A step towards a multidirectional 2D model for large scale traffic networks

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Extented Abstract

1 Introduction

The modeling of traffic propagation in a large city network can be tedious. Thus, there is a need for aggregated models that can simplify the representation. One approach is the Macroscopic Fundamental Diagram (MFD) i.e. the idea that there exists a relation between the average flux and the average density over a road network as it has been observed with real data in [Daganzo and Geroliminis, 2008], [Geroliminis and Daganzo, 2008]. This relation justifies the creation of models of accumulation which describe dynamically the number of vehicles in one area (or several [Leclercq et al., 2015]) with an Ordinary Differential Equation. However, these models are not tuned to capture precisely how traffic flow propagates spatially.
Another type of aggregated models is two dimensional models, see [Aghamohammadi and Laval, 2018] for a literature review of some of these models. Several models have been proposed. In [Romero Perez and Benitez, 2008], the 2D model considered does not use the Fundamental Diagram i.e. the dependency between vehicles speed and density. In [Della Rossa et al., 2010], authors suggest a 2D model that includes a diffusion term. In [Khoshyaran and Lebacque, 2018], a bi-dimensional model that can consider major arterial in the urban network is described in a discrete setting. Other models as [Jiang et al., 2011], [Du et al., 2013] describe the direction of the flow by solving an Eikonal Equation such that vehicles take the path of the lower cost. This approach is similar to what is done in pedestrian modeling. In [Jiang et al., 2015], authors investigate a second order model in order to have a more accurate prediction of traffic emissions.
2D models are mathematically identical than models developed for pedestrian [Hughes, 2002]. However, if pedestrian evolve in general in the 2D space, traffic flow are in practice constraint on a network. Then, representing traffic flow as a continuum is an approximation, but this is why information from the physical network can be very important to use. Thus, in [Mollier et al., 2018b], authors introduce a 2D model that uses the geometry of the road to estimate the flow direction. In [Mollier et al., 2018a], a model considering space dependent parameters estimated also from the road network is presented. In [mol, 2019], the authors suggest a method to compare these 2D models with the results of microsimulation over the network. However, in all the 2D models introduced previously, the direction of the flow at a given space and time position is unique. Thus, these models are not able to describe the multiple origin-destination of real traffic scenario. A possible solution is to consider multiple layers of density such that each layer can represent a main direction of flow. A first and interesting study in this direction of research is done is [Lin et al., 2017]. Nevertheless, the model suggested, inspired from pedestrian models, is not based on the road network and thus, for instance, does not assume that traffic flow density has a maximal value.
The idea of multi layer models come from mixed traffic models that exist in 1D as in [Wong and Wong, 2002], [Benzoni-Gavage and Colombo, 2003]. These models aim to represent different populations of vehicles/drivers driving on the same road and in the same direction but with different behaviors that can be for instances different speed limits. Two other studies for pedestrian [Goatin and Mimault, 2015], [Tory et al., 2011] respectively in 1D and in 2D investigate the case where the flows go in opposite directions. The authors remark that in these cases, the systems are not hyperbolic but hyperbolic-elliptic.
The contribution of this article is to introduce a two dimensional and multi-layer traffic model and to point out that this model is unlikely to be hyperbolic. The article is organized as follows. First, we present the two dimensional and multi-layer model. Then, we introduce the estimation methods for the parameters and study the condition for hyperbolicity of the model. Finally, we show some numerical results.

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2 A 2D multi layer model

2.1 General model description

In the one dimensional case, multipopulation models have mainly been developed in order to describe mixed traffic with different maximum speeds of vehicles. In our two-dimensional case, we consider multiple layers of density in order to represent different directions of flow on a network contained in a domain $\Omega \subset \mathbb{R}^2$. Each layer $\ell \in [1, \dots, L]$, represented by a density ρ^ℓ can describe a specific direction $\vec{d_\theta^\ell}$ of flow for a global density $\rho(t,x,y) = (\rho^1(t,x,y), \dots, \rho^L(t,x,y))$. Then, we construct a system of conservation laws in two space dimension as follows:

$$\begin{cases} \dfrac{\partial \rho^\ell(t,x,y)}{\partial t} + \nabla \cdot \vec{\Phi^\ell}(\rho(t,x,y),x,y) = 0 \\ \rho^\ell(0,x,y) = \rho_0^\ell(x,y) \end{cases} , \forall t \in \mathbb{R}^+, \forall (x,y) \in \Omega, \forall \ell \in [1, \dots, L] \tag{1}$$

By definition, the flux can be split as the product of density time velocity:

$$\vec{\Phi^\ell}(\rho(t,x,y)) = \begin{cases} \Phi_x^\ell(\rho(t,x,y)) \\ \Phi_y^\ell(\rho(t,x,y)) \end{cases} = \rho^\ell(t,x,y) \cdot v^\ell(\Psi^\ell(\rho(t,x,y)),x,y) \cdot \vec{d_\theta^\ell}(x,y) \tag{2}$$

where:

- $\rho^\ell(t,x,y) : \mathbb{R}_+ \times \Omega \to [0, \rho_\mathrm{m}]$ is the density of the layer ℓ. They are two dimensional densities i.e. number of vehicles per area.
- $\vec{d_\theta^\ell}(x,y) = \begin{cases} \cos(\theta^\ell(x,y)) \\ \sin(\theta^\ell(x,y)) \end{cases}$ is the unit vector of velocity direction for the layer ℓ.
- $v^\ell(\Psi^\ell(\rho),x,y) : [0,\rho_\mathrm{m}] \times \Omega \to [0, v_\mathrm{m}]$ is the velocity magnitude of the layer ℓ. It expression is given by the Fundamental Diagram: $v^\ell(\Psi^\ell(\rho),x,y) = v_\mathrm{m}(x,y)\left(1 - \dfrac{\Psi^\ell(\rho)}{\rho_\mathrm{m}(x,y)}\right)$ with $v_\mathrm{m}(x,y)$ and $\rho_\mathrm{m}(x,y)$ respectively the space dependent maximum density and maximum speed that we assume to be equal for the different layers.
- $\Psi^\ell(\rho = (\rho^1, \dots, \rho^L) : [0,\rho_\mathrm{m}]^L \to [0,\rho_\mathrm{m}]$ is a mapping function from the whole density to the considered density that expresses which layer has an impact on the velocity of the layer ℓ. In this paper, we define this function as the sum of the density of the different layers: $\Psi^\ell(\rho) = \sum_{\ell_i=1}^{L} \rho^{\ell_i}, \quad \forall \ell \in [1, \dots, L]$. An important remark is that this choice of interaction Ψ and of velocity involves that, if the two layers of density belong to $[0,\rho_\mathrm{m}]$, the sum of the density also remains between zero and ρ_m. This is guaranteed by the fact that the velocity of the two layers goes to zero when the sum of the density goes to ρ_m.

3 Parameters estimation

3.1 Presentation of the network

The network considered is an 10×10 artificial Manhattan grid of 1km square. This size can easily be extended but we consider first a small example for study purposes. The concentration of road is lower in the left part of the network. The speed limits of every road are 30km/h except for two main roads where it is 50km/h. The roads of the network are bidirectional. In order to describe the different directions of flow, we divide the network into two oriented sub-networks: one globally oriented in the North-East direction, the second globally oriented in the North-West direction.
There are three parameters in the models and they are space dependent. First, the direction fields of the two layers are estimated using each of the sub-network. The general idea is to use only the geometry of the road because we do not have knowledge of the trajectories of drivers. Thus, the estimation of direction field is done by interpolation of the road direction. More details on this method can be found in [Mollier et al., 2018b]. The second parameter is the maximum speed v_m which is also done by spatial interpolation of the speed limits of the roads. Finally, the estimation of the maximum density is also done using the network such that the area of the network with a lower density of road have a lower capacity. More details on the estimation of these parameters can be found in [Mollier et al., 2018a]. The result of estimation is represented in 1.

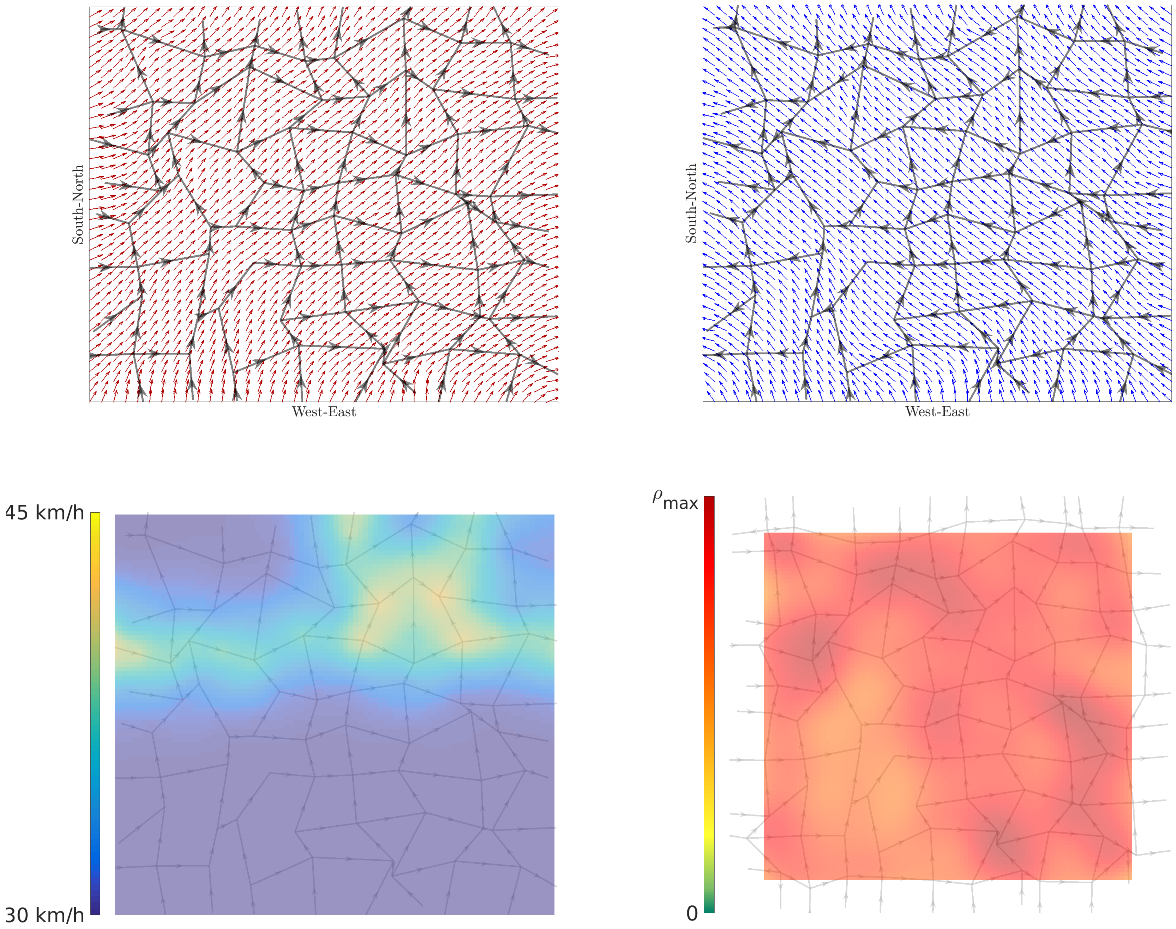

Figure 1: Estimation of the parameters of the model: on the top are represented the direction field of the two layers and on the bottom there is the estimated maximum velocity and maximum density

3.2 Study of the hyperbolicity of the model

Hyperbolic partial differential equations (PDE) correspond to a class of PDE that describes phenomena propagating in finite speed. Models developed in traffic flow, as the LWR model, are hyperbolic and many numerical methods have been developed for this class of equation. Thus, it can be interesting to study the hyperbolicity of the presented model. In practice, the investigation of hyperbolicity is done by the analysis of the Jacobian matrix of the flux see [Chen, 2011], [Toro, 2013].

Definition: We denote by Φ_x and Φ_y the flux function for the x and y dimension. Then, the system is hyperbolic if $\forall \alpha \in [0, \dots, 2\pi[$, the matrix:

$$A = \cos(\alpha)\mathrm{J}_{\Phi_x} + \sin(\alpha)\mathrm{J}_{\Phi_y} \tag{3}$$

with J_{Φ_x} and J_{Φ_x} the Jacobian matrices of the flux, has real eigenvalue i.e. if the discriminant Δ is positive:

$$\Delta = \left(\cos(\alpha - \theta^1)\left(v^1(\Psi^1(\rho,\theta)) + \rho^1 \frac{\partial \Psi^1}{\partial \rho^1}\frac{\partial v^1}{\partial \Psi^1}\right) - \cos(\alpha - \theta^2)\left(v^2(\Psi^2(\rho,\theta)) + \rho^2 \frac{\partial \Psi^2}{\partial \rho^2}\frac{\partial v^2}{\partial \Psi^2}\right)\right)^2$$
$$+4\rho^1\rho^2\cos(\alpha - \theta^1)\cos(\alpha - \theta^2)\left(\frac{\partial \Psi^1}{\partial \rho^2}\frac{\partial v^1}{\partial \Psi^1}\right)\left(\frac{\partial \Psi^2}{\partial \rho^1}\frac{\partial v^2}{\partial \Psi^2}\right)$$

In practice, Δ is positive for all possible densities if and only if the direction field are collinear and with same direction. In the case of arbitrary direction, the system is mixed hyperbolic-elliptic and remains hyperbolic for low densities.

4 Numerical results

4.1 Numerical method

The numerical method is based on splitting method. First, we apply a splitting to each layer, then we use dimensional splitting to compute separately each dimension. Finally, the flux at the cell interfaces for each direction and for each layer is computed by scheme using demand-supply as it is done in [Mollier et al., 2018a]. For more detail about space dependent flux function and the stationary waves that it could create, we refer to [Holden and Risebro, 2015]. We consider ghost cells at the boundaries.
However, these numerical methods to compute the flow at the interfaces (Godunov, Lax-Friedrich) are valid for hyperbolic partial differential equations. As it is shown in [Goatin and Mimault, 2015], the use of these methods in the elliptic case can lead to oscillation observable in simulations. Thus, we are able to simulate some examples where the densities remain in the free flow state for which the model is hyperbolic.

4.2 Simulation results

The scenario considered is a free flow scenario where a small amount of vehicles start in each layer.
The initial conditions are given by equation (4) and (5):

$$\rho_0^1(x,y) = \begin{cases} \frac{2}{16}\rho_{\max} & \text{if } \frac{2}{16}L_{\mathrm{x}} \le x \le \frac{7}{16}L_{\mathrm{x}} \quad \text{and} \quad \frac{2}{16}L_{\mathrm{y}} \le y \le \frac{7}{16}L_{\mathrm{x}} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$$\rho_0^2(x,y) = \begin{cases} \frac{2}{16}\rho_{\max} & \text{if } \frac{9}{16}L_{\mathrm{x}} \le x \le \frac{14}{16}L_{\mathrm{x}} \quad \text{and} \quad \frac{2}{16}L_{\mathrm{y}} \le y \le \frac{7}{16}L_{\mathrm{x}} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

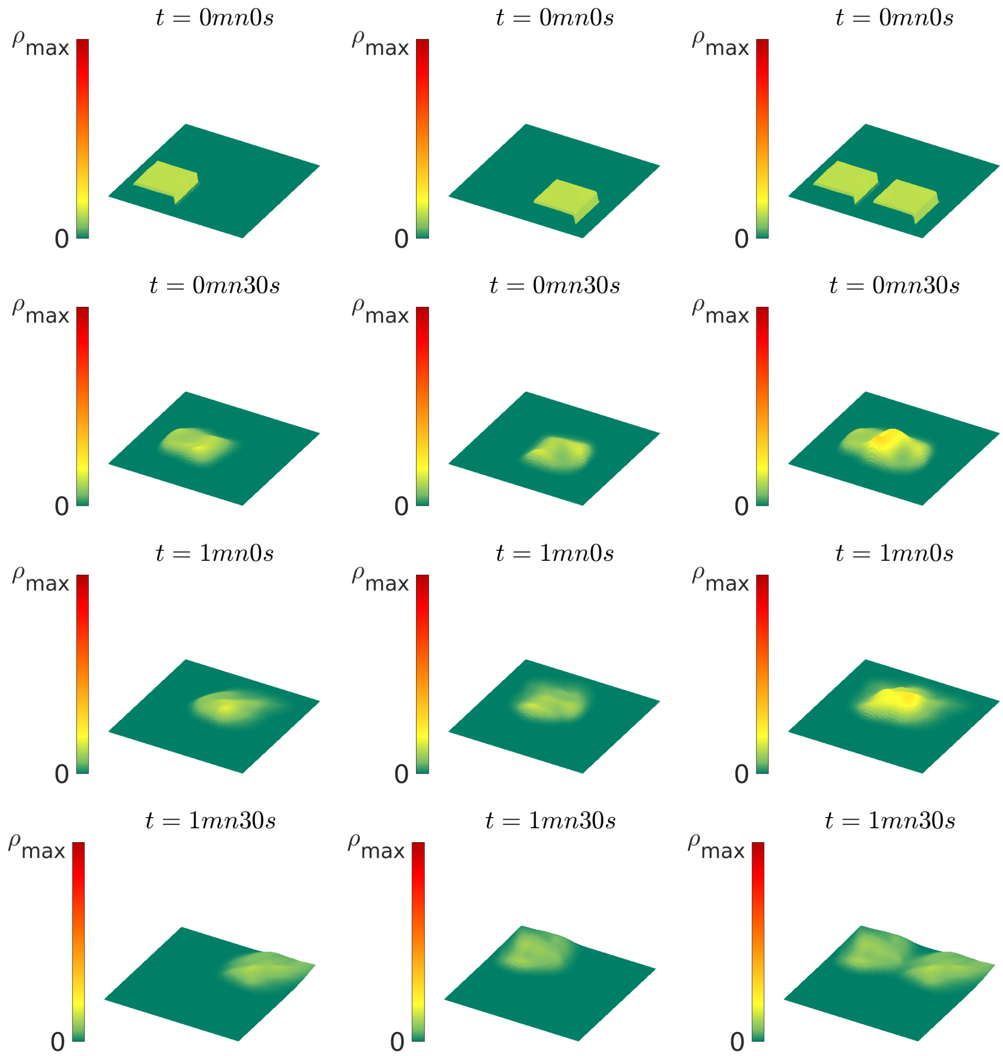

Figure 2: Results of simulation for the free flow scenario with density of the first layer on the left, density of the second layer on the middle and total density on the right. Videos of the simulation with the view from above https://youtu.be/zo1ScLKI9yk and the lateral view https://youtu.be/F2GD3ZWOCt4 are available online.

The results of the simulation can be seen in Figure 2 with a lateral view. We can see that the densities inside the two layers cross in the middle of the network and then interact with each other. However, as the global traffic state remains in free flow there are no major perturbation.

Conclusions and future works

In this article, we present a two-dimensional model with several layers that describe different directions of traffic flow in a large network. We give an estimation method of the space-dependent parameters using the road network. Then, we point out that this model is not hyperbolic due to the interaction between the different directions of the flow and we show that this can induce some instability in the simulation. In the future, we plan to investigate a way to transform the model in a fully hyperbolic system or to study a numerical method applicable for a mixed hyperbolic-elliptic system of partial differential equations. After that, it can be interesting to test the model with an existing road network and to compare the results with real or synthetic data.

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Author Contribution Statement

The authors confirm contribution to the paper as follows: study conception and design: S. Mollier, M.L. Delle Monache, C. Canudas-de-Wit; analysis and interpretation of results: S. Mollier, M.L. Delle Monache, C. Canudas-de-Wit; draft manuscript preparation: S. Mollier, M.L. Delle Monache, C. Canudas-de-Wit. All authors reviewed the results and approved the final version of the manuscript.

References

[Aghamohammadi and Laval, 2018] Aghamohammadi, R. and Laval, J. A. (2018). Dynamic traffic assignment using the macroscopic fundamental diagram: A review of vehicular and pedestrian flow models. *Transportation Research Part B: Methodological*. In Press. Online at https://doi.org/10.1016/j.trb.2018.10.017.

[Benzoni-Gavage and Colombo, 2003] Benzoni-Gavage, S. and Colombo, R. M. (2003). An n-populations model for traffic flow. *European Journal of Applied Mathematics*, 14(5):587–612.

[Chen, 2011] Chen, G.-Q. G. (2011). Multidimensional conservation laws: overview, problems, and perspective. In *Nonlinear conservation laws and applications*, pages 23–72. Springer.

[Daganzo and Geroliminis, 2008] Daganzo, C. F. and Geroliminis, N. (2008). An analytical approximation for the macroscopic fundamental diagram of urban traffic. *Transportation Research Part B: Methodological*, 42(9):771–781.

[Della Rossa et al., 2010] Della Rossa, F., D'Angelo, C., and Quarteroni, A. (2010). A distributed model of traffic flows on extended regions. *NHM*, 5(3):525–544.

[Du et al., 2013] Du, J., Wong, S., Shu, C.-W., Xiong, T., Zhang, M., and Choi, K. (2013). Revisiting jiang's dynamic continuum model for urban cities. *Transportation Research Part B: Methodological*, 56:96–119.

[Geroliminis and Daganzo, 2008] Geroliminis, N. and Daganzo, C. F. (2008). Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. *Transportation Research Part B: Methodological*, 42(9):759–770.

[Goatin and Mimault, 2015] Goatin, P. and Mimault, M. (2015). A mixed system modeling two-directional pedestrian flows. *Mathematical biosciences and engineering*, 12(2):375–392.

[Holden and Risebro, 2015] Holden, H. and Risebro, N. H. (2015). *Front tracking for hyperbolic conservation laws*, volume 152. Springer.

[Hughes, 2002] Hughes, R. L. (2002). A continuum theory for the flow of pedestrians. *Transportation Research Part B: Methodological*, 36(6):507–535.

[Jiang et al., 2011] Jiang, Y., Wong, S., Ho, H., Zhang, P., Liu, R., and Sumalee, A. (2011). A dynamic traffic assignment model for a continuum transportation system. *Transportation Research Part B: Methodological*, 45(2):343–363.

[Jiang et al., 2015] Jiang, Y.-Q., Ma, P.-J., and Zhou, S.-G. (2015). Macroscopic modeling approach to estimate traffic-related emissions in urban areas. *Transportation Research Part D: Transport and Environment*.

[Khoshyaran and Lebacque, 2018] Khoshyaran, M. M. and Lebacque, J.-P. (2018). Homogeneous bidimensional traffic flow model. *IFAC-PapersOnLine*, 51(9):61–66.

[Leclercq et al., 2015] Leclercq, L., Parzani, C., Knoop, V. L., Amourette, J., and Hoogendoorn, S. P. (2015). Macroscopic traffic dynamics with heterogeneous route patterns. *Transportation Research Procedia*, 7:631–650.

[Lin et al., 2017] Lin, Z., Wong, S., Zhang, P., Jiang, Y., Choi, K., and Du, Y. (2017). A predictive continuum dynamic user-optimal model for a polycentric urban city. *Transportmetrica B: Transport Dynamics*, 5(3):228–247.

[Mollier et al., 2018a] Mollier, S., Delle Monache, M. L., and Canudas-de-Wit, C. (2018a). 2D-LWR in large-scale network with space dependent fundamental diagram. In *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*, pages 1640–1645.

[Mollier et al., 2018b] Mollier, S., Delle Monache, M. L., and Canudas-de Wit, C. (2018b). A simple example of a two-dimensional model for traffic: Discussion about assumptions and numerical methods. *Transportation Research Record*, 2672(20):249–261.

[Mollier et al., 2019] Mollier, S., Delle Monache, M. L., Canudas-de-Wit, C., and Seibold, B. (2019). Two-dimensional macroscopic model for large scale traffic networks. *Transportation Research Part B: Methodological*, 122:309 – 326.

[Romero Perez and Benitez, 2008] Romero Perez, L. and Benitez, F. G. (2008). Outline of diffusion advection in traffic flow modeling. In *Transportation Research Board 87th Annual Meeting*, number 08-1503.

[Toro, 2013] Toro, E. F. (2013). *Riemann solvers and numerical methods for fluid dynamics: a practical introduction*. Springer Science & Business Media.

[Tory et al., 2011] Tory, E., Schwandt, H., Ruiz-Baier, R., and Berres, S. (2011). An adaptive finite-volume method for a model of two-phase pedestrian flow. *Networks and Heterogeneous Media*, 6(EPFL-ARTICLE-170240):401–423.

[Wong and Wong, 2002] Wong, G. and Wong, S. (2002). A multi-class traffic flow model–an extension of lwr model with heterogeneous drivers. *Transportation Research Part A: Policy and Practice*, 36(9):827–841.