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The perspective of indexicality: How tool-based actions and gestures contribute to concept-building

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In this paper we apply the theoretical perspective of indexicality to gesture use on a digital place value chart on the iPad and show that this perspective allows for explaining how mathematical meaning is accumulated linking specific gestures to the actions performed on the digital device. Thus, practical dragging leads to structural dragging via operational dragging, resulting in a shift of the representational function of gesture (gesture-of) to the epistemic function of gesture (gesture-for).

Keywords: Digital tool, indexicality of gesture, gesture-of and gesture-for, modes of dragging.

Introduction

De Freitas and Sinclair (2013) have proposed to adopt a new materialistic view in research on learning mathematics, specifically for technology use. In this respect, the indexicality of signs is relevant (Sinclair and de Freitas, 2014). This paper investigates the added value of the theoretical perspective of indexicality while adopting it to the analysis of technology-based epistemic processes in the design project DeciPlace, to understand more deeply how acting based on a multi-touch surface contributes to building mathematical knowledge.

DeciPlace is a design-based research project. Its main goal is to develop a task sequence for conceptualizing decimal fractions as structures in small groups of students by the use of a digital place value chart (DPC) on the iPad (Behrens, 2016; Behrens & Bikner-Ahsbahs, 2016). The core approach is to act tool-based with the DPC. Thereby, conceptualizing is not considered as a pure individual cognitive process but as a collective communicative process of constructing mathematical structures in an embodied and multimodal way (see Krause, 2016). For this kind of learning, the instrumental approach (see Drijvers, Kieran & Mariotti, 2010), which is often used for research on technology learning settings, fails to attain insight into the epistemic process in all its aspects. Our focus is on tool-based acting and interacting. However, the way this contributes to knowledge construction is not yet understood in-depth. In this paper, we will first present the tool, then outline what we mean by indexicality and finally apply this perspective in the analysis of some episodes from the DeciPlace data corpus to show how the adoption of this perspective deepens insight into tool-based collective learning.

The digital place value chart (DPC)

The digital place value chart on the iPad (designed by Ladel & Kortenkamp, 2013; itunes-App: “Place Value Chart”) can represent a number on three different levels in parallel: In the bottom row of the chart tokens can be displayed by tapping on the screen. In the upper row the name of the place value as well as the number of tokens in each column of the place value chart is indicated. Additionally the standard notation of the represented number can be displayed above the chart. When the user drags a token to the next (after next) column to the right, the token is de-bundled

automatically into ten (hundred) tokens and so on (see Figure 1). The other way around, by dragging a token to the next (after next) column to the left, either nine (99) tokens move along with the dragged token bundling together to one token in the new column or – if there are not enough tokens to come along with – the dragged token slides back to its origin. Hence, in contrast to traditional place value charts (paper and pencil or material tokens) this digital version de-bundles and bundles (if possible) automatically, while the represented value is kept constant, and it gives feedback if the intended manipulation is impossible.

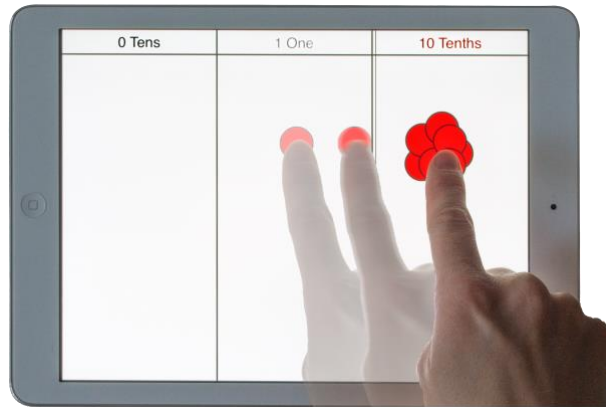


Figure 1: De-bundling a token from ones to tenths in the digital place value chart

Recent results: Three modes of dragging

In the analysis of activities on the DPC by a pair of grade 6 students who were introduced to the extension of the place value system from natural to decimal numbers, we noticed a shift in the students' activities from dragging-actions on the iPad to dragging-gestures (incorporated by the characteristic movement to the right or left), becoming more and more independent from the representation on the screen during the course of interaction with the DPC (see Behrens & Bikner-Ahsbabs, 2016).

Using an epistemic analysis based on ideas developed by Krause (2016) we were able to distinguish three modes of dragging:

- *Practical dragging* comprises actions of dragging tokens performed directly on the digital place value chart, when students use the function of bundling or de-bundling by dragging with a practical aim without scrutinizing the underlying principle.
- *Operational dragging* can be observed when students are able to foresee the result of bundling or de-bundling or when they want to test something by dragging a token intentionally in the chart, so that they use both transformations to fulfil a particular goal. This mode of dragging can be manifested either as a direct act of dragging in the digital place value chart or as a dragging-gesture referring directly to the chart.
- *Structural dragging* becomes apparent, when a particular mathematical structure is being described generally and the movement of dragging is represented in a gesture from left to right or vice versa performed independently from any concrete representation.

Aim of this paper

Using these three modes of dragging, we were able to describe the epistemic role of gestures in processes of building the decimal fractions' concept, which is mainly based on the principle of bun-

dling and debundling (see Behrens & Bikner-Ahsbahr, 2016). However, we were not able to understand in detail how these three modes of dragging contributed to processes of learning based on the digital artifact. In this paper, we will address this topic by answering the following research question:

How do actions and gestures “regarded as indices” contribute to conceptualizing the decimal fractions’ concept based on the digital place value chart on the iPad?

Describing the theoretical approach: Actions and gestures regarded as indices

In this paper, we focus on the connection between the digital place value chart and a pair of individuals interacting with each other with regard to the device. To examine this interaction we focus on signs which are produced in the setting, such as gestures, inscriptions, tokens on the display, the artifact itself and so on.

The students’ collective epistemic process is manifested in their actions (based on the tool), their verbal utterances and other semiotic resources. These actions can be analyzed in a multimodal way based on the concept of the semiotic bundle (Arzarello, 2006), which consists particularly of gestures, speech, inscriptions and relations among each other. To emphasize the influence of tool-based actions on speech, gesture and inscriptions, we adapt the perspective of the indexicality of actions on multi-touch devices described by Sinclair and de Freitas (2014). This perspective draws on Peirce’s notion of semiotics, “in which signs (icons, indices and symbols) differ in terms of the nature of the relationships between the signifying sign and the signified” (Sinclair & de Freitas, 2014, p. 355). According to Peirce, a sign is defined by a triadic relation between sign, object and interpretant:

A sign, or *representamen*, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the *interpretant* of the first sign. The sign stands for something, its *object*. It stands for that object, not in all respects, but in reference to a sort of idea, which I have sometimes called the *ground* of the representamen. (Peirce, 1932, 2.228, emphasis in the original)¹.

As a consequence, a sign comes into being when there is an individual who produces an interpretant according to the relation between the sign and the object. This relation distinguishes a sign to be an icon, an index or a symbol (Peirce, 1994, p. 239).

While an icon is characterized by producing the idea of resemblance of sign and object in individuals and symbols are defined to be conventionalized signs, an index

refers to its object not so much because of any similarity or analogy with it, [...] as because it is in dynamical (including spatial) connection both with the individual object, on the one hand, and with the senses or memory of the person for whom it serves as a sign, on the other (Peirce, 1932, 2.305, cited in Sinclair & de Freitas, 2014, p. 355f.).

¹ This refers to Peirce in terms of ‘(section/page)’, where 2.228 stands for ‘volume 2, paragraph 228’.

By this, indexical signs “show something about things, on account of their being physically connected with them” (Peirce, [1894] 1998, p.5; cited in Sinclair & de Freitas, 2014, p. 355).

Sinclair and de Freitas (2014) emphasize that also the action that resulted in the emergence of another sign may be included in the concept of indexical signs:

For instance, the chalk drawing of a parallelogram on a blackboard is often considered to be an iconic reference to a Platonic conception of parallelogram, but it is also an indexical sign that refers to the prior movement of the chalk. This latter indexical dimension is usually not emphasized in the semiotic study of mathematical meaning making, since we tend to focus on the completed trace and dislocate it from the labour that produced it. (Sinclair & de Freitas, 2014, p. 356)

Taking this assumption into account, we can further assume that every process of producing a sign is an indexical sign referring to the sign and the sign itself refers indexically back to the effort which produced it.

They conclude that “indexation becomes part of an experience that exceeds itself, and is thus self-generative” (p. 359). Thus an action on multi-touch screens leaves traces, hence, these traces as well as a hand gesture may refer to the original action when this gesture is produced in a similar way to the action on the device. Taking up this theoretical perspective we want to examine the above described modes of dragging with respect to “how they function as indexical, material actions” (p. 360) trying to explain how dragging movements contribute to build the concept of decimal fractions.

Applying the theoretical approach: Indexicality in dragging modes

We are re-analyzing episodes of our design-study DeciPlace (Behrens & Bikner-Ahsbabs, 2016) in order to answer the above posed research question. This way we will investigate the added value of this indexicality perspective.

As described above, practical dragging takes place when tokens are dragged directly in the digital place value chart from one column to the other either to the right or to the left without observable intention. The digital place value chart reacts to this practical dragging of one token to the next column on the right by de-bundling this token into ten tokens. Likewise, the DPC can bundle ten tokens to one in the next column, when one token is dragged into the next column on the left. Bundling and de-bundling by dragging keeps the value of the decimal number the same. This way, dragging can be linked to bundling or de-bundling in a material way being performed as an action of dragging from right to left or vice versa on the DPC. According to Sinclair and de Freitas (2014) an action of dragging leaves traces – e.g. the new arrangement of tokens within the chart – which refer back to the original action of dragging and vice versa.

In the following scene, two students are asked to find different representations for the number 101 in the place value chart:

- 1 Bella: I’ll just try (*drags a token from hundreds to tens within the digital place value chart on the iPad (in the tens’ column ten tokens emerge)*, see Figure 2) Woah
- 2 Hanna: Ten and One.



Figure 2: De-bundling as dragging from hundreds to tens

In this scene of practical dragging the newly emerged bunch of ten tokens in the tens' column can be seen as an iconic sign representing ten tens and therefore the number 100. Additionally the bunch of ten tokens refers back to the action of dragging a token from the hundreds' column to the tens' column on the DPC and at the same time to the DPC's reaction by letting the token explode into ten tokens without changing the represented number (concept of de-bundling). Thus, a new sign emerged by indexation linking the action of dragging on the DPC with a visual representation of de-bundling. This in turn produced a reaction of astonishment expressed by Bella ("Woah"), again referring back to the DPC's reaction on her dragging.

When the students get more and more familiar with the DPC in phases of practical dragging, they may apply movements of dragging intentionally for a specific purpose, for example while making a conjecture or predicting what will happen in the case of dragging directly in the DPC. Operational dragging can take place as an action of dragging *on the surface of the device* and also as a gesture *directly above the surface* referring to the DPC but without touching it (the so-called potential level, see Krause, 2016, p. 134–139). Because of the material link between dragging actions and dragging gestures both being performed by a similar movement from left to right or vice versa, dragging gestures may refer to previous actions of dragging and what is already linked with them. In this respect, operational dragging being conducted as a gesture on the potential level of reference is materially linked to the performance of dragging in the DPC and at the same time linked to the traces which are potentially and materially produced by that.

In the following situation the place value "tenth" is just introduced by the interviewer adding the new column named "tenth". The students are asked to find further representations for the number 4. At first, the students tap in four tokens into the ones' column of the DPC.

- 3 Bella: Can I drag over one (*moves her right hand at the bottom of the iPad from the ones' column to the tenths' column; see Figure 3*) and see what gets out?
- 4 Interviewer: Try.
- 5 Bella: (*drags a token from the ones' to the tenths' column, where ten tokens emerge; see Figure 4*)
- 6 Hanna: Ohh.
- 7 Bella: So ten are (1 sec.) one one (*pointing to the ones' column*) are ten tenths (*moves her hand to the right flexing and extending her index finger pointing to the tenths' column*).

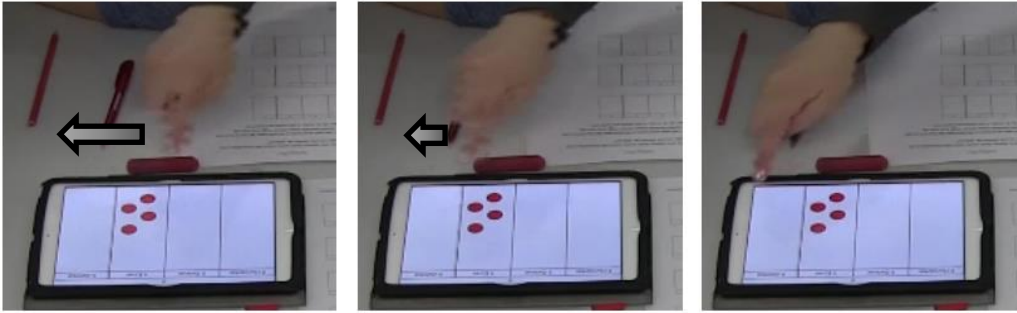


Figure 3: Dragging-gesture to the right from ones to tenths above the chart



Figure 4: De-bundling ones to tenths by dragging

In this scene Bella intentionally exploits the DPC's function of de-bundling to get insights into the relation between ones and tenths (lines 3 & 7). Thus, the two dragging-gestures (lines 3 & 7) as well as the dragging-action (line 5) represent operational dragging. From the perspective of indexicality, both dragging-gestures from left to right (acc. to the view of the students) can be assumed to be linked to previous actions of dragging from left to right on the DPC and their traces, because of the close resemblance between gesture and action. By this, dragging-gestures become indices of dragging-actions including the experiences and assumptions that have been made by dragging tokens on the DPC from left to right, e.g.: "when I drag a token to the right, the number of tokens changes / increases" or "when I drag a token to the right, the represented number remains the same".

Although both dragging-gestures are executed more or less equally, in relation to speech they function differently. The first dragging-gesture seems to focus on the movement of dragging to the right from ones to tenths specifying "what" and "where" (Krause, 2016, p. 125) to "drag over" (a wording frequently used by the students). In contrast, the second dragging-gesture adds the way by which the insight that "one one are ten tenths" (line 7) was gained, referring back again to the experiences and assumptions made by the action of dragging rightwards on the DPC just before (line 5). Similar to the notion of model-of and model-for (van den Heuvel-Panhuizen, 2003, p. 14) we have identified the development from a gesture-of (representing the action of dragging) to a gesture-for (representing the procedure of arriving at this particular conclusion) mediated by operational dragging.

Structural dragging is done when a dragging-gesture is conducted in the gesture space without visible references to any concrete representation of a place value chart. This was used particularly when describing the concept of bundling within the digital place value chart, which is a main step of conceptualizing the decimals' structure. Assuming that the characteristic movement from left to right or vice versa indicates the material link between dragging-actions and dragging-gestures, we can con-

sider structural dragging to be an indexical sign on the traces left by practical and operational dragging including all experiences, intentions, and conjectures made before.

During the whole design experiment, the two students established a shared context where they observed the other student dragging and negotiated shared answers to the tasks. In the first situation here, Bella performs the dragging (line 1), while Hanna sums up the emerged result (line 2). In the second scene Hanna reacts to Bella's dragging on the chart (line 5) and the chart's reaction by astonishment (line 6) and is therefore likewise involved in conceptualizing de-bundling. Thus, a dragging-gesture of one student can also be taken as an index to previous dragging-gestures by the other student. This way, both students and the device constitute an ecology of tool-based interaction to build the concept of decimal fractions.

Discussion: Reflections and consequences

Applying the theoretical perspective of indexicality we have reconstructed how the action and the gesture of dragging can accumulate more and more aspects about bundling and de-bundling. This process of mutual reference between indexical actions and gestures brought forward the conceptualization of bundling and de-bundling as the basis for the concept of decimal fractions. At the same time the dragging-gestures detached more and more from the concrete dragging-actions on the DPC and became shared signs by enriching indexical references.

Similar to the shift from model-of to model-for (van den Heuvel-Panhuizen, 2003, p. 14), a shift from gesture-of to gesture-for was observed, that is: A dragging gesture first represents the action of dragging (gesture-of), later the dragging-gesture is used as an epistemic means to structure the description of the base-ten structure (gesture-for). Operational dragging can be considered as an intermediate state. It produces a change of view from dragging tokens to bundling and de-bundling as the underlying concept. Hence, adding indexicality to gesture analysis may improve our understanding in how epistemic processes progress.

Whether or not this theoretical perspective keeps being fruitful for tool-based learning in general can only be answered by further empirical research. The main issue will be how this perspective can be fruitfully linked with local theories and models for learning specific contents, such as expanding natural numbers to decimals fractions. For that, we will apply the indexicality perspective to additional data from design experiments with another 15 student pairs in the DeciPlace project, attempting to prove our results and gain further insight into the role of tool-based dragging-actions and -gestures for contributing to conceptualize the decimal fraction's structure.

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