Nature and characteristics of digital discourse in mathematical construction tasks

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This paper introduces the concept of digital discourse in Mathematics using a philosophical framework by Alexander Galloway. The notion of the digital is discussed and the concept of digital discourse is elaborated on that basis. The empirical data shows its value by reporting on transitions in language when working with digital tools on geometrical tasks. Existing research findings show effects of DGS on language changes referring to geometrical objects and actions. The present study analyses qualitatively both students' language referring to mathematics as well as to the digital tool in the context of geometrical constructions. The empirical results give insights into processes and transitions in the language use (by students) from a tool-oriented language (e.g. referring to buttons) to a mathematical-oriented language (referring to mathematical concepts) and aim to explore the nature and the characteristics of digital discourse.

Keywords: Language, digital tools, geometry, discourse.

There are substantial research results concerning the changes in language used for describing mathematical actions and objects when working with DGS (Kaur, 2015; Sinclair & Yurita, 2008). This paper shows results focusing on empirical phenomena concerning a language that students use in order to describe actions and objects referring to the digital tool. The interplay between these two layers referring to the tool as well as to the mathematics is analyzed in detail. The analysis will give empirical insights into the transition-process between the language of mathematics and technology and, by doing so, will examine a central facet of digital discourses.

Etymological and philosophical aspects of digital discourse

What is digital discourse? Initially one might think of SMS-chats, of instant messaging via Skype or WhatsApp, of blogging or the like of tweets, of posting messages on social network sites, or of video-based online-discussions (e.g. Llinares & Valls, 2009). There is no doubt that all these forms of discursive practices have their roots in the way new media is used. But is it adequate to use the predicate *digital* for such discursive practices? For the examples above, this paper will rather use the term discourse "in the New Media" (Thrulow & Mroczek, 2010) to differentiate it from the term digital. But what is the *digital*? This paper wants to stress the notion of the digital in the mathematics classroom, especially the notion of what can be seen as digital discourse in mathematics. To do so, we will follow a philosophical path drawing on a work by the philosopher Alexander Galloway (2014), in which he gives an introduction to the work of the philosopher François Laruelle and—by doing so—tracing back the notion of the digital in philosophy to Plato and Sokrates and, especially, to Hegel's work. For Galloway, rather than distinguishing zeros and ones (the digital) compared to continuous variation (the analog), the "digital is the basic distinction that makes it possible to make any distinction at all. The digital is the capacity to divide things and make distinctions between them. Thus not so much zero and one, but *one and two*." (Galloway, 2014, p. xxix) In that sense, the digital is closely connected to the notion of difference. This philosophical perspective on the digital is used here to discuss its value for the mathematics

classroom and especially for discursive practices and processes of concept formation in it. By doing so, this approach does not claim to adopt Galloway's perspective on philosophy and his non-standard philosophical approach drawing on Laruelle (c.f. Laruelle 2010). It rather uses perspectives he offers to introduce the concept of digital discourse in mathematics and to better understand its nature and characteristics.

The understanding of discursive practices and the underlying norms have been a major subject of study in mathematics education. And yet there is a need for reactivating such analytical approaches in the light of the use of digital tools since digital tools "give rise to new ways of thinking that may conflict with the established discourse of formal mathematics" (Sinclair et al., 2016a). In their analysis, Sinclair and Yurita (2008) outline the way in which the use of dynamic geometry changes discourse, e.g. transitions from static to dynamic forms of discourse. Also, Schacht (2015a; 2015b) reports on shifts in language regarding student's documentations, in which the students use a language that clearly refers to the digital tool and not to mathematics. Both examples show—in different ways—that the digital tool affects the discursive practices in class. And still we know little about normative rules affecting the discursive practices: the conceptual (mathematical) norms involved, the social- and socio-mathematical norms, the norms established and the norms brought in by the technology in use. By introducing the concept of digital discourse, this paper approaches an understanding of discursive processes by using Galloway's perspective on the digital in order to distinguish different discourses referring to the tool or to mathematics with specific underlying norms. Therefore, we will first briefly highlight Galloway's (2014) notion of the digital, then introduce the definition of digital discourse in mathematics before applying it to the empirical data.

What is *the digital*?

Building on the broad definition of the theoretical concept of the digital that points at the notion of difference (see above), Galloway describes the operation of the digital as follows: "the making-discrete of the hitherto fluid, the hitherto whole, the hitherto integral. Such making-discrete can be effected via separation, individuation, exteriorization, extension, or alienation. Any process that produces or maintains identity differences between two or more elements can be labeled digital." (Galloway, 2014, p. 52) Although the digital can be seen as an archetype of philosophical thinking in general (tracing it back to Plato and Sokrates), Galloway (2014) describes the digital as fundamental to the dialectics of Hegel with its two moments: "the (digital, F.S.) moment of *analysis*, where the one divides in two (1→2, F.S.), and the (analog, F.S.) moment of *synthesis*, where the two combines as one (2→1, F.S.)" (Galloway, 2014, p. xxix). Galloway also attributes analog and digital to both moments (p. xxxi). Although Galloway (2014) does not focus on computers or new media in this work but rather on discussing the theoretical concept of the digital in general, the (digital) process of discretization and of computation however, of extending *the one* (or, in a mathematical context: the mathematical concept) "beyond its own bounds, thereby branching the one, splitting it" (Galloway, 2014, p. 52) is inherent to computers and hence to digital tools. In this sense, the dialectic idea is existentially present and closely related to the digital: "Hegel is dead, but he lives on inside the electric calculator." (Laruelle, Introduction aux sciences génériques, 28, cited in Galloway, 2014, p. xxxiv)

Digital discourse in mathematics

Following Galloway's discussion of the digital, the following definition is used here: Any discourse that produces or maintains differences between two or more elements can be labeled digital. This definition does not necessarily focus on technology. Also, the notion of digital discourse presented here certainly differs from studying discourses in the new media (Llinares & Valls, 2009). As a theoretical concept, this notion can be applied to any discursive practice. However, the analysis of the empirical data will demonstrate that the concept of digital discourses can be used to structure and describe conceptual processes and transitions underlying the work with digital tools because the (*digital*) distinction between expressive reference to the mathematics and to the digital tool and, in line with that, the corresponding underlying norms seem to play a central role in these processes. In this sense, the approach follows the "need to study the transition phases in the progress of geometrical concept formation" (Sinclair et al., 2016b, p. 696).

The term discourse is used here in a pragmatic (more precise: from an inferential) perspective (Schacht & Hußmann, 2015): Individual conceptual processes and mastering mathematical concepts is understood as being able to give reasons for the use of concepts within discursive practices (=master the inferential relations), similar to Wittgenstein's idea of mastering the rules of the language game. In this perspective, individual conceptual acting is highly normative since the individual acknowledges the reasons one has for applying a concept to be true or at least to be adequate in a certain situation. Hence, it is one of the tasks of this analytic approach to reconstruct the (normative) rules that the individuals (as concept-appliers) follow. Discursive practices, in this perspective, give access to individual conceptual processes and the underlying social and individual norms. Galloway's notion of the digital is used here for digital discourses to differentiate between different (normative and conceptual) discursive layers, and especially between the following two layers discussed in this paper concerning the mathematics and the tool. In this sense, it is the aim of this paper to introduce the concept of digital discourse and to better understand its characteristics and nature when working with digital tools exemplified by an example from geometry class.

Language, written documentations and digital tools in geometry

Before analyzing the digital discourse in the empirical example from a dynamic geometry environment (DGE), this paragraph will briefly highlight research findings on the changes in language when working with digital tools. Such tools affect the conceptual thinking and acting with the mathematical objects as well as the written solutions (Ball et al., 2005; Weigand, 2013). In geometrical contexts, DGS can support the grasping of both the geometrical objects as well as the actions conducted with these objects (Jore & Parzysz, 2005). Also, the use of DGS changes language and discourse in various ways. Hölzl reports that students tend to use active verb forms while working with the dragging mode (Hölzl, 1996) which mirrors the movement and the dynamic actions. Also young children (ages 7–8) tend to change forms of reasoning using action verbs influenced by dynamic and temporal elements of the DGS like the dragging tool (Kaur, 2015). In line with that, changes in discourse and vocabulary could be observed regarding the transition from static to dynamic forms of discourse about geometrical objects: "[S]hapes were discussed as if they were a multitude of objects, changing over time, rather than as a single object" (Sinclair & Yurita,

2008). Hence, not only the understanding of the objects and actions change, but also the language used in order to make actions and objects explicit.

Whereas most of these findings report on transitions of language regarding *mathematical actions and objects* when working with DGS, Schacht (2015) shows that students also use a rather naturalistic language to describe the *manual actions* precisely to get to their solution as well as *objects referring to the tool* like buttons or commands. This paper focuses on the way in which students describe actions and objects both referring to the mathematics and referring to the digital tool as well as on the transitions between these layers when working on geometrical constructions. These different referential units—the mathematics and the digital tool—each reflect certain conceptual—and, following the pragmatic approach, also normative—layers e.g. it makes a difference whether one describes the solution process in terms of the mathematical process or in terms of the manual actions conducted with the digital tool.

This paper studies the transitions of using *tool language* to using *technical language*. This paper reports on results of a qualitative study, in which processes like these were studied based on students' involvement on construction tasks using DGS (GeoGebra) with respect to the following underlying research question and the overarching interest to explore the nature and characteristics of digital discourses: *Which language transitions can be observed between a tool- and a maths-oriented language when working on construction tasks?*

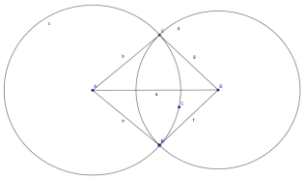
Methods and design

The empirical study was conducted with N=20 students (age 13–15) from different schools. All students worked in pairs with GeoGebra within clinical interviews. This paper uses some of the data of this bigger project in order to demonstrate the potentialities of digital discourses as a way to understand lexical transition processes better . A systematic analysis of such lexical processes is given in Schacht (in revision). All pairs that did the construction tasks were videotaped during the construction and afterwards. The interviews were analyzed qualitatively focusing on the oral and the written language by using lexical categories (Schacht, 2015). The written documents were also analyzed. Three to four geometrical tasks were given to each pair; the duration of working on these tasks ranged between 40 and 80 minutes. All students were introduced to the DGS first and they were encouraged to explore some main functionalities since most of the students had not have much experience working with GeoGebra. Most tasks had an explorative character, meaning for example that the students were asked to construct certain objects and try to formulate a description. Within a task aiming at exploring the concept of symmetry, the students were asked to describe and give reasons for their findings. In this sense, the information the experimenters were trying to obtain from the interviews focused on the way the students work on such geometrical tasks and the language they use when speaking and writing. A detailed description of the tasks relevant to this paper is given below.

Results and discussion: Transitions in language

The pairs of students were given a geometrical configuration and they were first asked to (reconstruct) the given Figure 1 with ruler and compass (on a sheet of paper) and formulate a written description of the given construction by using ruler and compass for themselves (line 1

below shows the description of student 1 (S1)). After that the students worked in pairs with a DGS. They were then asked to construct the same figure with GeoGebra and then formulate a description of the construction together (lines 2 & 3 shows the common description of S1 and S2 with the DGS).

Line 1	S1 (r&c): Draw a 5,7cm straight line (German: Linie) from A.	 Figure 1: The given geometrical configuration
Line 2	S1&S2 (DGS): Click on the button "segment" at the top.	
Line 3	S1&S2 (DGS): Segment from the center to each point of intersection.	

The analysis focuses on the language the students use to describe the objects they deal with. Using ruler and compass, S1 refers to a term from everyday language (line (*German: Linie*)) which is—in the German translation—not considered to be a proper mathematical term. When the students work with the DGS, they document their manual action (line 2) precisely by describing which button to click on. In this case, they choose the segment-button. In the version of GeoGebra they use, the name of the button "segment" is shown. Hence, the term *segment* refers to a *button* (c.f. Schacht 2015a) which is an object that refers to the digital tool. It is important to note that although *segment* can be considered as a mathematical object, the students describe a button, hence an object referring to the tool. On the other hand, in line 3 the students refer to the segment from the center to the intersection points. This description refers to the mathematical object of segment.

This analysis shows two transitions in the students' language within their documentations. First, there is a change from the description of a given object in everyday language (line 1) to the description of a certain button (line 2). For the students in line 2, *segment* is a signifier of the button that they use for their documentation. The second transition shows in which way this signifier is used as a mathematical term to refer to the mathematical object of the *segment* (line 3). The students do not refer to a button. They rather use the term segment within a mathematical description. Hence, the analysis gives insights into the process of the description of buttons (as objects referring to the tool) to the description of segments (as mathematical objects). This is a central characteristic of digital discourses: The distinction between the tool-oriented and the mathematical language offers a possibility for the students to use terms common in mathematical discursive practices. The example shows that digital discourses can follow a linear structure in which the difference between the tool oriented layer and the mathematical layer can even be bridged by the digital tool since it offers expressive resources that students adopt.

This first example has limitations though: The students can easily adopt the term *segment* because it connects to a colloquial use of the term. As the following contrasting example will show, obstacles may occur when students use terms that do not connect easily to a colloquial understanding. In this task, the students explored the phenomenon of symmetry. The students were given a GeoGebra file with two quadrilaterals, where one quadrilateral was the image of the other under a line reflection. The line itself was not depicted. By dragging the one quadrilateral, the other one moves according to the line reflection. The students were asked to construct a new geometrical figure by starting with the two rectangles (Figure 2) and the two students S3 and S4 formulated a description for a construction he made with the DGS afterwards.

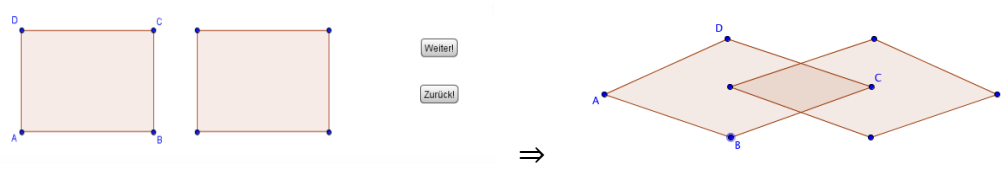
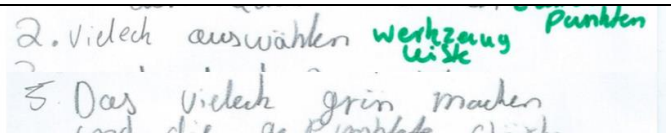


Figure 2: Exploring the properties of a line reflexion. By dragging ABCD (left) the image moves correspondingly

In the next step, only the written description was given to the other student who then had to (re-)construct the original figure with the DGS. In the following example, student S3 draws a triangle by using the *polygon-button* 

. He had seen the signifier of the button during his construction. In both descriptions below the student S3 refers to the triangle using the term *polygon*.



line 2	S3 (DGS): (toolbar)	2. Choose polygon
lines 6-8	S3 (DGS):	5. Colour the polygon green

Transcript of the oral description:

Turn 1 S3 (DGS): By pulling at the left polygon, here at the points, coordinates, in order to have three rectangles.

First, the student refers to the polygon-button (line 2) in order to describe a manual action precisely and the objects (referring to a *button*) needed in order to handle the DGS. Next, the student adopts the signifier in order to refer to the mathematical object (lines 6–8). In lines 6–8, the polygon is seen as a static object that has to be colored in green. Third, this mathematical object is used dynamically since the student describes how to pull at it (turn 1). Hence, this example shows a transition from the description of an object that refers to the tool (*polygon button*) to the description of first a static and then a dynamic mathematical object (*polygon*).

Although this transition seems to mirror the process that could be observed in the first example, there are significant differences. It is important to note that student S3 does not refer to the term *triangle* at all. Instead he sticks to the term polygon throughout the interview. When (his partner) S4 has difficulties with the description and asks S3 to describe precisely what he meant by *choose polygon,* S3 does not refer to the three points that his specific polygon (as a triangle) has (turn 1). Hence, this example shows that although S3 uses the signifier to deal with the mathematical object, he has difficulties with grasping the concept to distinguish between a polygon and a triangle since he has no conceptual understanding of this term. This analysis shows interesting obstacles. First, S3's description to *choose a polygon* is viable, but it is not precise enough for S4 to understand that he has to create a triangle. There are several other scenes in this interview, which lead to the conclusion that S3 does not grasp this concept as he cannot connect it to other mathematical concepts. Especially, it is not obvious for S3 that a triangle is a certain polygon. This example raises the question of how it is possible to support a *semantic* connection to the students' knowledge by using the digital tool and the given signifier. In terms of the characteristics of the digital discourse, the student does not manage to make the mathematical concept explicit by giving reasons for it in the course of the interview. He rather incorporates the expression *polygon* offered by the user interface. The second example gives insights into a divergent digital discourse because there is no mediation between the two layers in order to establish a commonly shared mathematical

understanding of the construction. The reason for that is the missing conceptual transition between the two layers mathematics and tool.

Final remarks

The two examples discussed in this paper show similar transitions in students' language when working with digital tools and give insights into digital discourses. They give insights into the process of the descriptions from objects related to the tool (e.g. buttons) to mathematical objects. Regarding the research question, the examples show different lexical phenomena when working on construction tasks. Example 1 reveals that digital tools can support lexical processes by adopting the signifiers' names in the description of the construction. The students manage the shift from a language that focuses on technology to a rather mathematical language. In other cases this can lead to obstacles which show the need for a conceptual connection to concepts the students already know (example 2). These different lexical transitions show that the use of a rather calculator-oriented language has potentialities as well as limitations. Although it is important for the mathematics classroom to support the use of both language referring to the tool (e.g. in exploration situations) as well as to the mathematics, it remains a challenge to develop means to bridge and support language processes in a way that students not only adopt signifiers but rather grasp a conceptual notion of the mathematical objects and actions. Hence, studying language processes like these shows the importance of a lexical consciousness of the different lexical norms of adequacy when language is used in specific situations (Schacht, 2015).

Studying such language transitions illustrates the role of the notion of difference in discursive practices when working with digital tools. The two examples in this paper show two different digital discourses, understood as discourses that produce or maintain differences between the mathematical and the technological discursive layer. On the one hand, a digital discourse can have a linear structure in which the difference between the mathematical and the technological discursive layer is bridged. On the other hand though, that difference between these two layers maintains throughout the discussion. Regarding the characteristics of such digital discourses, the examples show that it makes a difference if the students use expressions referring to the mathematics or to the digital tool. One the one hand, each layer implies specific underlying norms and specific conceptual aspects, which refer either to the mathematics or to the digital tool. On the other hand, the processes above show the dynamic of such discourses and the way in which these—linear or divergent—transition processes can have implications on the individual concept formation processes.

The results also show possibilities for further studies of the digital discourses in respect (i) to the underlying conceptual processes, (ii) to a deeper insight into the nature and characteristics of digital discourses and (iii) to the potential for classroom practices to foster digital discourses within conceptually supporting environments.

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