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# Learning mathematics with technology. A review of recent CERME research

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*This theoretical paper reports about our perception of the contributions that the working group TWG 16 about the learning of mathematics with digital media have made during the last CERME 9, taking into consideration the previous and the upcoming ERME conferences. Our analysis highlights the evolution of research questions, methodologies and theories through the lenses of the “didactical tetrahedron” metaphor and the networking strategies and methods. Finally, we point out themes that are, to our opinion, insufficiently addressed and need further discussions within the technology group.*

*Keywords: Mathematics, learning, technology, didactic tetrahedron, networking strategies.*

## Introduction and rationale

‘State of the art’ is a common expression used in surveys, review papers and up to date books reporting on the newest achievements in the research. This is also the ambition of the tenth Congress of the European Society for Research in Mathematics Education (CERME 10) TWG 16 leaders as they have announced:

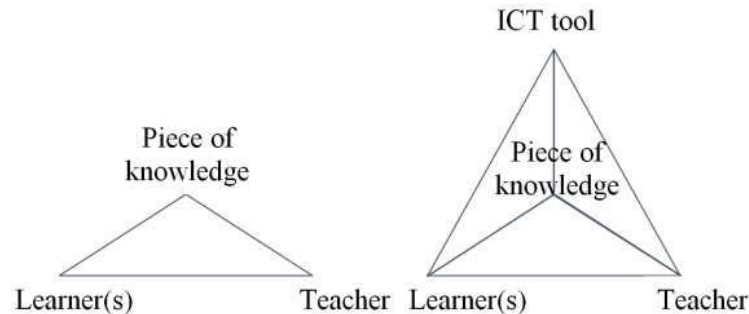
We want to establish an overview of the current state of the art in technology use in mathematics education, including both practice-oriented experiences and research-based evidence, as seen from an international perspective and with a focus on student learning [...] (Call for papers, CERME 10 TWG 16).

There are studies trying to establish such overviews (e.g., Drijvers et al., 2016), but also some claiming to report on the ‘state of the art’ research without sufficient argumentation and full justification of their statements throughout the text. The phrasal adjective ‘state of the art’ fits to advertise a ‘product’ but has our community become mature enough to respond to a challenge of offering ‘state of the art’ descriptions of complex phenomena like the use of technology in mathematics education which has a characteristic of enormous dynamism?

In this article, we do not claim that we have undertaken a meta-research beyond the scope of the CERME although we are aware of the variety of working groups on similar themes at other conferences as ICME, ICTMT, CADGME or ATCM and special issues of journals. Aiming to investigate how the CERME TWG 16 could capitalize knowledge of discussions regarding the learning with technologies, we have rather devoted ourselves to focus on two main issues: 1) *how have the research questions and methodologies about the learning of mathematics with technologies evolved* and 2) *is there a substantial progress regarding the use of the theories*. We begin discussing these two issues through the relations in a “didactic tetrahedron”.

## The “didactic tetrahedron” metaphor

The “didactic tetrahedron” metaphor (Fig. 1 right) was introduced by Tall (1986, p. 6) as an enlargement or adaptation of the “didactic triangle” (Fig. 1 left) commonly used before the advent of technology to analyze the teaching and learning of mathematical knowledge.



**Figure 1: From a “didactic triangle” (left) to a “didactic tetrahedron” (right)**

The integration of an artefact, e.g. an ICT tool, introduces a new component into the teaching/ learning system and creates new relationships between the components of the didactic triangle. Thus, for example the face ALK (A for Artefact or ICT, L for learner and K for Knowledge) represents phenomena related to learning mathematics with technology, such as students’ conceptualizations of given mathematical concepts mediated by technology, or the edge AK highlights phenomena related to new approaches to given mathematical concepts offered by the affordances of a given digital artefact. The didactic tetrahedron has by now been used for analyzing mutual participation of artifacts and their users in a socio-cultural context (e.g. Rezat & Sträßer, 2012) or as a heuristic for studying the implementation of digital media in the teaching and learning praxis (Ruthven, 2012). In this paper, we use it to position the scopes of technology groups at the ERME conferences.

Until CERME 8, issues related to any vertex, edge or face fell within the range of a unique technology group, initially called “Tools and Technologies”. The subsequent changes of the name into “Tools and technologies in mathematical didactics” from CERME 2 to 5 and “Technologies and resources in mathematics education” from CERME 6 to 8 indicate the appearance of enhanced specifications. The growing interest in the theme and the amount of research have led to splitting the technology group into two groups at CERME 9 which have progressed discussing topics focusing on edges and faces having “teacher” and “learner(s)” as a vertex, respectively.

## Research method

In this paper we propose an analysis of the two issues 1) and 2) stated above based on the “didactic tetrahedron” through the: a) Calls for papers of the CERME 8-TWG 15, CERME 9-TWG 16 and CERME 10-TWG 16, b) Introductions to papers and posters of the groups published in the proceedings of the CERME 8 and 9, and c) Papers of these groups published in the proceedings of the CERME8 and 9. In this analysis we also refer to “networking strategies and methods” (Prediger et al., 2008, p. 170).

## Findings and discussion

### a) Evolutions tracked through the Calls for papers since the CERME8

The Call of the CERME 8-TWG 15 guided the discussions by posing three themes referring to design and uses of technologies, students' learning, and teacher professional development in presence of technologies. These three themes clearly refer to the three vertices of the triangular face "ALT (T for Teacher)" in the didactic tetrahedron (Figure 1). Although such structured shape for questioning the themes of interest may not appear straightforward by reading the text in the CERME 9 and 10 Calls, they are indeed meant to contribute to research related to the face "AKT" (TWG 15) and to the face "AKL" (TWG 16). Besides the split of the technology group in two groups, the relation between learning, teaching and digital tools is still present in the issues of the CERME 9-TWG 16 Call, as stated for example in the items "designs of teaching experiments with software and technologies concerning student learning" or "results of empirical studies and investigations especially concerning long-term learning with ICT, massive courses, national programmes of teachers' professional development". Thus, the face "ATL" remains relevant to both groups.

### b) Evolutions tracked through the Introductions to the papers and posters of the CERME 8-TWG 15 and CERME 9-TWG 16

The Introduction of the CERME 8-TWG 15 corresponds to the Call and is structured according to the three themes (stated in a), i.e. the face "ATL". Moreover, it goes beyond the affirmed issues by raising a general one for "capitalization of research results" (Trgalová et al., 2013, p. 2500). This general issue has been addressed in an overview for mathematics, technology interventions and pedagogy based on systematic literature review by *Bray* (CERME 8, 2013) and in a survey reporting about undergraduate, master and doctoral studies for promoting the use of technologies in mathematics education by *Scheffer* (CERME 8, 2013). Further on, in this Introduction, it is claimed that a development of "specific methodologies enabling to assess the effectiveness of ICT in learning processes" (Trgalová et al., 2013, p. 2501) is required. The call for a "proper usage of research methods, which are informed by contemporary theories" (Lokar et al., 2015, p. 2438) is present in the Introduction of the CERME 9-TWG 16.

This paper builds on this claim and attempts to further investigate the usage of theories referring to the learning of mathematics in technology-rich environments in the next subsection.

### c) Evolutions tracked through the Papers published in the proceedings for CERME 8-TWG 15 and CERME 9-TWG 16

#### *Evolution of research questions (RQs) and methodologies*

Unlike the frequent use of several methodologies and theories for exploring teaching (e.g. TPACK or instrumental approach), a large assortment of RQs and methodologies comes out from the papers regarding learning phenomena with technologies. We organize them in the following two categories:

- Category 1: RQs referring to at least two of the edges of the face "ALK"

While the most of the papers from this category discussed at CERME 8 focus on the impact of using technology on students' behavior, learning or performance, there is a greater variety of research issues addressed in papers at CERME 9. For example, the qualitative-empirical study by *Kaya, Akçakın, & Bulut* (CERME 8, 2013) related to the RQ: "does the use of Geogebra via interactive whiteboards as

an instructional tool affect students' academic achievement on transformational geometry?" (p. 2596) seems to meet all edges in this triangle. Likewise, a quasi-experimental study by *Kilic* (CERME 8, 2013) considers concepts in geometry (K), a development of geometric thinking and ability of proving in geometry (L) by using a Dynamic Geometry Software (A). Based on teaching experiments with high school students and prospective teachers, *Bairral and Arzarello* (CERME 9, 2015) have raised the RQ: "which domain (constructive or rational) of manipulation touch screen could be fruitful to improve student's strategies for justifying and proving?" (p. 2460). In this contribution, there is evidence not only of the three edges of the face "ALK" but also of the teaching component of the "didactic tetrahedron" by pointing out a lack of research about the teaching of mathematics with the use of touch screen devices besides task design concerns and cognitive implications (p. 2464-2465).

- Category 2: RQs referring to one of the edges of the face "ALK"

Exemplary studies addressing the edge "AL" are: a design based study by *Misfeldt* (CERME 8, 2013) about the students' instrumental genesis with GeoGebra board game, a study by *Persson* (CERME 8, 2013) grounded on students' interviews and teachers' questionnaires about instrumental and documentation genesis, or empirically based case study by *Storfossen* (CERME 8, 2013) about instrumented action of primary school students. It seems that the emphasis on RQs and methodologies studying instrumental genesis regarding the relation "AL" has slightly decreased from CERME 8 to CERME 9.

A paradigm which is noticeable in the CERME 9-TWG 16 papers and was not present before, except for one paper, is the online learning. Although the significant amount of RQs referring to learning through the Web (e.g., peer learning, collaborative learning, networking, flipped classroom) is visible (e.g., *Biton et al.*, CERME 9, 2015; *Triantafyllou & Timcenko*, CERME 9, 2015), many specific questions related to the face "ALK" remain unanswered. For instance, what is the most relevant mathematical content available on the internet and how to locate it or what is a good quality of online teaching/ learning materials for mathematics and how to measure it. Another such question referring to the edge "AL", is about "students' perceptions if and how online resources contribute to mathematics learning and motivation" (*Triantafyllou & Timcenko*, *ibid.*, p. 1573). The diverse nature and the complexity of these questions about online learning, in addition to the methodological approaches applied, mainly small scale studies or online surveys, do not allow generalizing conclusions about its truthful effects for the mathematics education.

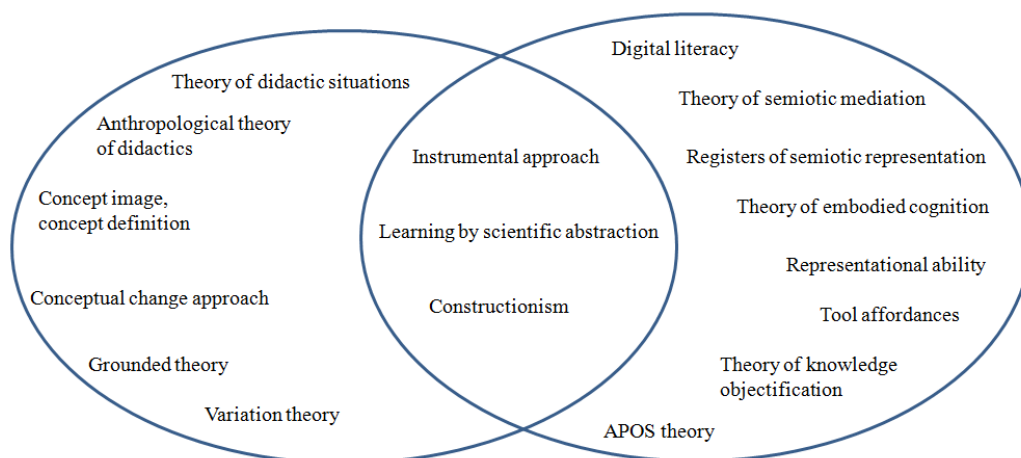
Looking at the face "ALK" of the "didactic tetrahedron", an interesting question that could be worth exploring is whether a possession of a "(piece of) mathematical knowledge" leads to gaining an "other (piece of) knowledge" embedded in an ICT tool, e.g., knowledge in computer engineering. Except for one contribution by *Misfeldt & Ejsing-Duun* (CERME 9, 2015) about learning mathematics through programming and algorithms, we have not found others which would report on any kind of connections between learning mathematics and computer science or informatics. Neither have RQs about the learning of mathematics in relation to robotics, augmented reality and artificial intelligence been proposed in any of the calls, the introductions to papers or the papers in the technology group for the learning of mathematics at the CERME 8 and 9. This issue is neither mentioned in the CERME10-TWG16 Call, although we could expect that it may become an emerging one due to curricular changes in some European countries (e.g., France) highlighting algorithms in mathematics education.

### *Evolution of theoretical frameworks*

Several observations can be drawn about theories and their networking in the papers.

First, the *instrumental approach* (Rabardel, 1995) appears as a widespread theoretical framework at CERME 8, while it is seldom mobilized at CERME 9. The hypothesis that may explain this fact is related to the shift in research questions reported above. However, in the terminology of “landscape of networking strategies and methods” (Prediger et al., 2008), it appears that the instrumental approach has been used for local organization and coordination, rarely combined with other theories. The heterogeneity of research questions at CERME 9 may be related to a greater diversity of ICT tools usage. Besides the commonly used technologies as dynamic geometry systems (DGS), computer algebra systems (CAS) or spreadsheets, innovative artefacts, such as multi-touch screen, Arbol software for developing combinatorics thinking or non-digital Fraction board, raise elderly and new concerns akin to those of tool affordances and multiple representations (“AK” edge of the didactic tetrahedron). Two main frameworks are called for exploring such questions: the *theory of semiotic mediation* (Bartolini-Bussi & Mariotti, 2008) and the *approach of registers of semiotic representation* (Duval, 1993). These two theories seem to go along one with another and have a relatively high degree of integration founded on the strategies for understanding and making understandable, comparing and synthesizing (Prediger et al., 2008). Original digital devices, and possible novel teaching methods enabled by them (e.g., flipped classroom, learning on the Web) may lead to modifications of learners’ perceptions of their efficiency or performance. These are explored through the Vygotskian perspective of object/meaning ratio.

Further observation leads to an assumption that there is a greater variety of theoretical frameworks used in CERME 9 compared to CERME 8 papers (Fig. 2). This seems to correspond to the previous argumentation. Besides the recognizable continuity of the usage of three theoretical frameworks, *instrumental approach*, *constructionism* and *learning by scientific abstraction*, there is a vivid occurrence of numerous others. Yet, “the multiplicity and isolated character of most theoretical frames used in technology enhanced learning in mathematics”, brought to the fore by Artigue (2007) and considered by the author as “an obstacle to the exchange and mutualisation of knowledge” (p. 75), is still not overcome. The heterogeneity of the networking space may further be analyzed by using the flexible triple of principles, methodologies and paradigmatic questions (Radford, 2008).



**Figure 2: Theories used in paper at the CERME8-TWG15 (left) and CERME9-TWG16 (right)**

It is worth noticing that most of the theoretical frameworks considered in the papers are not technology specific. In fact, the *instrumental approach*, *human-with-media concept* (Borba and Villareal, 2005) and the *theory of semiotic mediation* are rare frameworks addressing the interactions between learners and artifact(s), digital or not, besides those between learners and teachers. A widely used technology non-specific theoretical framework is the *theory of didactical situations* (Brousseau, 1997), which is occasionally combined or integrated locally with other theories.

Finally, we wish to draw attention to theoretical concepts that are not mentioned in the papers, although they are particularly relevant for addressing the relation “AK”. Some of them, such as *computational transposition* (Balacheff, 1993) and *epistemological domain of validity* (Balacheff & Sutherland, 1994) are powerful means for ICT tool analysis in reference to a given field of knowledge and in terms of their possible contribution to the teaching and learning.

## Conclusion

Looking through the lenses of the “didactic tetrahedron”, the split of the CERME 8 technology group in two groups since the CERME 9 is not only a practical, organizational necessity due to the rapid growth of the number of scholars interested in the theme. It rather seems as a temporary solution to tackle and deeply investigate challenging questions about each of the faces of the tetrahedron before fabricating ‘state of the art’ reports.

Thinking about the capitalization of knowledge disseminated by the CERME 8-TWG 15 and the CERME 9-TWG 16 relating each of the two main issues in this survey paper, we may conclude the following.

1) *Evolution of RQs and methodologies*. Miscellaneous RQs are emerging rapidly, before the previous are being sufficiently explored. On the one hand, it seems that the trend of publishing findings about the influence of the World Wide Web including social networks and online educational platforms will continue in a relatively large amount despite an apparent lack of specific methodological and theoretical frameworks that could be commonly used to approach topical issues in the field of technology in mathematics education. Applied methodologies for approaching these questions belong within the frame of small scale qualitative empirical studies. On the other hand, research questions, appropriate methodologies and theories about attitudes, accomplishments and inclusion of specific groups of learners as low achieving, gifted and/or disabled students in technology supported learning environments remain urgent in the research agenda.

2) *Evolution of theories*. Is the use of current general theories like those referring to the “didactic triangle” sufficient or is there a need for a development of new ones, which would allow addressing issues specific to technology enhanced teaching and learning of mathematics? The latter seems to be more likely, as shown by a new item in the call for papers in the theory working group welcoming contributions on “theories for research in technology use in mathematics education” (CERME 10-TWG 17 Call for papers), which has not been part of the previous call of the group. Our analysis also shows that exploitation of the networking strategies and methods for understanding, comparing, contrasting, coordinating, combining, synthesizing and integrating theoretical frameworks (Prediger et al., 2008) may be beneficial for further truthful studies of the learning mathematics with technologies.

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