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Networked theories for a didactical study of communities of mathematics teachers

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Existing research about communities of mathematics teachers mainly draws on sociocultural theories. In complementarity, our project consists in studying more deeply the place of mathematics within communities of teachers. We also define the notion of didactical study of communities of mathematics teachers to focus on a general research project on the place of mathematics in teacher community dynamics. We propose a theoretical model to analyze the problems of the community stemming from the network of several theoretical frameworks and approaches. The theoretical model is particularly based on arguments given by the members of the community and mathematics teaching resources. Two contrasted case studies show the relevance of our focus on mathematics.

Keywords: mathematics teaching, teacher communities, problem of a community, argumentation, mathematics teaching resources.

Introduction

This paper focuses on communities of mathematics teachers. We are particularly interested in the role of mathematics in community dynamics, its epistemological characteristics, its validation modes and its conditions of teaching. We wonder about the singularity of a community of mathematics teachers: in what way the study of a community of teachers in mathematics education is different from the same study in physics education, in geographic education and so on?

In this contribution, we discuss a general question which is the focus of our research project: “What network of theoretical approaches is needed to determine the singularity of communities of mathematics teachers?” We are in an exploratory stage in which we develop theoretical constructs. In the three last CERME workgroups about networks of theoretical approaches, two authors discussed the issue of communities of mathematics: Palmer (2013) and Castela (2013). Palmer notes that Wenger’s (1998) theory of communities of practice does not focus on mathematics education and/or teaching. Castela (2013) stresses the fact that the activity of communities of teachers and different modes of validation of mathematical knowledge in these communities are determined by the institutional conditions and constraints. For instance, teacher’s communities do not have the same role as the researcher communities in the knowledge validation process. These researchers mainly draw on sociocultural theories in order to understand some phenomena. In complementarity to these research works, we aim to determine the singularity of communities of mathematics teachers. Our research also contributes to the French ReVEA¹ project, which concerns

¹ ReVEA is a project funded by the French National Agency for Research www.anr-revea.fr/

the interactions between teachers (individual and collective aspects of their work) and their resources in four disciplines, including mathematics.

Before presenting the kind of networking needed to study the communities of mathematics teachers, we shall define the way we use some words in the following. Many researchers use *communities of teachers* to talk about the forms of collaboration between teachers (Robutti et al., 2016). In the same line, we use the term *community*, loosely defined and not aligned with particular theoretical ideas. A community of teachers designates several teachers engaged together professionally to achieve a common project related to mathematics teaching. We propose also to characterize “*community dynamics*” as the role of the members and their individual projects, the verbal interactions, and the interactions with the resources for teaching.

Communities of mathematics teachers: Networking of theories needed

For a didactical study of communities of mathematics teachers

Several researchers on the practices of mathematics teachers have already studied the issue of communities of teachers (Graven, 2004; Gueudet, Pepin, Sabra, & Trouche, 2016; Jaworski, 2006; Krainer, 2003).

Some researchers explore the place of collective work in teacher training (Llinares & Krainer, 2006; Jaworski, 2006) and more recently its place in online teacher training (Borba & Llinaras, 2012). Furthermore, relying on the theory of CoP, Jaworski (2006) develops the *communities of inquiry* approach. She stresses that developing critical thinking on teaching practice takes place in long-term processes of collaboration between teachers, trainers and researchers. This approach takes the specifics of mathematical epistemology in teaching and learning practices into account. However, from the standpoint of research, the communities of inquiry approach only allows the study of communities determined by the particular operational process of this approach.

Other researchers in mathematics education characterized different forms of communities but the place of mathematics is not always considered as central in the theories mobilized and developed (for example, see Krainer, 2003).

In order to identify the singularity of communities of mathematics teachers, we define the notion of *didactical study of a community of mathematics teachers* as the study of the conditions and the constraints of learning and sharing knowledge related to mathematics education through the design of resources and interaction between members of this community. Let us specify each element of this definition:

- The conditions and constraints of learning and sharing knowledge are those that allow, encourage, restrict or inhibit learning in the community.
- Knowledge related to mathematics education consists of *subject matter knowledge* and *pedagogical content knowledge* (Ball, Thames, & Phelps, 2008) that affect the condition of mathematics diffusion.
- Resources are anything that is developed and used by teachers (and students) in their interaction with mathematics to teach or learn inside/outside the classroom (Pepin,

Gueudet & Trouche, 2013). This element is preponderant in the study of a community and plays the role of a mediation and communication tool.

- By “interaction between members”, we do not only mean simple verbal, oral or written, exchanges but also sharing resources and experiences through online activities.

The next sections discuss the theories and approaches that help us to take into account each of the elements above.

Learning and sharing knowledge in CoP: through problems of the community?

Wenger (1998) proposes a general theory built in the context of knowledge management to study communities of practice (CoP) defined as apprenticeship communities. He emphasizes that learning takes place during the exchanges between members, especially those exchanges related to their joint (community) problems. The treatment of these problems leads to new knowledge emerging from the participation process.

Wenger’s theory does not give specific tools to consider mathematics, though we rely on this general idea proposed by Wenger of considering the problems of the community. We distinguish between *problems of the community* – identified by the researcher – and *community problems* – identified by the members – even if both can sometimes be the same. In this contribution, according to our focus on mathematics, a *problem of the community* is a phenomena or event that prevents or hinders the achievement of the common project and that is linked with mathematics as a science or as a teaching field.

At this point of our contribution, we hypothesize that the study of the problems of a community permits to determine a part of the community dynamics. We propose to realize it by following up the resources and the argumentation process.

Resources for teaching mathematics

The documentational approach of didactics (Gueudet & Trouche, 2012) attempts to frame the process of the design of teaching from the point of view of the teachers’ work. It is a socio-cultural approach that permits us to study the interleaving between the community resources and the teachers resources that are involved (Gueudet et al., 2016). The teacher interacts with resources, selects them, and works on them (adapting, reviewing, reorganizing, etc.) in some processes where *design* and *enacting* are intertwined. The intertwining of new resources and interactions with other actors in the educational system creates new teaching knowledge and carries teachers’ professional development (Gueudet & Trouche, 2012).

In order to achieve common projects, the different individual documentational geneses interact (Gueudet et al., 2016). The consideration of resources in the community of teachers should take into account: 1) the gathering, creating and sharing of resources in order to achieve the mathematics teaching goals of the community; 2) the result of this process, the shared resources and shared associated knowledge (what teachers learn together from conceiving, implementing, and discussing resources).

In the documentational approach, we use the definition given by (Pepin, Gueudet, & Trouche, 2013): “all the resources which are developed and used by teachers (and pupils) in their interaction

with mathematics in/for teaching and learning, inside and outside the classroom”. The advantage of this definition is that it determines the documentation work in terms of interaction with mathematics. In this perspective, the resources of the community could be considered like a highly structured system, where resources are linked according to the level of teaching, the mathematical topic, the teaching mode, and the evolution of the curricula.

The documentary approach is a framework designed to articulate with some other ones. To answer our question, we need a complementary framework to consider the epistemological characteristic of mathematics.

Interaction between members: the place of argumentation

Taking into account the interaction between the members of a community implies taking the oral or written verbal exchanges into account. We have chosen to identify the arguments given by the members during the exchanges and to interpret them with the help of the resources that were mobilized or designed. We define argument as the reason that one presents to defend a certain point of view (Plantin, 1990). It is always oriented toward the decision that speaker wants to take (Ibid.).

Pedemonte (2007) stresses, in the case of mathematics, what can be considered an argument. She notes that it is necessary to look at the proposition and at the context, which allows us to remove misunderstanding. More specifically, she distinguishes between arguments to convince – based on rationality – and arguments to persuade – for example based on authority.

Plantin (1990) and Pedemonte (2007) considered argumentation in the learning processes. Which kind of transfer could we make in the case of teachers? Some aspects need to be developed and clarified in our future work: typology of arguments, impact on the community dynamics.

Emerging model for analyzing a problem of the community: criss-crossing of several theories and approaches

Problems of the community that we consider are linked to mathematics as subject matter knowledge and as pedagogical content knowledge (Ball et al., 2008). We hypothesize that the treatment of these problems is related to: (1) the role of members in the community of teachers; (2) the arguments given by members; and (3) the use of resources that the teachers mobilize or produce to support the process of argumentation. We aim to determine the articulation between these components that could emerge (or not) in the treatment of these problems (members, arguments, resources) and the decision-making. We consider two case studies.

Problem of the community treatment: two contrasted case studies of a didactical study of a community

We developed the model on two contrasting cases to: 1) highlight the importance of considering the problem of the community; 2) extend the validity of the model.

We conducted the first case study at high school level. The community is spontaneous. The mathematical knowledge considered is the notion of “function”. The researcher is outside of the community.

We conducted the second case study at the primary level. The community is intentional. The mathematical knowledge considered are research situations and proof. The researcher is within the community.

In both cases, we present some elements of context, the arguments given by the members of the community and their role in the treatment process of the problem of the community. Finally, we present the interpretation of the arguments exchanged by specifying the determining factors in the decision-making. We provided the details of the analysis in another paper (Georget & Sabra, 2015).

First case study: grade 10 textbook project

Sesamath is a mathematics teachers association founded in 2001. It aims to design and disseminate free and open resources. Interactions between the Sesamath members take place on online platforms and by mailing lists. Sesamath is structured around several project communities. We consider one of these communities: the grade 10 textbook project.

The chosen corpus corresponds to a thread of discussion about the progression of the topic “functions” in the textbook it is intended to design. “Functions” is the main topic to introduce in mathematical analysis at secondary level in France. The curriculum introduces functions for the first time at the grade 10 (15-16 years old), which generates a problem of the community that led to a controversy that we formulate by the following words: “What progression of the topic 'functions' must be adopted?”

Three members of the community are mathematics teachers with more than ten years of experience each. They play a particular role in the discussions: Mr W as a designer and a commentator and reviewer of resources, Ms A as a designer and a reviewer and a commentator of resources, and Mr H as a coordinator for several Sesamath projects and coordinator between Sesamath and other external communities.

This first case study illustrates arguments given to treat a problem of the community: (1) mathematical arguments with a discussion on the mathematical contents (connectivity between some concepts); (2) epistemological arguments about “functions” with a level of abstraction, depersonalization, and decontextualization of associated concepts; (3) didactical arguments coming from a personal experience or by consideration of the topic “functions” in the different level of teaching; and (4) arguments mainly linked to the design of resources.

The interpretation of the arguments during the discussions cannot be made independently of the context of the problem (structuring of the "functions" topic in the textbook). The mathematical argument takes importance in the formulation of the decision. We consider some arguments made by Mr H (argument to persuade) as a strong determinant factor in the decision. Mathematical arguments launched by Mr W and confrontation of his proposal with existing textbooks are also a second determinant factor in the decision. The members formulated the decision in terms of mathematical arguments.

Second case study: CoP and situations of research and proof at the primary level

The second case study is about an experiment of an intentional CoP during three years. We implemented a CoP to develop the practice of situations of research and proof between peers in

some classes at the primary level. It was composed of teachers and the researcher-coordinator of the CoP. We proposed several “mathematical situations” on online resources to the teacher and the design of these resources has been a key point in this CoP.

The data analyzed are about an extract of transcription of a meeting at the end of the second year with three teachers and the researcher-coordinator. One of the teachers, Mr D, does not understand the proof of a mathematical problem. He brings the usability of the resources into question by pointing out that the resources do not take his lack of mathematical knowledge into account.

For Mr D, the introduction of the proposed situations in his practice is rather exceptional. Ms S is voluntary in her implication in the CoP and for practicing these situations with her pupils. Mr H is more experienced than Mr D and Ms S, who have also problems of understanding for several resources, especially Mr D about a certain resource which Ms S understands well. Some exchanges help Mr D to understand the proof of the mathematical problem after which Mr H closes the debate implicitly. Mr D and Ms S do not rekindle it. Following some controversies about the usability of several resources, there is therefore an implicit decision not to modify them.

During the interactions, both kinds of arguments are exchanged: (1) resource design arguments about the usability of the teaching resources given by Ms S and Mr D who launch the interactions; and (2) mathematical arguments given by Mr H and the researcher/coordinator that permit the understanding of the proof by Mr D.

At the end of the exchanges, the decision is implicitly validated by mathematical arguments: the proof presented by the resource is valid and understood by the majority of those present at the meeting, so it is not necessary to modify the resource once the explanations have been given.

Results and discussions

In both cases studied, community members use mathematical arguments to validate the final decision. Mathematics appears to have at least two different roles. In the first case, the validation by some mathematical arguments seems to be essential to legitimate the decision-making even if the determining factor concerns the design of teaching resources. In the second case, the decision-making is implicit and validated by mathematical arguments while the common project of the community concerns the design of “pertinent” teaching resources.

Thus, from a methodological point of view, it is relevant to analyze the specific role of mathematics in the exchanges about a problem of a community. From the study of both case studies, a model emerges (see figure 1) as a proposal for understanding the community dynamics. We base this model on three components: (1) The members of the community depending on their role and their implication in the treatment of the problem, (2) the arguments, and (3) the resources for mathematics teaching that support the argumentation. The model gives the opportunity to analyze the problem of all type of communities.

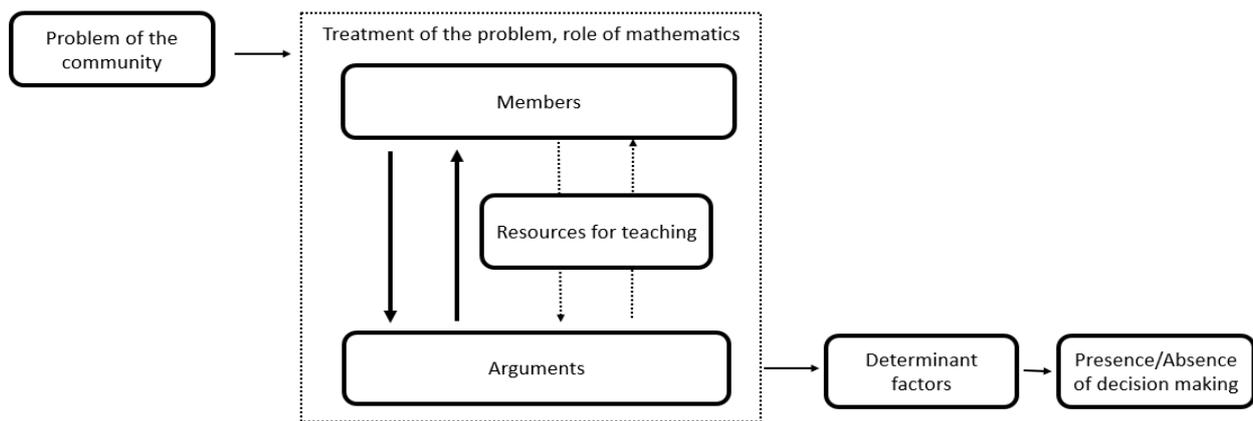


Figure 1: Model to analyze the role of mathematics (didactical, epistemological, etc.) in the treatment of a problem of the community

Conclusion

Existing research about mathematics teacher communities draw mainly on sociocultural theories. In complementarity, our project consists in studying more deeply the role of mathematics within communities of teachers. We propose a model of analysis resulting from articulation of several theories and approaches. We have also defined the notion of *didactical study of communities of mathematics teachers* to clarify the frame of this general research program. We have presented a model based on (1) the identification of a problem of the community, (2) the analysis of this problem in terms of community dynamics, (3) the characterization of the specific role of mathematics among arguments and resources for mathematics teaching. Both contrasted case studies contribute to show the relevance of our proposals. The model of analysis allows us to distinguish two roles of mathematics in the treatment of problems: 1) to explicitly formulate a decision already taken; 2) to implicitly validate a decision that might even be against the goal of the community. However, the model must now be tested on other data and on other kinds of communities. We also aim to test it in the perspective of identifying other roles of mathematics in community dynamics.

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