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# Meaning-generation through an interplay between problem solving and constructionism in the C-book technology environment

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*Starting from Silver's (1997) approach for the importance of the interplay between problem solving and posing in the agenda of creativity, a new kind of e-book, aiming to promote creative mathematical thinking to students, in which its designers enriched the posing element with a constructionist approach, is used and examined in a real classroom. The paper follows a pair of Grade-8 students while they are working on this book. The contribution of this new interplay in the meaning-generation process around the concept of covariation is examined and the change in the creativity landscape by the analogy between problem posing and constructionism is discussed.*

**Keywords:** C-book technology, meaning making process, constructionism.

## Introduction

According to Silver (1997) deep flexible knowledge is closely related to creativity and emerges during the interplay between problem solving and problem posing. On the other hand, new exploratory and expressive digital media provide users with access to and potential for engagement with creative mathematical thinking and meaning-generation activities (Hoyles & Noss, 2003). However, education systems fail to rise this challenge due to restrictions stemming from the emphasis given on conformity and standardization in testing (Chevallard, 2012). So, new designs are needed to support students' engagement with dynamic digital media that aim to foster creative mathematical thinking. In this spirit, a pair of students are working collaboratively to solve a problem using a new digital medium, we call 'c-book', ('c' for creativity), a new genre of authorable e-book, extending e-book technologies to include diverse dynamic widgets, interoperability and collective design. In this paper we draw on end-users' interactions examining their interplay between problem solving and constructionism as enabler of meaning-making and try to analyze the effect of this affordance of the medium to the meaning making process of the pair as well as to comment on this new role of Constructionism as facilitator/substitute of problem posing.

## Theoretical framework

The close connection of mathematical knowledge with the interplay between problem solving and problem posing flows from the fact that most of the mathematicians do their research, mainly by formulating their own questions and problems and then trying to solve them, rather than solving problems posed for them by others (Borwein, Liljedahl, & Zhai, 2014). In this sense, the generation of new mathematical meanings for students, as an action, may be related with this kind of interplay. Cai and Cifarelli (2005) further refined this link between problem solving and problem posing, considering the posing and solving process to be mathematical exploration structured by this

recursive process. In the context of technology Abramovich and Cho (2015) found that a technological environment facilitates problem posing and turns it into discovery experience. And it is possible for meaning-generation processes to take place during such experience. Obviously, the affordances of the digital environment determine so much the kind of interplay that takes place as well as the meaning-generation process. Papadopoulos, Diamantidis and Kynigos (2016) describe how specific affordances of an expressive digital medium (c-book) led students to meaning-generation process around the concept of angle. However, in their study a possible relation between constructionism and posing is hardly examined. Constructionism is a theory that examines design and learning processes focusing on the ways in which these are part of individual or collective construction of digital artefacts. It illuminates how the representations, the affordances, the rules behind the behaviour of digital objects and the fields in which they reside and the ways in which these representations can be manipulated can all constitute representational registers around which meanings are generated, shared and developed (Kynigos & Pscharis, 2003). It thus provides an analytical lens to study the design and construction process in close interaction with the changes made to the artefact in question and the meanings those changes carry (Papert & Harel, 1991). In the case of a jointly constructed artifact by a group of students, the changes made to the artefact constitute externalization of the group's knowledge. Microworlds are such environments, allowing at the same time personal construction of objects and new meaning. C-books exploit half-baked microworlds which are incomplete by design, challenging students to fix them fostering thus learning through tinkering (Healy & Kynigos, 2010). Students have to solve problems that they encounter, in between and may come up, as a result of students' efforts to make new constructions, in order to fix the initial bug of the microworld. So, the question now is: How the affordance of the c-book technology to support the interplay between problem solving and problem posing/'Constructionism' might contribute to a process of meaning-generation?

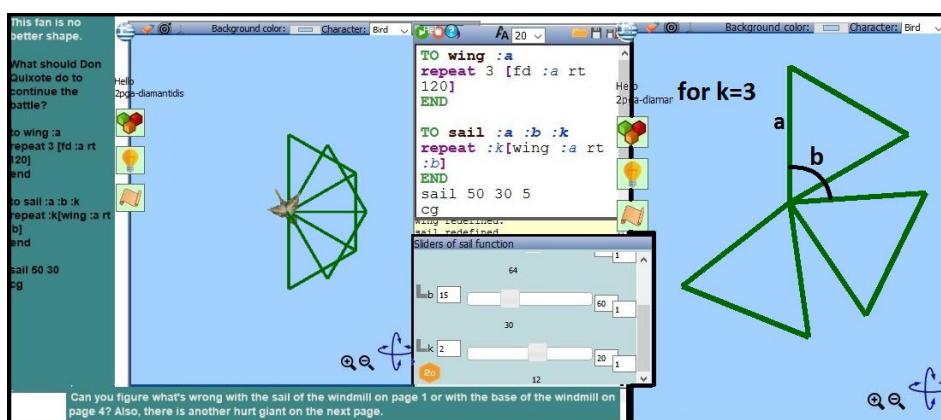
## **The digital medium and the Don Quixote c-book unit**

C-book is a new expressive medium that affords the design of modules named c-book units. Each c-book-unit is based on a storyline, and includes diverse 'widgets' between the lines of the narrative. The term 'widgets' is used for objects, such as hyperlinks, videos and mostly instances, or activities, from a range of educational digital tools such as Geogebra and MaLT2, a web-based Turtle Geometry environment that affords Logo-mathematics symbolic notation and dynamic manipulation of 3D geometrical objects, using sliders as variation tools. Most of the widgets refer to mathematical inquiries, constructions and problems. Students can navigate through the pages of the c-book unit and be involved in the included tasks through experimentation, reconstruction and problem solving.

The c-book unit used in this study presents a different twist of Don Quixote's story. It begins with Don-Quixote confronting 30-40 windmills he mistakenly considers giant enemies (first pages of the c-book unit). But, after being close to them he realizes that they are damaged windmills and he wants to repair them. Half-baked logo codes in MaLT2 represent the windmills' fans and sails in various geometrical figures and Don Quixote has to modify the codes so as to repair and reconstruct the fans and the sails.

## The study

This study presents an educational intervention designed and implemented in a classroom. Adopting the methodology of “design experiments” (Collins et al., 2004) the focus was on seeking relationships between the learning process and the use of digital media used by the students during the implementation phase. Twenty-four students (18 from Grade-8 and 6 from Grade-9) from a public Experimental School in Athens participated in the study which took place in the pc-lab of the school during after-class mathematics courses for totally eight teaching hours within four weeks. The students were divided into pairs. Most of them were familiar with the usage of 2D E-slate Turtleworlds. Two teachers served as facilitators for technical issues, whereas two researchers undertook the role of observers recording instances of the students’ interactions with the digital medium. Voice recorders and a screen-capture software (HyperCam2) were used to record students’ interactions with the c-book unit tools and their discussions, since both of them constituted our data. The students’ interactions were transcribed and the protocols were parsed into episodes with emphasis on the transitions between episodes since these were the points at which the change from solving problems to creating new ones used to happen (Schoenfeld, 1985).



**Figure 1: The ‘buggy windmill’s fan’ task in the c-book unit environment**

In this study, we follow two students as they are coping with a task asking them to fix a broken windmill. A Logo program was already developed producing a buggy and half-complete fan of windmill (Figure 1, left). It was needed to make changes in the Logo program, to fix the bug and shape up the fan.

## Results

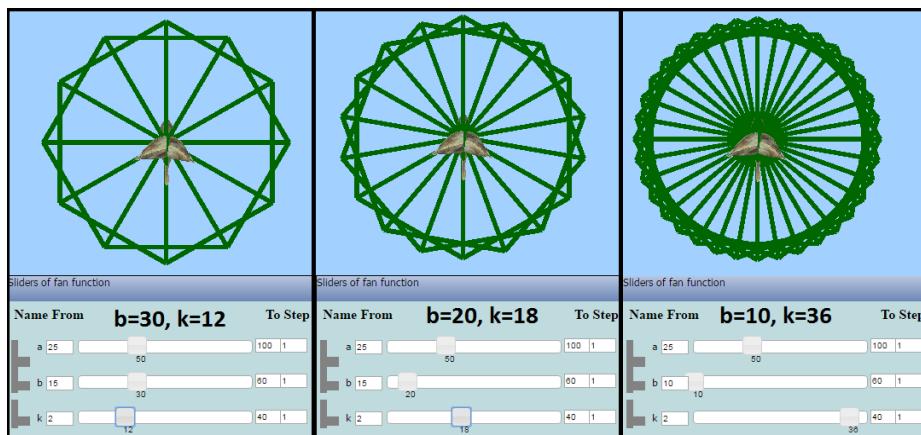
The students initially had to fix the bug on the windmill (Figure 1, left). The fan was ill-constructed since its wings had not been joined in a proper way. So, they started using the variation tool to observe changes and identify the role/function of each slider/variable. The initial Logo-code construction contained three variables  $a$ ,  $b$  and  $k$ , for the ray of the fan, the angle between two consecutive wings of the fan, and the total number of wings, respectively (Figure 1, right).

There are two procedures in this code. The “wing” which uses variable “ $a$ ” to make an equilateral triangle with side length “ $a$ ”, and the “sail” (main procedure) which constructs the whole fan using “wing” as sub-procedure. Fixing the bug, is an open-ended problem with a variety of solutions (for example, for a polygon-shaped fan a feasible solution would be to replace  $b$  with  $360/k$ ).

After some back-and-forth of changing dynamically the values of all variables in the code, and examining the results of their actions on the screen the students found a pair of values that made the figure to look like a windmill's fan:

- S1-23: We managed to make it well shaped, but only for a certain pair of values; 12 for k and 30 for b. We must put certain values instead of variables.
- S2-24: It is not a fair solution; we should find a way to keep the fan well shaped, for any set of values. Is there a possibility that a, b and k vary analogous to each other?
- S1-25: What do you mean by “analogous”?
- S2-26: I mean that the change of only one value through the variation tool, results to changes for all of them, without our intervention.
- S1-27: Let's see [*she changes dynamically the value of a*]. It is not worth dealing with a. It only changes the length. We should find a relation between k and b.

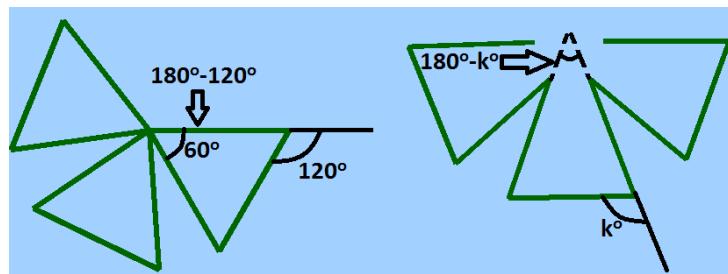
In the extract above, it seems that according to Student-2 the specific pair of the variable values cannot be considered as a proper solution. The references to “analogous” and “without our intervention” are indicative of the student's confidence that a more generic solution such as a relation between the variables, is needed. Therefore, they started actually talking about covariation.



**Figure 2: Three pairs of values that make the shape look like a fan.**

Thus, in order to find the relation between “b” and “k”, students went on with their investigation through dynamic manipulation, identifying pairs of values for b and k that made their construction to look like a proper fan (Figure 2). Their investigation resulted to the conclusion that “b” and “k” might be inversely proportional. Although they reached a conclusion about the kind of relation between “b” and “k”, they did not take the next step to express this finding as a formula, so as to use it for fixing the bug, reducing thus the number of the necessary variables. On the contrary, they decided to go on with their investigation, adding a new variable in the sub-procedure “wing”:

- S1-33: We found the solution. But I think that we must go further. You see, in the program “wing”, there is a right turn by 120 degrees, which means that our solution works only for this amount of turning.
- S2-34: Yes, we should put a variable instead of 120, let's use the letter “k” again, in order to find out what is going on, and solve the problem for every case of turning right.

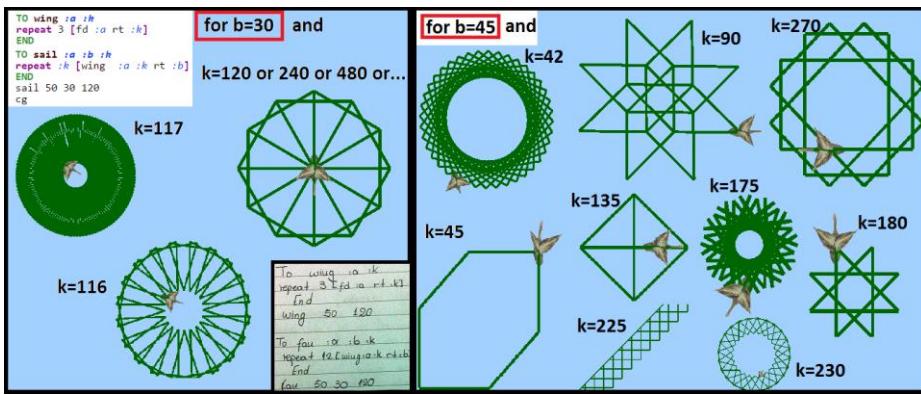


**Figure 3: The role of right turn by 120° (left) and by  $k^\circ$  (right)**

Variable “ $k$ ” now refers to the right turn for each new wing (instead for the total number of wings). Students made their own construction, by adding a variable in the ‘wing’ sub-procedure, which actually made the problem more complicated. The choice of a constant right turn by 120° is crucial for having the wings evenly delivered across the fan, since right turn by 120 degrees means that the sails will be equilateral triangles. Substituting the constant right turn by the new variable “ $k$ ”, has an impact on the angles of the triangular wing (Figure 3). Technically, this choice results to a fan even buggier than the original one. However, students did not see it as an obstacle. On the contrary, they accepted the challenge to solve a new problem that seemed to be more challenging to them:

- S1-46: Let's use the same variable  $k$ , for both, the number of wings in ‘sail’ and the amount of right turn in ‘wing’. [*They ran the program and moved hastily the slider that stands for  $k$ . This action changed not only the number of wings, but the shape of each wing of the fan as well.*] What a strange shape!
- S2-47: Are  $b$  and  $k$  still inversely-proportional or proportional amounts? [*They moved the sliders, in order to find pairs of values, as they had done before* (Figure 4).]
- S1-48: Variables  $b$  and  $k$  do not seem to be proportional.
- S2-49: Nor inversely proportional. This is not fair!
- S1-50: Is it possible that there is no connection, no relation between  $b$  and  $k$ ?
- S2-51: What other kind of relation other than proportional and inversely proportional may exist between them?

This question became the starting point for the students to be engaged in a new inquiry, about a new meaning that seemed to emerge. They started speaking about the notion of covariation in a more abstract sense than before (S1-25, S2-26). The spirit of this negotiation is mirrored in the final remark they made in order to solve the problem: “We think that there must be a relationship between  $b$  and the new  $k$ . We found that for  $b=30$ , if  $k$  equals to 120 or 240 or 480 or 960 the sail stays well-shaped, so there is a relation like  $k=120 \cdot 2^x$ . We also discovered a pattern for the values of  $b$  that is much more complicated.” They refer to their observation that if for example  $b=45$  then the most ‘acceptable’ shapes are the ones with  $k$  multiple of 5 (Figure 4).

Figure 4: Snapshots for pairs of values of  $b$  and  $k$ 

## Discussion

The Don Quixote c-book unit is designed in alignment with the view that creativity lies in the interplay between problem-solving and problem-posing, an idea which is very much in accordance with Silver's approach (1997), arguing that it is in the interplay of formulating, attempting to solve, reformulating, and eventually solving a problem where creative activity may lie in. Indeed, as Papadopoulos et al. (2016) describe, students who used this c-book were able to show creative mathematical thinking that did not emerge instantly but was the result of the above mentioned continuous interplay combined with the provided affordances. In this paper we focus on the meaning-generation processes that took place before the creative moment, relating them with the entrance of constructionism in the agenda of creativity. The members of the designing team of the c-book unit showed an inclination to connect creativity with constructionist activities due to their background and familiarization with this theoretical tradition (Papadopoulos et al., 2015). This resulted to a fostering of the problem-posing element by a constructionism view. So, in this c-book unit the students were working in a context that enabled a continuous and more distinct interplay between problem solving and posing/constructionism and this interplay is examined in relation to whether it operates as enabler of meaning-making in mathematics. As we presented above, the students had initially to explore the problem of an ill-structured windmill and find the part of the problem that is ill-defined. The problem seemed to be solved for a specific set of values (S1-23, S2-26) but this does not ensure the generality of the solution. So, the new problem was to keep the fan well-shaped for any set of values. To solve this task, it was deemed necessary to identify the role of each variable in the solution of the problem which resulted to the knowledge that variable 'a' is not related to the bug (S1-27). This actually transformed the last problem to a new one, asking for the relation between variables 'b' and 'k'. This new problem contributed to the shift of the focus towards the notion of covariation. In order to find the relationship between the variables, a series of new constructions took place. They resulted to a collection of pairs of values for 'b' and 'k' that made the fan look like a proper one. This made the students think that the two variables were inversely proportional. However, the formula was still missing and this became their next problem. Therefore, a new variable was added to the code (S1-33, S2-34). A new, more complex construction took place. The feedback on their screen from their constructions made them doubt their claim for the proportionality of the variables (S1-48, S2-49) and the phrase "This is not fair!" (S2-49) opened the discussion about possibly another kind of relationship (S1-50, S2-51). That was the new

problem which resulted to the more focused discussion on covariation and the possible formula that might fix the bug.

It seems that students made a step further. They tried to answer two interim questions they themselves posed and which came up as they tried to fix the buggy windmill, reconstructing it in a way consistent and meaningful for them. So, they reformulated the initial problem, starting from their reconstructing efforts and came back to solve it anew in a process where new and perhaps more creative aspects of mathematical knowledge were expected to emerge. Thus, the interplay between problem solving and constructionism was apparent, while the interplay between problem solving and posing was not direct. The formulation of new inquiries by students indicates that constructionism facilitated problem posing. Actually, this view of constructionism is close to what Brown and Walter (1990) argue about the problem-posing process: the solver first makes a list of all attributes included in the statement of the original problem, and then, he proceeds in negating each of them formulating thus an alternative proposal, a new problem. However, negating an attribute makes the original problem ill-defined (or ‘half-constructed’), and so the solver is challenged to proceed to the ‘construction’ of a ‘new’ problem. It is in this sense, we argue, that an interesting connection between problem-posing and the Constructionism perspective arises within the context of the c-book technology and c-book units, never having been identified in the literature so far. At the same time there is evidence that during this interplay a meaning-generation process takes place. The problem of finding the bug of an ill-structured windmill’s fan, which seemed to be mostly related to spatial observation and Geometry, turned to be investigated by the students through algebraic procedures, use of symbols and looking for relationships between variables. Students while trying to understand what was going on with the shape of the fan by reconstructing it, actually moved back and forth between processes of problem solving and construction. It was during this interplay that students started moving from the specific notions of proportional and inversely proportional variables to the more abstract notion of covariance.

## **Conclusions**

The problem solving and posing approach in creativity (Silver, 1997) attributes creative moments in the interplay between them. The entrance of Constructionism in the agenda of creativity seems to have an impact in the creativity landscape and perhaps opens new research challenges. The new c-book units that aim to foster creative mathematical thinking in students are based on a design principle that is characterized by ‘Constructionism fostering/substituting problem posing’. Then the whole story is evolved around the continuous interplay between problem solving and posing/constructionism. So, on the one hand some new research questions arise about the role of Constructionism in fostering creative mathematical thinking. On the other hand, there is evidence that this interplay between problem solving and posing/constructionism in the path towards creative moments facilitates meaning-generation processes by the students.

## **Acknowledgment**

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