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# Abstraction in Context and Documenting Collective Activity

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*In this report we advance the methodological and theoretical networking for documenting individual and collective mathematical progress. In particular, we draw together two approaches, Abstraction in Context (AiC) and Documenting Collective Activity (DCA). The coordination of these two approaches builds on prior analysis of grade 8 students working on probability problems to highlight the compatibility among the epistemic actions that ground each approach and drive the respective methodologies. The significance of this work lies in its contribution to coordinating what might otherwise be viewed as separate and distinct methodologies.*

*Keywords: Methodology, theories coordination, individual cognition, collective meaning making.*

## Background

In this report we advance the methodological and theoretical networking (Bikner-Ahsbahr, & Prediger, 2014) of two different approaches, the Abstraction in Context (AiC) approach with the RBC+C model commonly used for the analysis of knowledge construction by individuals or small groups; and the Documenting Collective Activity (DCA) approach with its methodology commonly used for establishing normative ways of reasoning in classrooms. In previous work related to this goal (Hershkowitz, Tabach, Rasmussen, & Dreyfus, 2014; Tabach, Hershkowitz, Rasmussen, & Dreyfus, 2014) we demonstrated how this coordination can illuminate the processes by which ideas shift from individuals and small group to the classroom community as a whole or vice versa. This combination revealed that some students functioned as “knowledge agents,” meaning that they were active in shifts of knowledge among individuals in a small group, or from one group to another, or from their group to the whole class or within the whole class.

We take the coordination between AiC and DCA a step further by explicating theoretical and methodological commonalities between the two approaches. These commonalities, which we first pointed to at CERME9 (Tabach, Rasmussen, Hershkowitz, & Dreyfus, 2015), drives further the integration of the two approaches, including what we refer to as environmental, underlying, and internal commonalities. The analysis in the present case led us to enhance the theoretical commonalities with data driven ones. We explicate these commonalities to set the stage for the analysis of students’ work, but first begin with a brief summary of the AiC and DCA approaches.

## Abstraction in Context and the RBC+C model

Abstraction in Context (AiC) is a theoretical framework for investigating processes of constructing and consolidating abstract mathematical knowledge (Hershkowitz, Schwarz, & Dreyfus, 2001). Abstraction is defined as an activity of vertically reorganizing previous mathematical constructs within mathematics and by mathematical means, interweaving them into a single process of

mathematical thinking so as to lead to a construct that is new to the learner. According to AiC, the genesis of an abstraction passes through three stages (ibid): (i) the arising of the need for a new construct, (ii) the emergence of the new construct, and (iii) the consolidation of that construct. AiC includes a theoretical/methodological model, according to which the description and analysis of the emergence of a new construct and its consolidation relies on a limited number of epistemic actions: Recognizing, Building-with, Constructing, and Consolidating (RBC+C).

These epistemic actions are often observable as they are expressed by learners verbally, graphically, or otherwise. Recognizing takes place when the learner recognizes a specific previous knowledge construct as relevant to the current problem. Building-with is an action comprising the combination of recognized constructs in order to achieve a localized goal, such as the solution of a problem or the justification of a claim. The model suggests Constructing as the central epistemic action of mathematical abstraction. Constructing consists of assembling and interweaving previous constructs by vertical mathematization to produce a new construct. It refers to the first time the new construct is expressed by the learner. Recognizing actions are nested within building-with actions, and recognizing and building-with actions are nested within constructing actions. Therefore, the model is called the nested epistemic actions model of abstraction in context, or simply the RBC+C model. The second “C” stands for Consolidation. The consolidation of a new construct is evidenced by students’ ability to progressively recognize its relevance more readily and to use it more flexibly in further activity.

### **Documenting Collective Activity**

The methodological approach of documenting collective activity (DCA) is theoretically grounded in the emergent perspective (Cobb & Yackel, 1996), a basic premise of which is that mathematical progress is both an individual constructive process and a process of enculturation into the emerging norms and practices of the local classroom community. That is, the personal and collective mathematical progress can be seen as two sides of the same coin. Collective activity of a class refers to the normative ways of reasoning that develop as students work together to solve problems, explain their thinking, represent their ideas, etc. These normative ways of reasoning can be used to describe the mathematical activity of a group and may or may not be appropriate descriptions of the characteristics of each individual student in the group. A mathematical idea or way of reasoning becomes normative when there is empirical evidence that it functions in the classroom as if it is shared. The empirical approach makes use of Toulmin’s model of argumentation (1958), the core of which consists of Data, Claim, and Warrant. Typically, the data consist of facts or procedures that lead to the conclusion that is made. To further improve the strength of the argument, speakers often provide more clarification that connects the data to the claim, which serves as a warrant. It is not uncommon, however, for Rebuttals or Qualifiers to arise once a claim, data, and warrant have been presented. Backing provides further support for the core of the argument.

The following three criteria are used to determine when a way of reasoning becomes normative: 1) When the backing and/or warrants for particular claim are initially present but then drop off, 2) When certain parts of an argument (the warrant, claim, data, or backing) shift position within subsequent arguments, or 3) When a particular idea is repeatedly used as either data or warrant for different claims across multiple days (Cole et al., 2012; Rasmussen & Stephan, 2008).

## Environmental commonalities

The use of both methodologies, RBC+C and DCA, requires quite specific classroom social norms (Yackel & Cobb, 1996). First, they require classrooms in which students routinely explain their thinking, listen to and indicate agreement or disagreement with each other's reasoning, etc. If such norms are not in place, then evidence is unlikely to be found of challenges, rebuttals, and negotiations that lead to ideas where knowledge is constructed and starts functioning as if shared by the whole class. We call such classrooms "inquiry-oriented classrooms" (Rasmussen & Kwon, 2007). Second, these classrooms require the intentional use of tasks designed to offer students opportunities for constructing new knowledge by engaging them in problem solving and reflective activities allowing for vertical mathematization. Both methodologies focus on the ways in which mathematical progress is achieved and spreads in the classroom. RBC+C focuses on individuals or small groups working in the classroom and DCA focuses on group or whole class discussions. In this sense, the two methodologies complement each other in analyzing a sequence of lessons including individual and group work and learning in whole class discussion and in tracing how knowledge is constructed and becomes normative.

## Underlying commonalities

Other characteristics of a classroom culture in which DCA and RBC+C methodologies might be enacted together are that the tasks are designed to afford inquiry and the emergence of new constructs from previous constructs by vertical mathematization (Treffers, & Goffree, 1985); such learning materials allow for interweaving collaborative work in both small-group work and whole-class discussions, where the teacher adopts a role that encourages inquiry in the above sense. Another underlying characteristic relates to the centrality of the shared knowledge. RBC+C characterizes shared knowledge as a common basis of knowledge which allows the students to make further progress. We find its counterpart in sociological terms, in the phrase "function as if shared" used by the DCA approach. What is common between the two constructs is the point that each operationalizes when particular ideas or ways of reasoning are, from a researcher's viewpoint, beyond justification for participants. At the collective level, ideas or ways of reasoning that function as if shared have the status of accepted mathematical truths for the group. At the individual level, consolidation results in individuals accepting something as a mathematical truth.

## Internal commonalities

DCA analysis helps illuminate what is happening on the social plane, while RBC+C analysis helps illuminate what is happening on the cognitive side. To elaborate, we highlight relationship between constructs suggested by the cognitive RBC+C analysis and their sociological counterparts suggested by the DCA analysis. We do that from a theoretical perspective and from an empirical perspective. To achieve this goal we begin with the following excerpt 1, used also in Hershkowitz et al. (2014) but for different purposes. It is a discussion between Noa and Gil, two eighth grade students working on a probability problem (see turn 95) during a group work period taken from the third lesson on this topic, and a bit of whole class discussion. This excerpt includes a DCA analysis, in particular classification of the marked parts of students talk (*shaded*) according to Toulmin's model as data [D], claim [C], warrant [W], backing [B], rebuttal [R] or qualifier [Q]. In addition, RBC+C actions were identified in students' talk (*italic*), and marked as recognizing (R), building-with (B),

end of the constructing action (C) or consolidating (CC) with respect to two knowledge elements:  
 $E_{xp}$  - experiment is needed in order to determine the chances and  $E_{xd}$  – experiment detailed.

No.		Utterance [DCA analysis]	RBC +C
95	Noa	(reads) ‘Is it possible to determine without experimenting what the chances are that we will take out a defective match from a matchbox? If yes, what is it?’ <i>You can’t know!</i> [D1] <i>Unless ... you have to experiment</i> [C1]! <i>You can’t know!</i> You need to experiment! I’m writing “You need to experiment!”	RB
96	Gil	<i>You don’t have to!</i> [C2, counterclaim]	B
97	Noa	Of course you do!	
98	Gil	“What the chances are of taking out a defective match from a matchbox?” <i>It’s 1 out of the number of matches in the box.</i> [D2]	R
100	Noa	<i>Right, so you take many boxes, how many, if, in the box</i> [W1]...	B
101	Gil	<i>Noa, it depends on how long you have been using the box, if you used it once then maybe it will be less ...</i> [Q1]	B
102	Noa	<i>No! If it’s defective! You have to take many boxes</i> [D1] <i>and see in each one if there is ... if there are let’s say 50 matches in each box and 1 is defective so it says on the box 1 out of 50</i> [W1], <i>so you have to experiment!</i> [C1, referring back to turn 95]	B
103	Gil	<i>So it’s 1 per the number of matches in each box</i> [W2].	R
104	Noa	<i>Not 1, there may be 2 defective matches in the box</i> [R2].	B
105	Gil	But what are the chances?	
106	Noa	But with 2 defective ones?	
107	Gil	But Noa, you are speculating ... <i>you can say 50 out of 50</i> [R to 104].	B
108	Noa	<i>But you can’t say 1 out of 50! Out of ... whatever!</i> [W to 104] What is the probability? It’s not correct what you are saying!	B
109	Gil	What isn’t correct?	
110	Noa	<i>Because just like you can’t say 2 out of the matches because you don’t know that it’s 2 or that it’s 1</i> [W1 = R2].	B
111	Gil	(writes) “ <i>can’t determine without experimenting.</i> ” [C3]	$C_{xp}$
112	Noa	<i>We can, if we experiment.</i> [C, slightly new claim of how to do the experiment]	$CC_{xp}$
113	Noa	<i>Ok, so what is the probability? It’s, we have to write that we won’t know</i> [D1] <i>until we experiment</i> [C1].	RB
114	Noa	Let’s write at the bottom, that <i>we need a few boxes</i> [D4], <i>suggest an experiment (dictates: “we need to take a few boxes of matches and see out of them</i> [D4]...” [Dictate together].)	B
115	Noa	No, wait! <i>How many matches does the box contain, and see how many defective matches are in it</i> [D4]... [Dictate together].	B

116	Gil	(continues to dictate) “ <i>then, check how many defective matches are in the box</i> [W4]” [Dictate together].	B
117	Noa	Then we will write “ <i>the probability is the number of defective matches in the ...</i> [C4, together with turn 123]” [Dictate together].	C <sub>xd</sub>
...			
122	Gil	Noa, <i>each box will come out differently.</i>	R
123	Noa	<i>So it's average</i> [C, note Data is previous argument], <i>not precise</i> [Q4]!	CC <sub>xd</sub>
		<b>Back to whole class discussion</b>	
135	Noa	In my opinion <i>you need to experiment</i> [C10]!	
136	T	Why?	
137	Noa	I don't know. I can <i>suggest an experiment</i> [Q10]	
138	T	Friends, listen, you need to express your opinion on what they said	
139	Gil	[addressing Noa] Why can't you say why you need an experiment, <i>you can't know how many matches there are in the box</i> [D10].	B <sup>1</sup>
140	T	Let's say I can reveal to you that there are 45 matches in the box.	
141	Gil	<i>And inside you have to [check].</i>	B
142	Noa	[you need to <i>take some</i> ] matchboxes [D11], you need to see <i>how many matches are in each box, and how many of them are defective</i> [W11].	
143	T	Let's say we know that information, what do I do with it?	
144	Noa	So ...	
145	Gil	So I do <i>the average</i> [C11, with 147]	B
146	T	What average?	
147	Gil	<i>Of the defective matches in each box</i> [C11, with 145]	C <sub>xd</sub>
148	T	And how is that going to help us know what the probability is that we take out a defective match?	
149	Noa	Let's say we have 2 defective matches in a box with 50 matches, so it's 2 divided by 50.	
150	T	2 to 50, what do you think?	
151	Gil	We are saying that <i>you can't do it without an experiment</i> [C10]. <i>You can't know how many defective matches there are because we don't know how many matches are in the box and we don't know either ... We can't speculate how many defective matches there are</i> [W10]. <i>We wrote that we need to take a number of matchboxes and see how many matches they contain, then count how many out of them are defective and do an average of how many defective matches in each box</i> [C11]. If we got 3 then it's 3 divided by 50.	CC <sub>xd</sub>

Table 1: Excerpt 1, Transcript from the class

<sup>1</sup> From this point on it is Gil who does the B and C actions

## RBC+C and DCA analysis

We begin by relating elements of the RBC+C and DCA analyses to each other, and then we relate the three criteria of the DCA approach to consolidation.

*Relationship between Recognizing and Data.* Theoretically, we argue that Recognizing actions are largely associated with Data. One uses some piece of information as Data because that piece of information makes sense to him/her. That is, this piece of information is relevant for the person; it is what the person selects for use (as Data). In the above excerpt, we see that Recognizing actions are primarily associated with Data. In some cases (e.g., turn 103), Recognizing actions can be associated with Warrants. In carrying out the DCA analysis, disentangling Data and Warrant is at times non-trivial, in which cases Recognizing actions can be sensibly associated with Warrants.

*Relationship between Building-with and Warrants.* Theoretically, Warrants establish a connection between data and claim; in order to establish such a connection, one needs to build-with what one has recognized. In the above excerpt this commonality is largely the case with some exceptions that need clarification. In turns 95/96 we had claims associated with building-with. These are the first building-with actions of this excerpt and thus the first ones of the part where the students are working on the present task. As a consequence, the building-with actions are somewhat shallow and make only claims without really warranting them. As such, this example does not pose a substantive threat to the theoretical conjecture. Similarly, turns 114 - 116 and 139 do not pose a substantive threat to the conjecture. These are the final utterances belonging to a constructing action; as such, they complete the constructing by explicitly stating the claim that was constructed. As we noted above, at times data and warrant are difficult to disentangle with certainty, hence building-with can be associated with data. Empirically, this is the case for turns 114 and 115.

*The relationship between Constructing and Arguments as a whole.* Constructing requires vertical mathematization. Constructing actions are usually much more extended than Recognizing or Building-with actions; they incorporate sequences of interweaving Recognizing and Building-with actions (plus the 'glue' between them). Similarly, arguments interweave data-claim-warrants and backings as a whole. Hence, in a line by line coding it is not feasible to indicate the holistic nature of an argument and it is typically indicated after a line by line coding (see for example Tabach et al., 2014). Moreover, arguments are usually co-constructed by several participants over several turns. Such interaction is also frequent in constructing actions.

*Consolidating and the three criteria for identifying function-as-if-shared ideas.* In consolidating actions as well as across the three DCA criteria for identifying when an idea functions as if shared, repetition, reuse, revisiting, or repurposing of earlier ideas frequently occurs. To clarify, in Criterion 1 there is a repetition, but the repetition is partial in the sense that some parts of the argument (Data, Warrants) cease to be explicitly stated. In Criterion 2 there is repurposing of previous part of an argument (e.g., Claim) as either Data or Warrant. In this sense there is a repeating and reusing, but for a different purpose. In Criterion 3 there is a revisiting of either Data or Warrants to establish new Claims. In consolidation, previous constructs are recognized as relevant (i.e., revisited), and then built-with, which means they are reused, often for a new purpose such as a new constructing action. Hence there are strong parallels between consolidation and the three DCA criteria. For example, in 151, DCA analysis shows that W10 (the warrant for Claim 10) turns into D11 (i.e. the

data for Claim 11); hence Criterion 2 is satisfied: the same part of the argument is reused with a different function. RBC+C analysis shows that knowledge construct *xd* is consolidated by being used again, and at the same time elaborated.

Further commonalities between consolidating and the DCA criteria can be seen by considering the characteristics of consolidation: awareness, self-evidence, flexibility, immediacy, and confidence (Dreyfus & Tsamir, 2004). Self-evidence links to Criterion 1 because the evidence is the Data, which drops off in subsequent arguments. The subsequent argument also then relates to immediacy and confidence in the validity of the idea. Flexibility links to Criterion 2 because components of an argument are being reused and repurposed (as sign of flexibility) in subsequent arguments. Similarly, Criterion 3 relates to flexibility, but in a different way. Here the flexibility lies in the fact that one is able to use an idea (e.g, Build-with) as Data or Warrant for a variety of different Claims. Hence close relationships exist between the criteria and characteristics of consolidation.

We conclude this report by returning to vertical mathematization, which was highlighted as an Environmental commonality. We also see vertical mathematization as an Internal commonality. Both methodologies work from the premise that vertical mathematization is core to mathematical progress. In the RBC+C approach, consolidation is vertical mathematization and, as we argued above, the consolidation is closely linked to the three criteria.

## Conclusion

We now turn to discussing some implications for research. In addition to offering a theoretically and empirically grounded approach for coordinating methodologies for individual and collective mathematical progress, there exist specific ways in which this coordination can play out. For example, one could choose an individual student within the classroom community and trace their constructing actions for the ways in which they contributed to the emergence of various normative ways of reasoning. Alternatively, when considering a normative way of reasoning, a researcher could investigate who the various individual students are that are offering the claims, data, warrants, and backing in the Toulmin analysis used to document normative ways of reasoning. How do those contributions coordinate with individual student constructions? For instance, does a student ever utilize an utterance that a different student authored as data for a new claim that they are authoring, and in what ways may that capture or be distinct from other students' individual mathematical meanings? Future research could take up more directly the role of the teacher in relation to individual and collective level mathematical progress. More generally, however, this report contributes to an emerging discourse on theories and ways in which different theoretical approaches can be profitably networked (e.g. Bikner-Ahsbabs & Prediger, 2014).

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