that the recorded points are its vertexes.”
	Teaching methods	14	“I would have gone for the description in their own words first, only then would I ask for the mathematical formulation of the theorem.”
	Constructivist teaching methods	7	“The teacher should have given pupils more space when experimenting. They should have formulated themselves that the result was a circle, where its centre lies and what its radius is.”
	Teaching aids	4	“I would work with the drawing longer. They could have seen that the two endpoints of the segment should be omitted.”
	Management	3	“I would rather call more pupils to come to the blackboard.”
	Pupils’ mistakes	2	“I would concentrate more on pupils’ mistakes, such as, what will happen when the point does not lie on the circle and the like.”

**Table 3: Types of theorising comments and proposed alterations**

## Discussion and conclusions

In our previous study (Vondrová & Žalská, 2015), we found out for a different group of future mathematics teachers at the end of their master study ( $n = 53$ ) that they did not notice the moments experts in mathematics education deemed important for the success of the lesson (the median for

the number of expert phenomena per student was 2, the expert rate was 7). In the present study, the success rate of the students was higher (median is 3.5, the expert rate is 6). Even though we cannot directly compare the two results as the students saw different lessons (and the choice of the lesson matters, Simpson, Vondrová, & Žalská, in press), we can tentatively suggest one possible cause for it. The expert analysis was done in cooperation with teacher educators from other fields and thus, chances are that the phenomena selected by experts are more visible than in our previous study in which the phenomena were selected by experts in mathematics education only and were rather subtle for student teachers to notice. It raises the question of explicitness of phenomena to be seen in the video (seen by experts in mathematics education, by experts in other fields, by pre-service teachers) which impacts the nature of observations and consequently their perceived quality.

All 26 students commented on at least one expert phenomenon. However, only 15 students were able to elaborate on them in a professional way. Thus, 11 students, just before entering teaching profession, did not show knowledge-based reasoning in their observations. A third of the 90 units were of a descriptive or evaluative nature. It might have several reasons. First, (and it is the limitation of any study on noticing investigating written observations), we only reason from the written work, we cannot really say that students do not have sufficient knowledge. For example, taking into account the fact that during their mathematics education courses the instructor emphasises the use of theory on examples from teaching, it is somewhat disappointing that only 6 students could use theoretical notions for the analysis of events in the lesson. Students did not theorise spontaneously. But if we asked them to justify their subjective evaluations, they might be able to use theory to do so. Second, it may give more credit to what Star, Lynch and Perova (2011) say that student teachers have to first be able to attend to the trivial features of the classroom in order to notice important moments.

On the other hand, 22 students suggested at least one alternative for events deemed important by experts. Kersting et al. (2010) claim that suggestions for instructional improvement might be a sign of expertise of practising teachers. They found that “students of teachers who included suggestions for instructional improvement that they connected to mathematical content showed greater learning gains than did students of teachers who included either general pedagogical suggestions or no suggestions at all” (p. 178). Our students suggested alternatives mainly for the core activity of the lesson seen as important by experts. However, they failed to comment on and suggest alternatives for the time of the lesson in which the teacher used two “problems to think” in an insufficient way. Also in (Vondrová & Žalská, 2015), we found an insufficient attention on the choice of tasks and its cognitive demands. This is something which should be stressed in their university studies as the choice of tasks is one of the key features of a successful mathematics lesson (Hiebert et al., 2003).

The limitation of our study is that a) we presumed that a noticed phenomenon is a recorded phenomenon which does not have to be the case and b) the phenomena seen as important by the six experts do not have to be seen as important by experienced teachers. This will be pursued further. Our study provided more insight into the knowledge-based reasoning of future mathematics teachers at the end of their master study. Unlike in most studies on noticing, it focused on features of the lessons which are considered by research as important. It shows that students need more explicit support in order to be able to connect theory and practice, namely to interpret what they see

in the lesson in terms of theory, and to notice and interpret the choice of tasks made by the teacher and the nature of their implementation in the lesson. It remains to be seen how the attention to expert phenomena will be influenced by teaching experience.

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