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Mathematical gestures: Multitouch technology and the indexical trace

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This paper is about the threshold between gesture and touch in mathematical activity, focusing on the role of multitouch technology. Drawing on the work of gesture theorist Jürgen Streeck, we propose and discuss the notion of the tangible gesture, in the context of mathematical explorations of young children with a novel, multitouch iPad environment called TouchCounts designed to promote counting on and with the fingers.

Keywords: Technology, multitouch, number and operations, gesture.

Introduction

With the advent of multitouch technologies, gestures have become an essential feature of user interface. Touch technologies break with previous computer-based norms, where the hand's actions were *indirectly* related to changes on the screen through mouse and keyboard manipulation. Touch technology invokes earlier drawing technologies in which the hand's actions had a more *direct* relation with a given surface. The digital nature of contemporary surfaces, however, significantly alters the relation between touching and meaning-making.

In this paper, we draw attention to the distinctive gestures new media elicit and produce and the way these new manual activities are changing the way we perform mathematics. We also interrogate the taken-for-granted distinction between the touchscreen gestures common in the technology world and the in-the-air gestures that have been the focus of study in mathematics education research. At first blush, they may seem quite distinct—albeit having the same word—but we follow Streeck (2009) in seeing them as being on a continuum, a perspective that enables us to better appreciate the role that touchscreen gestures might have in mathematics learning.

In the context of mathematics education research, many studies have focused on student and teacher use of gesture in classrooms, but this work tends to code and sort gestures insofar as they are *representations* of thinking. These studies tend to divorce the motoric hand from the feeling swipes and swishes of fingers on screens. Material semiotic approaches to the study of interaction, on the other hand, consider gesture less as representations and more in terms of the material effects they achieve (Roth, 2001; Radford, 2002; Nemirovsky, Kelton & Rhodehamel, 2012). Our goal in this paper is to unpack the implications for understanding mathematics learning in relation to new media. In order to illustrate how these new media gestures operate as expressions of numeracy, we draw on research involving a novel multitouch App in which fingers and gestures are used to count (Jackiw & Sinclair, 2014).

Gesture as movement

The predominant line of research in gesture studies focuses on movements of the body (especially the hand) and their interactions (i.e. correlations) with speech in communication (Kendon, 2000, 2004; Kita, 2003; McNeill, 1992; 2005). Researchers have identified different categories of gestures

(icon, metaphoric, deictic and beat) so as to distinguish different relations between gesture and speech. McNeill has drawn on Peirce's (1932) semiotics in which signs (icons, symbols, and indices) differ in terms of the nature of the relationships between the signifying sign and the signified. These categories have been used extensively in mathematics education research.

Research that codes gesture only in terms of linguistic potential tends to overlook the physicality of the hand movement, except insofar as such movement contributes to or obscures linguistic meaning (Rossini, 2012). As Streeck (2009) indicates, "it is common to treat gesture as a medium of expression, which meets both informational and pragmatic or social-interactional needs, but whose "manuality" is accidental and irrelevant" (p. 39). Streeck (2009) defines gesture:

... not as a code or symbolic system or (part of) language, but as a constantly evolving set of largely improvised, heterogeneous, partly conventional, partly idiosyncratic, and partly culture-specific, partly universal practices of using the hands to produce situated understandings. (p.5)

Thus he studies gesture for how it is "communicative action of the hands" with emphasis on the term *action* (p.4). This focus on action allows Streeck to study gesture for how it couples with and intervenes in the material world in non-representational ways. Researchers often distinguish between hand movements *in the air* and hand movements *that make graphic marks*, where the former is deemed a gesture and the latter an act of inscription. However, such distinctions become fuzzy when we study the movement of the hand across and through media, where 'media' can be more or less receptive of trace or mark. In other words, all hand movements traverse and incorporate media. We see a trace in certain media, and not in others, but since the logic of new media is to break with current conventions of perception, this distinction is provisional. New media allow for new kinds of traces. This insight allows for new ways of studying numeracy and multitouch technologies. In the next section, we discuss a case study of children working with such technologies, showing how this material encounter entails a very different concept of number *precisely because of the indexical aspect of gesture*.

The question of trace and inscription returns us to how the indexical is different from the iconic. The contiguity aspect of indexicality (the smoke is materially caused and coupled with the fire) is aligned with touch and the way our body connects with another through touch. This approach to gesture supports the systems approach to bodies by which bodies become coupled with the environment they inhabit (Maturana & Varela, 1987). Streeck studies gestures as part of a nested scaling approach to this system, beginning in the world that the "hand knows best", and then examining how gestures operate at greater distance or remove from that world (p. 58). The touch or haptic factor of hand movement is precisely how the fragile interface of inscription or trace is currently produced. Again, we emphasize that this production of a trace is contingent on current configurations of sensory perception and material media.

Most of the hand's features (digits, degrees of freedom of movement, fatty palms, flexure lines, papillary ridges) evolved to facilitate grasping (prehension), and thus the hand "became a 'compromise organ'" in serving multiple purposes (Streeck, 2009). Prehensile "postures" are formed as the hand reaches its target (in our case this will be a screen), during which a pre-conscious calibration of speed and collective force determines the particular movement of the digits, hands and arm. As the hand moves towards the target, there is a strong reliance on peripheral vision

rather than vision directed at the target (Streeck, 2009, p.47). The speed of the gesture reduces as the hand reaches its target. But the moment of contact entails the forming of a new assemblage, when the entire body of the gesturer links up with that which it touches. Thus, we are focusing on how gesture is a hand action that does more than identify or code particular aspects of an object.

The video data discussed below is part of a larger project exploring the power of touchscreen technologies in teaching and learning mathematics in early childhood. Several research studies have already been carried out concerning the way that children learn various concepts using *TouchCounts*, including ordinality (Sinclair & Coles, 2015), place value (Coles & Sinclair, 2017) and finger gnosis (Sinclair & Pimm, 2015). The focus of this paper is less on the learning process using *TouchCounts* than on the various and distinctive forms of hand actions that are involved in creating and manipulating number in this environment.

What is distinctive about the index is that it is a sign that is materially linked or coupled to “its object”. According to Peirce (1932), an index “refers to its object not so much because of any similarity or analogy with it, (...) as because it is in dynamical (including spatial) connection both with the individual object, on the one hand, and with the senses or memory of the person for whom it serves as a sign, on the other” (2.305). For instance, the chalk drawing of a parallelogram on a blackboard is often considered to be an iconic reference to a Platonic conception of parallelogram, but it is (also) an indexical sign that refers to the prior movement of the chalk. This latter indexical dimension is usually not emphasized in the semiotic study of mathematical meaning making, since we tend to focus on the completed trace and dislocate it from the labour that produced it. This focus on the completed sign neglects how the activity of the body and various other material encounters factor in mathematical activity.

TouchCounts: A multitouch early number App

In this paper, we discuss an application that author Sinclair has been involved in creating in which the digital gesture plays an even more central role in the mathematical activity. *TouchCounts* (Jackiw & Sinclair, 2014) is an application that permits young learners to coordinate simultaneously various forms of number: number names like ‘three’, number of taps on the screen, number of discs on the screen and number symbols like 3. It enacts a multimodal correspondence between finger touching, numeral seeing and number-word hearing (a one-to-one-to-one correspondence of touch, sight and sound). The App has two worlds: the Enumerating and the Operating worlds. In this paper, we focus on the Enumerating world, which is the one that children usually first experience.

In the Enumerating World, the screen starts almost blank, except for a horizontal bar called a shelf. In this world, a learner taps her fingers on the screen to summon numbered discs. The first tap produces a new yellow disc on which the numeral “1” appears. Subsequent taps produce sequentially higher numbered discs. As each tap summons a new numbered disc, *TouchCounts* audibly speaks the name of its numeral (“one,” “two”). As long as the user’s finger remains on the glass, it holds the numbered disc, but as soon as she “lets go” (by lifting her finger) virtual gravity makes the number object fall to and “off” the bottom of the screen. If the user releases her numbered disc above the shelf, or “flicks” it above the shelf on release, it falls only to the shelf, and comes to rest there, visibly and permanently on screen, rather than vanishing out of sight “below” (see <https://www.youtube.com/watch?v=7xD-pqnsce0>). Since each time a finger is placed on the

screen, a new numbered object is created, one cannot “catch” or reposition an existing numbered disc by retapping it. We note that, at least initially, the eye plays a prominent role in directing the finger above or below the shelf, but that if one does not care where the disc alights, the tapping of the finger needs little visual direction.

If we take the finger tap as a gesture involving the placement of a finger on the screen, and the subsequent production of an event featuring visual, mobile and aural aspects, then we might say that the gesture is iconic in its relation to the production of unitary quantities, or perhaps even metaphorical for the children for whom such unitary quantities are still “abstract”. But what seems much more pertinent for the children as they engage with this application is the indexical nature of the gesture. The tap both points to the screen, designating one place of contact with it, but also creates a new numbered disc under the soft skin of the finger-pad, a disc which often falls with gravity-like weight. In addition, each tap produces a simultaneous sound. The children can also tap the Reset button, which makes all the numbered discs disappear and resets the count to 1.

While *TouchCounts* was designed to support the development of one-to-one correspondence between number and hand movement, by drawing on the tangible dimension of counting, its use by young children has prompted us to examine both the particular ways in which they use their hands and the implications of their hand actions on the meanings they make around counting, in particular the concepts of ordinality and cardinality (see Sinclair & Pimm, 2015).

A case study

This case study is drawn from a broader research project that was conducted in daycare and primary school settings over the course of three years. In the excerpt we present, co-author Sinclair was engaged in a clinical interview with a five-year-old kindergarten child named Katy, who is interacting with *TouchCounts* for the first time. (Indeed, it was the first time she was using a touchscreen tablet.) The interview occurs in June and therefore close to the end of the school year. We have chosen the excerpt because it illustrates a range of gestures that have been observed over the course of the research study, while also showing hand motions that have not been explicitly taught. In this case study, the hand actually operates very close to the surface of a screen: pointing to objects on the screen by tapping them; sliding objects along on the screen so as to leave visual and aural traces of the finger’s path; pinching objects together in order to make new ones.

The room is quiet. Without prompting, Katy’s hand approaches the screen, and her finger touches the top of it and slides down to the bottom. A yellow disc appears under her finger with the numeral ‘1’ on it and the sound ‘one’ is made. The index finger moves back to the top of the screen, slowly swimming downwards. A chorus of ‘two’ comes both from her mouth and the iPad. This happens repeatedly, although sometimes only the iPad can be heard announcing the new numbered disc while Katy’s lips move in synchrony ([Figure 1a](#)). The appearance of ‘10’ on the tenth yellow disc attracts attention, perhaps because of its double digits, and Katy bends over to look closely.

Katy looks up again and her finger resumes touching the screen, but now only the iPad counts the numbers ([Figure 1b](#)): she no longer says them aloud herself. After ‘seventeen’, several fingers fall on the screen at once, and then ‘twenty-one’ is heard (since she has tapped the screen with several fingers, only the sound of the final number is said aloud, but the four discs all appear under where she has touched). This produces a pause in the action, and Katy’s lips spread into a smile. All but

the index finger are tucked away, as the rhythmic tapping continues along with the chorus of named numbers. At ‘twenty-seven’ Katy looks up, no longer watching the screen (see [Figure 1c](#)), and she continues swiping and saying numbers. This continues until a finger accidentally land on Reset.

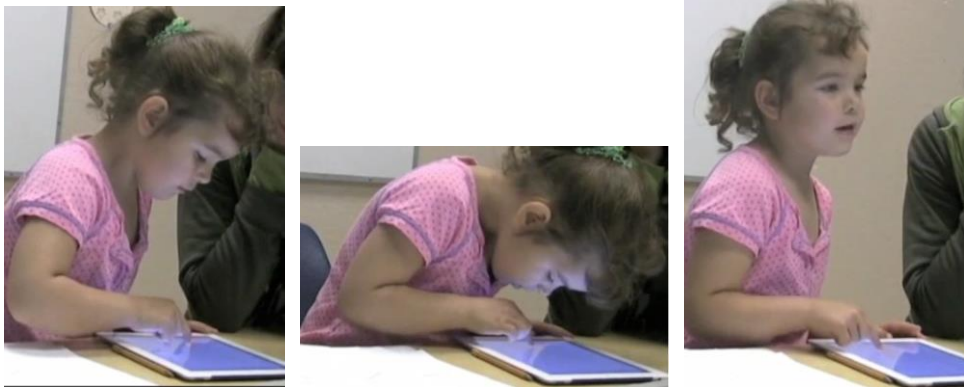


Figure 1(a). Katy swiping; (b) Following the yellow disc; (c) Tapping while looking up.

Katy’s finger – as the main organ of touch in this encounter – takes on new capacities through the reset button. It is no longer the organ that can only move or drag the yellow circle. The power of the reset button to recalibrate the tempo and rhythm of this encounter, becomes part of the finger’s potentiality, thereby redefining what is currently entailed in the sense of touch.

Discussion

Fingers can serve as both a physical extension of what Rotman (1987) calls the ‘one-who-counts’ (p. 27) (usually with an extended pointed finger reaching out to the world) as well as the thing-to-be-counted (in which the gaze is directed towards one’s own fingers): fingers are thus simultaneously subject and object, both of the person and of the world (Alibali & diRosso, 1999). And this is what makes the finger actions of Katy so interesting; the mathematical act of counting with *TouchCounts* fuses this duality and in so doing changes the relationship between the hand and eye, as well as the ears.

Katy’s hand actions change over the course of the episode, not only in the particular muscular form they take, first sliding down the screen as if lingering on the yellow discs to produce or partake in their falling off the screen, and then tapping impetuously so that each new touching of the screen follows the end of the sounds of the voiced numerical. The swiping gesture seems more exploratory while the tapping gesture seems to concatenate into a unit the touch-see-hear bundle of sensations involved in making a new disc-numeral-name. As Streeck writes, tapping is also “characteristic of ritualized behavior” (p. 76), which suggests that Katy has moved from exploration to practice. In both the swiping and the tapping, the finger can be seen as making an indexical gesture, with the trace being both visible and audible, not to mention tangible for Katy.

Although the initial movement and touch of her finger is what produces the disc, it is the disc that drives the swiping movement of her finger. Indeed, both her finger and her eyes *follow* the yellow disc as it heads down the screen. In shepherding the numbered disc off the screen, Katy sees when it’s time to lift her finger and start making a new disc. But with the tapping, the eyes attend to the numerical sign on the disc—indeed, when 10 appears, Katy notices the change from the previous one-digit numerals. In this sense, the eye and the finger do very similar things in the swiping, the

visible trace is followed closely by Katy's eyes as the swiping takes place, so that the hand is subordinated to the watchful eye. With the tapping, the hand seems less subordinated, with the eye only interested when there's a novel situation. When Katy looks up, the hand is no longer subordinate. When Katy's eyes close, her fingers do the seeing and touching as they repetitively tap.

But of course, there is more than the eye and hand involved in this situation. The ear and voice feature importantly as well. Indeed, while the voice is subordinate to the touch (it only speaks while Katy taps), Katy's hand is also subordinate to the ear in that the ear judges the moment of the next tap. And the ear is disrupted by the hand, when several fingers touch the screen at once, causing the voice to jump from "seventeen" to "twenty-one." The eye, which was about to drift off, must return to survey the situation. And the hand returns to its single digit tapping. The importance of the aural and the vocal in this context is interesting in terms of the counting activity at play. Indeed, the ritual origins of counting are oral in nature, and counting with young children is often undertaken as the learning of a song that one memorises and chants. The involvement of the hand in this otherwise oral event provides a visual and tangible trace of the count, while also associating each counted number with a single swipe or a single tap.

One might question whether Katy's actions on the screen, which we might think of as touch-pointing, can really be thought of as gestures. Streeck argues that such touch-pointing gestures (and indeed all gestures) emerge from the touching and handling of things—the tracing (or other "data-gathering devices" such as caressing, probing, cupping) of objects that allows one to discover its texture and temperature (and, for young children, for instance, the difference between a cylinder and a pyramid). When the hand has done its exploring of the object, which fulfills an epistemic function in gathering information, it may then be lifted off the object and inclined to repeat the same movements 'in the air': "the hands' data-gathering methods are used as the basis for gestural communication" (p. 69). Streeck identifies such gestures as being communicative, which for him is the characteristic feature of a gesture. So perhaps Katy's touch-pointing becomes a gesture once she lifts her hand from the screen to do her tapping.

Distinguishing hand movements that explore from ones that communicate is problematic though. As Streeck writes, exploratory actions can become communicative when they are made visible to others, who may join in the action or infer tactile properties. If we look at Katy's swiping and tapping gestures, we might say that they are both exploratory, with the swiping gestures involving prolonged tactile contact that enables her to discover what would happen when her finger touches the glass—that a yellow disc would appear, with a numeral on it; that the disc would move down the screen; that the iPad would speak the numeral's name aloud, and that this could all be repeated as often as she wished. But Katy's swiping and, later, her tapping, are also communicative inasmuch as they tell *TouchCounts* what to do and say. The same might be said for clicks of the mouse or key presses of the keyboard, with the difference that the touchscreen is acted upon by direct hand motions. Instead of disentangling the tracing from the pointing (the exploration from the communication), we suggest that re-assembling them into an indexical enables us to see how Katy's hand movements can tap into the potentiality of the body by reconfiguring the relationships between sensations of touch, sight and sound that are at play. This potentiality mobilizes new mathematical meanings as Katy uses her fingers to count on, to count with and to count out one by one and indefinitely. Streeck recognises that hand-gestures "enable translations between the senses" (p. 70)

as tactile discoveries provide visual information for interlocutors. With Katy though, the tactile discoveries provide visual and auditory information to herself. She is her own interlocutor.

Conclusion

Streeck argues that hand-gestures cannot be taken only as components of a language system cast apart from the material world and used only to communicate *about* the world. Rather, they are *of* the world, and part of how we *feel* the world around us. This perspective requires us to see the moving hand as “environmentally coupled” (Goodwin, 2007), that is, as inextricable from the things it touches and engages with. But while Streeck implies a vector from the exploratory hand action to the communicative hand-gesture, our case studies reveal how the exploratory hand frees itself from the optic regime and invents meaning as much as it communicates it. This new kind of gesture is possible in large part because of the feedback mechanism of digital technologies, which can talk, push and show back. With the touchscreen interface, and particularly the multitouch actions, the hand is involved in a process of communicating that is also a process of inventing and interacting.

In the example we presented, we have shown that the gestures made by Katy in *TouchCounts* had an indexical nature both because they involved some kind of pointing (with one finger or more) and they left a trace that is both visible and audible. The trace is important in drawing attention to the material engagement of the gestures. The gestures arise out of movements of the hand, but they also result in material reconfigurations that can *give rise to* new movements of the hand. In discussing the effect of new digital technologies in mathematics, Rotman has written about the future cultural neoteny in which speech would “become reconfigured (as it was once before when transformed by alphabetic writing), re-mediated and transfigured into a more mobile, expressive, and affective apparatus by nascent gesturo-haptic recourses” (p. 49). We interpret these speculative comments as an indication that the future of the gesturing hand in relation to new media may involve all sorts of surprises, and that perhaps even pre-school children may count ‘on their hands’ to 100 as they engage with these media.

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