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Research paper

1D morphodynamic modelling using a simplified grain size description

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1 ABSTRACT

2 This paper introduces a 1D numerical code RubarBE for hydraulic and mobile-bed simulations. The code's ability to
3 reproduce the downstream fining of a gravel-sand mixture in response to bed aggradation is tested against laboratory
4 experiments. Unlike in most numerical models, grain size distribution in each sediment layer is not represented using a
5 multi-class model, but using the median diameter d_{50} and a sorting coefficient σ . The comparison of numerical results
6 with experimental data shows that the adaptation length L_a , classically used for non-equilibrium sediment transport,
7 is an essential parameter of the model to accurately reproduce the evolution of the deposit front. Empirical laws
8 for adjustments of d_{50} and σ are proposed to reproduce sediment sorting through two grain-size related adaptation
9 lengths (L_d , L_σ). They are scaled by the length of the reach in morphological equilibrium, which is a useful result
10 for the field applications.

11 *Keywords:* sediment Aggradation; downstream fining; median grain size; sorting coefficient; 1D numerical
12 model

13 1 Introduction

14 The numerical simulation of sediment transport and processes along rivers is essential to under-
15 stand river dynamics and predict river bed evolution. While the bed structure and the sediment
16 grain size distribution should be accurately reproduced in a numerical model, it may induce high
17 computational cost. Modelling strategy consists in finding the best compromise between a detailed
18 representation of driving processes and an acceptable computational cost. Both robustness and
19 parsimony are needed in simulation, since only scarce experimental data are available.

20 In order to reproduce the bed structure, different layers can be distinguished according to their
21 dynamics. Hirano (1971) introduced a three-layer model:

- 22 • the sediment transport layer is the upper layer where sediments are in motion;
- 23 • the substrate layer is the bottom layer, which is assumed to be permanently buried with no
24 sediment motion;

- the active layer is located between the two aforementioned layers and acts as a buffer zone to satisfy sediment continuity between the upper and bottom layers. Sediments within this layer are in constant interaction with the two other layers.

The Hirano (1971) active layer concept is used in many sediment transport models for simulating selective bedload transport (Deigaard, 1980; Mikoš, 1993; Hoey & Ferguson, 1994). di Silvio (1992) considers four different layers, dividing the transport layer in two sub-layers for suspension and bedload transport, respectively. Separate equations were also applied in di Silvio (1992) model to quantify horizontal and vertical sediment movement inside a layer or between two layers. Parker (1991) suggested that the active layer thickness δ_{AL} may be scaled with the grain size of the layer ($\delta_{AL} \approx d_{90}$ where d_{90} is the grain diameter at which 90 percent of the distribution in mass lies below). In case of sandy rivers with bedforms, Blom (2008) showed that δ_{AL} is scaled with the bedform height.

The characterisation of bed layer sediments can be done using different strategies. The common one consists in characterising the grain size distribution by a discrete number of classes. According to Belleudy (2001), the multi-class representation permits to better calculate the transport rate for each class of sediments, including possible interaction between classes. Interactions between classes are generally introduced through a modification of the critical bed shear stress for inception of movement including hiding or exposure coefficients (Egiazaroff, 1955; Parker, 1990; Wu, Wang, & Jia, 2000; Wilcock & Crowe, 2003). This representation has been used by many authors to simulate the sediment downstream fining due to selective transport (Parker, 1991; van Niekerk, Vogel, Slingerland, & Bridge, 1992; Vogel, van Niekerk, Slingerland, & Bridge, 1992; Langendoen & Alonso, 2008) or coarsening due to bed degradation (Langendoen & Alonso, 2008). For this purpose, several authors performed some numerical modelling calibration and validation using the test case presented by Seal, Paola, Parker, Southard, and Wilcock (1997). Wu and Wang (2008) focussed their discussion on the ability of their model to simulate transient flows over movable beds but simulation of grain size evolution was not presented. Cui, Parker, and Paola (1996) and Belleudy (2001) simulated grain size evolution using a multi-class representation but if the reproduction of the aggrading front was in good agreement with experimental results, downstream fining appeared to be sensitive to the grain size description. Fine sediments ($d < 2$ mm) were excluded from the simulation of Cui et al. (1996). Langendoen and Alonso (2008) presented accurate simulations for both degradation experiments by Ashida and Michiue (1971) and aggradation experiments by Seal et al. (1997) using the channel model CONCEPTS with 14 predefined size classes. In a similar way, Qian, Cao, Pender, Liu, and Hu (2015) presented a model that yields accurate results for the experiment of Seal et al. (1997) including some interesting qualitative results for the vertical and longitudinal description of the bed. However, in the multi-class representation, sediment transport has to be evaluated for each class of sediment; validation may therefore be difficult, and even often impossible to achieve (Camenen, Holubová, Lukač, Le Coz, & Paquier, 2011). Indeed, comprehensive data are needed to accurately describe each sediment class and little experimental information exists to properly describe the interaction between classes (Wilcock & Crowe, 2003). Hoey and Ferguson (1994) showed that a slightly size-selective bedload transport can produce rapid downstream fining, hence models may be strongly sensitive to parameters that are not fully understood such as hiding and exposure parameters. For suspended load, downstream fining may be reproduced through size-dependent variables such as bed concentration or settling velocity. This phenomenon may be predominant compared to bedload transport in case of sandy river (Wright & Parker, 2005). Blom (2008) stressed the importance of taking into account the variability in bed form geometry in modelling sorting and morphodynamics. It is particularly important for improving the prediction of the adaptation time-scales of the bed surface composition, the vertical sorting profile, and the composition of the bedload transport. However, such models are very sensitive to bed sediment description, and may not be robust enough to simulate real study cases.

75 In case of a unimodal grain size distribution such as a log-normal distribution, the distribution
 76 can alternatively be represented using two parameters only: the median value d_{50} and the standard
 77 deviation of this distribution $\sigma = \sqrt{d_{84}/d_{16}}$ (where d_x is the grain diameter at which x percent of the
 78 distribution in mass lies below). One difficulty is then to represent hiding or exposure phenomena
 79 or more generally selective transport in such a model. The objective of this paper is to evaluate
 80 the numerical code RubarBE (El kadi Abderrezzak, Paquier, & Gay, 2008; Paquier & El Kadi
 81 Abderrezzak, 2008; El kadi Abderrezzak & Paquier, 2009), which uses the d_{50} - σ representation
 82 for the grain size distribution and includes specific rules for representing selective transport. The
 83 performance of this model is assessed using the test case presented by Seal et al. (1997) for the
 84 downstream fining modelling. Eventually, the RubarBE code robustness, capabilities and limits are
 85 discussed.

86 2 Presentation of the numerical code: RubarBE

87 2.1 Description of the model

88 The RubarBE code is a 1D hydraulic mobile-bed numerical code, which solves the Barré de Saint-
 89 Venant equations (shallow water equations). These equations are solved using a second-order
 90 Godunov-type explicit and upwind scheme, which is shock capturing and robust enough to de-
 91 scribe transitions between subcritical and supercritical flows (El kadi Abderrezzak et al., 2008; El
 92 kadi Abderrezzak & Paquier, 2009). It should be noted that hydraulic parameters are resolved with
 93 a cell-centred scheme (computed in the middle of two cross-sections) while sediment parameters
 94 (sediment transport and bed evolution) are resolved with a node-centred scheme (computed at
 95 each cross-section) (El kadi Abderrezzak et al., 2008). This particular feature of RubarBE numer-
 96 ical scheme improves its stability especially when bed evolves dramatically. Sediment variables are
 97 computed at each time step as well as the riverbed geometry, which is updated using the Exner
 98 equation for the sediment continuity:

$$(1 - \phi) \frac{\partial A_b}{\partial t} + \frac{\partial Q_s}{\partial x} = 0 \quad (1)$$

99 where ϕ is the porosity, A_b is the cross-sectional area of the bed above a reference datum, Q_s is
 100 the volumetric sediment transport, t is the time, and x is the longitudinal direction.

101 An equation describing the non-equilibrium sediment transport Q_s , i.e. the spatial lag equation
 102 (Daubert & Lebreton, 1967) is introduced to tackle unsteady morphological processes (Bell &
 103 Sutherland, 1983; Armanini & di Silvio, 1988; Phillips & Sutherland, 1989; Rahuel, Holly, Chollet,
 104 Belleudy, & Yang, 1989):

$$\frac{\partial Q_s}{\partial x} = \frac{Q_{s*} - Q_s}{L_a} \quad (2)$$

105 where Q_{s*} is the equilibrium sediment transport (or sediment transport capacity) and L_a is the non-
 106 equilibrium adaptation length, which permits to reproduce spatial lag effects in morphodynamical
 107 adjustments of the bed.

108 The bed is represented using a three-layer model (Figure 1): the transport layer, which includes
 109 both bedload and suspended load; the active layer, and the substratum layer, which is divided
 110 in a given number of sub-layers. For each layer and sub-layer, the sediment size distribution is
 111 represented with the median diameter d_{50} and the sorting coefficient $\sigma = \sqrt{d_{84}/d_{16}}$. Exchanges
 112 between layers are calculated according to sediment balance (Balayn, 2001). At each time step Δt
 113 and for each cell, there is a sediment discharge coming from the upstream cell Q_s^{up} (with a mass
 114 $M^{up} = Q_s^{up} \Delta t$). These sediments interact with the sediments from the bed layer through the active

115 layer, which acts as a buffer layer (cf. Figure 1). This results in a sediment discharge toward the
 116 downstream cell Q_s^{dn} (with a mass $M^{dn} = Q_s^{dn} \Delta t$). Q_s^{dn} will correspond to Q_s^{up} for the next cell.

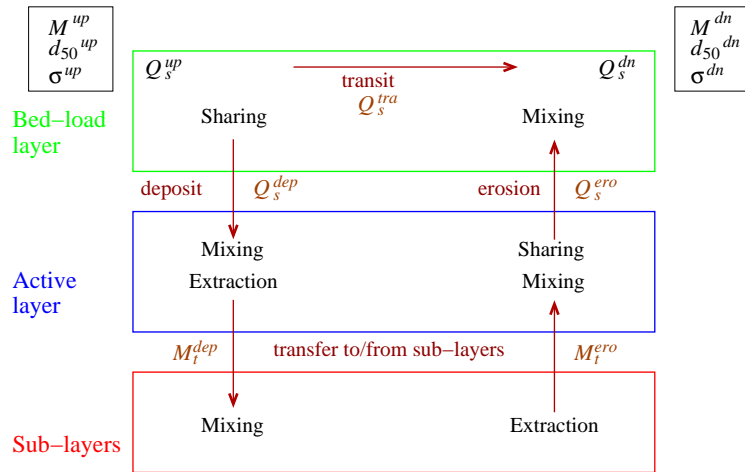


Figure 1 Illustration of the sediment exchanges within a cell of length Δx and over a time-step in the RubarBE code.

117 The sediment discharge transiting directly to the next cell is calculated using the spatial lag
 118 equation (Eq. 2 integrated over space without Q_{s*}):

$$Q_s^{tra} = Q_s^{up} \exp\left(-\frac{\Delta x}{L_a}\right) \quad (3)$$

119 where Δx is the length of the cell or mesh size. Then, the remaining part of the input sediment
 120 $Q_s^{dep} = Q_s^{up} - Q_s^{tra}$ is deposited and mixed with the content of the active layer.

121 Sediment discharge eroded from the active layer and entrained by the flow is also calculated
 122 using the spatial lag equation but based on the equilibrium sediment transport, which is calculated
 123 using local hydraulic conditions :

$$Q_s^{ero} = Q_{s*} \left[1 - \exp\left(-\frac{\Delta x}{L_a}\right) \right] \quad (4)$$

124 Finally, the resulting outward sediment discharge (toward the downstream cell) is $Q_s^{dn} = Q_s^{tra} +$
 125 Q_s^{ero} and the sediment mass in the active layer is $M_{AL}(t + \Delta t) = M_{AL}(t) + M^{dep} - M^{ero} =$
 126 $M_{AL}(t) + \Delta t(Q_s^{dep} - Q_s^{ero})$. The sediment mass in the active layer is defined such as $M_{AL} =$
 127 $M_{AL*} = \rho_s Q_{s*} \Delta x / U$ where U is the current velocity. M^{dep} is first mixed with M_{AL} , then, M^{dep} is
 128 extracted from M_{AL} (cf. Figure 1). Depending on the remaining mass in the active layer, there is
 129 a direct exchange of mass with the upper bed layer (extraction if $M_{AL}(t + \Delta t) < M_{AL*}$ or mixing
 130 if $M_{AL}(t + \Delta t) > M_{AL*}$).

131 Since the present model using d_{50} cannot reproduce selective transport through a sediment
 132 capacity formula or the non-equilibrium adaptation length, grain size evolution is modelled here
 133 through sediment exchanges. Sediment exchanges between each layer are reproduced using three
 134 main rules, which allow to reproduce effects of selective erosion or deposition (cf. Figure 1):

- 135 • mixing law: homogeneous mix of two sediment populations;
- 136 • sharing law: sharing of a sediment population into two different sediment populations follow-
- 137 ing an empirical spatial lag equation;
- 138 • extraction law: sharing of a sediment population into two sediment populations with identical
- 139 grain size characteristics as the initial one.

140 The mixing law for two samples is calculated using a mass weighted average between the geo-
 141 metric median diameter and standard deviation of the two amounts of sediment for each class (El
 142 kadi Abderrezzak & Paquier, 2009):

$$\left(\begin{array}{c} M_a \\ d_{50a} \\ \sigma_a \end{array} \right) \oplus \left(\begin{array}{c} M_b \\ d_{50b} \\ \sigma_b \end{array} \right) = \left(\begin{array}{c} M = M_a + M_b \\ d_{50} = d_{50a}^{\frac{M_a}{M_a+M_b}} \times d_{50b}^{\frac{M_b}{M_a+M_b}} \\ \sigma = \sigma_a^{\frac{M_a}{M_a+M_b}} \times \sigma_b^{\frac{M_b}{M_a+M_b}} \end{array} \right) \quad (5)$$

143 The sharing law is introduced to separate an initial sediment sample into two sub-samples, the
 144 first one, which is finer, is transported by the flow, whereas the second one, which is coarser, settles
 145 down. It is based on Eq. 2, which can be written as $\partial d_x / \partial x = \partial d_x / \partial Q_s (Q_{s*} - Q_s) / L_a$ where d_x
 146 is the grain diameter for which x percent of the distribution in mass lies below. Assuming the
 147 term on the right-hand side of the equation is a linear function of d_x , the spatial evolution of both
 148 median grain size and standard deviation can be described by an exponential function. The grain
 149 size characteristics of the finer class depends on the mass $M_f = Q_s^{tra} \Delta t$ or $Q_s^{ero} \Delta t$ taken from the
 150 total initial mass M , and is given by Balayn (2001):

$$\left\{ \begin{array}{l} d_{50f} = d_{50} \times \exp \left[-\frac{\Delta x}{L_d} \times \frac{\sigma - 1}{\sigma} \times \frac{M - M_f}{M} \right] \\ \sigma_f = \sigma \times \exp \left[-\frac{\Delta x}{L_\sigma} \times \frac{\sigma - 1}{\sigma} \times \frac{M - M_f}{M} \right] \end{array} \right. \quad (6)$$

151 where L_d and L_σ are the adaptation lengths related to the median diameter evolution and sorting
 152 coefficient evolution, respectively. Similarly, the coarser class (mass $M_c = M - M_f$) in the sharing
 153 law is given by :

$$\left\{ \begin{array}{l} d_{50c} = d_{50} \times \exp \left[\frac{\Delta x}{L_d} \times \frac{\sigma - 1}{\sigma} \times \frac{M - M_c}{M} \right] \\ \sigma_c = \sigma \times \exp \left[\frac{\Delta x}{L_\sigma} \times \frac{\sigma - 1}{\sigma} \times \frac{M - M_c}{M} \right] \end{array} \right. \quad (7)$$

154 One should note that a sediment sample that has been shared and mixed again comes back to
 155 the same sample. The coefficients $(\sigma - 1)/\sigma$ and $(M - M_f)/M$ were introduced to limit sediment
 156 evolution in case of a well sorted sediment or limited sediment transport. And if $L_d = L_\sigma = \infty$,
 157 the sharing law reduces to the extraction law: the grain-size characteristics of the two classes are
 158 the same.

159 2.2 Non-equilibrium adaptation length

160 The non-equilibrium adaptation length usually depends on the flow characteristics and sediment
 161 size (Armanini & di Silvio, 1986; Phillips & Sutherland, 1989). For bedload transport, Phillips and
 162 Sutherland (1989) suggested that L_a would be directly proportional to the average step length of
 163 grains, and eventually proposed:

$$L_a = \alpha_{L,PS} (\theta - \theta_{cr}) d_{50} \quad (8)$$

164 with $\alpha_{L,PS} \approx 5000$. For suspended load, it has been approximated as a function of the depth
 165 averaged velocity U , water depth h and settling velocity of the sediment W_s (Armanini & di Silvio,

166 1986):

$$L_a = \alpha_{L,AS} \frac{Uh}{W_s} \quad (9)$$

167 where the coefficient $\alpha_{L,AS}$ is a decreasing function of the ratio W_s/u_* . Both Q_{s*} and L_a are sensitive
 168 to the grain size for bedload when θ is close to θ_{cr} , and for suspended load. As a consequence, for
 169 a multi-class model, the use of Eq. 2 enhances the variation of the grain size distribution, which
 170 may be difficult to calibrate. Most of these models use a hiding-exposure function that limit these
 171 effects (Hoey & Ferguson, 1994; Wu, Viera, & Wang, 2004; Langendoen & Alonso, 2008; Viparelli,
 172 Sequeiros, Cantelli, Wilcock, & Parker, 2010; Stecca, Siviglia, & Blom, 2014). The use of Eq. 2 for
 173 the d_{50} - σ parametrisation impacts bed evolution only, not grain size distribution.

174 Based on their own model results with several degradation and aggradation cases, Wu et al.
 175 (2004) adopted the equation $L_a = 7.3h$, corresponding to the ripple wave length. As shown by
 176 Wu et al. (2004), L_a is an important parameter for numerical stability; and the smaller L_a values
 177 are used, the smaller grid sizes are required, inducing in turn a smaller time step for numerical
 178 stability purpose. In natural situation, because of the larger spatial and time scales, large mesh
 179 sizes and time steps are usually used limiting the effect of small L_a values. It may explain the large
 180 values suggested by Rahuel et al. (1989); Belleudy (2000, 2001); El kadi Abderrezzak et al. (2008),
 181 especially for 1D modelling that integrates 2D and 3D effects. L_a may thus be scaled by larger
 182 morphological characteristics such as meanders, gravel bar length, etc. and so may be scaled to the
 183 width of the river B .

184 2.3 Grain-size related adaptation lengths

185 The sharing law is based on Sternberg (1875), who observed that the characteristic grain size of a
 186 river follows an exponential law of the longitudinal distance:

$$d = d_0 \exp(-\alpha_d x) \quad (10)$$

187 where x is the longitudinal distance and d_0 is the grain size at $x = 0$. Both L_d and L_σ are parameters
 188 that drive this grain sorting; they can be scaled with a distance. As a first guess, L_d et L_σ would
 189 be taken larger than L_a since L_a is representative of one step length and may be scaled to the
 190 width of the river B for a 1D model as discussed above. Grain sorting could be observed after
 191 many step lengths only and is observed on relatively long distances (from several dozens of metres
 192 for a gravel bar to several km for a river main channel). Morris and Williams (1999) showed that
 193 $1/\alpha_d$ is proportional to the length L of the reach in morphodynamical equilibrium (see Figure 2).
 194 Since $L_d \propto 1/\alpha_d$ based on Eqs. 6 and 7, one can assume that $L_d \propto L$, which means that this
 195 coefficient should be scaled by the length of the reach in equilibrium. In Figure 2, some additional
 196 experimental values that include an estimation of α_σ were added (Seal et al., 1997; Toro-Escobar,
 197 Parker, & Paola, 2000; Ferguson, Church, & Weatherly, 2001; Gomez, Rosser, Peacock, Murray
 198 Hicks, & Palmer, 2001; Surian, 2002). It appears that $\alpha_\sigma \propto 1/L$ as well, although some larger
 199 uncertainties exist. Apart from the case of Toro-Escobar et al. (2000) with fine sands, one can
 200 observe that generally, $1/\alpha_d > 1/\alpha_\sigma$. Based on Eqs. 6, 7, and 10, one can first suggest that $L_\sigma \propto L$
 201 and $L_\sigma \geq L_d$.

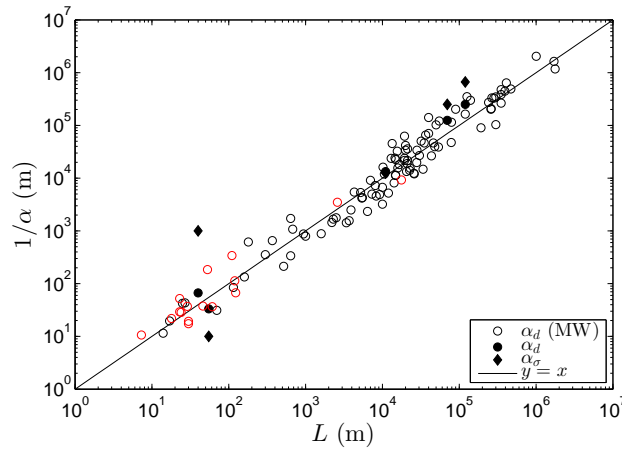


Figure 2 Relationship between coefficients α_d and α_σ and the length L of the reach in equilibrium (α_d (MW) stands for data given by Morris and Williams (1999), red dots correspond to negative values or coarsening reaches).

202 3 Downstream fining experiments

203 3.1 Experimental set up

204 The main objective of the experiments performed at St Anthony Falls Laboratory by Seal et al. (1997) was to reproduce the downstream fining of a gravel-sand mixture in response to bed aggradation. The three tests were conducted in a 45 m long, 0.305 m wide and 1.2 m deep flume with an initial bed slope set at 0.20% (Figure 3). The bed and the walls were covered with a smooth PVC membrane. The downstream boundary condition was set thanks to a tailgate (H_{tail}) imposing a constant downstream water height $z_{w,tail}$ for each of the three runs (Seal, Parker, & Mullenbach, 1995).

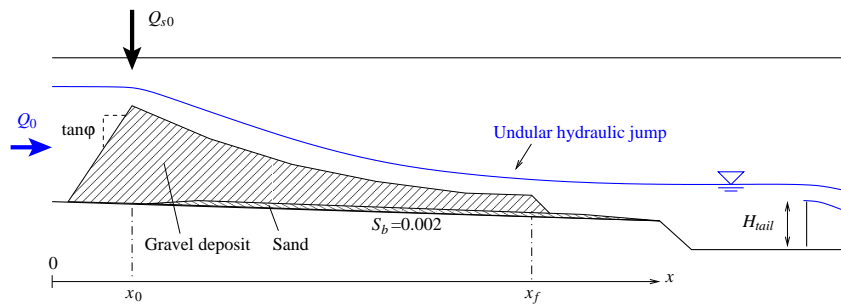


Figure 3 Schematic drawing of the experimental set up of Seal et al. (1997).

211 3.2 Description of the downstream fining experiments

212 The input mixture, ranging from 64 to 0.125 mm, was a mixing of three sediment samples with
 213 mean diameters equal to 22, 5 and 0.4 mm, respectively. The final experimental distribution was
 214 a poorly sorted sand and gravel mixture. However, since Seal et al. (1995) observed that very fine
 215 sediments were flushed away and deposited directly downstream of the channel, and represented
 216 approximately 4% of the total mass, the grain size distribution was truncated in the numerical
 217 model at $d = 0.15$ mm (Cui et al., 1996; Toro-Escobar, Parker, & Paola, 1997) and the solid
 218 discharge decreased by 4%. Both raw and adjusted grain size distributions for the sediment input are
 219 presented in Figure 4. Eventually, the sediment characteristics used in the model are $d_{50} = 7.5$ mm

220 and $\sigma = 6.0$. The distribution is properly represented by a median diameter d_{50} and a sorting
 221 coefficient $\sigma = \sqrt{d_{84}/d_{16}}$ corresponding to a log-normal distribution, although both recalculated
 222 d_{16} and d_{84} appear to be overestimated ($d_{16'} = d_{50}/\sigma = 1.3\text{mm}$, $d_{84'} = d_{50}\sigma = 45\text{mm}$ instead of
 223 $d_{16} = 0.8\text{mm}$, $d_{84} = 35\text{mm}$) (Figure 4).

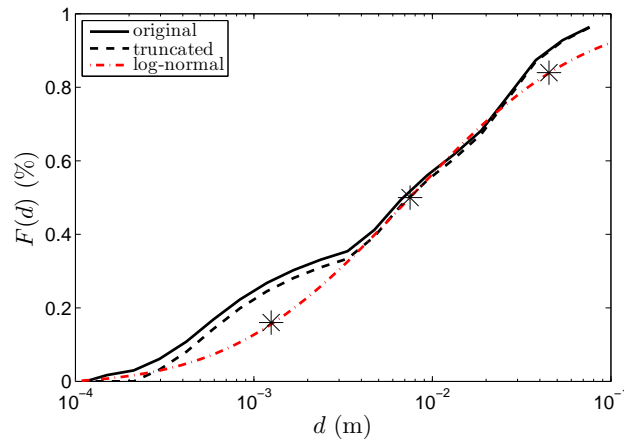


Figure 4 Grain size distribution of the input mixture in Seal et al. (1997) experiments ($F(d)$: cumulative mass (%); dash line corresponds to a readjustment of the grain size distribution of the sediment input, crosses to d_{50} and $d_{16'}$ and $d_{84'}$ obtained from the d_{50} - σ representation, and dash-dot line to the corresponding log-normal curve).

224 In the numerical model like in the experiment description, no sediment is present on the bed
 225 at the initial time. In the experiments, the sediment input is injected at the longitudinal distance
 226 $x_0 = 1$ m. In the numerical model, as the flume starts at the abscissa $x_0 = 1$, the backslope
 227 gravel transport at the flume entrance (cf. Figure 3) is not simulated. Nevertheless, according to
 228 Seal et al. (1997), only sediment involved in transport is defined as sediment input, making our
 229 approximation acceptable.

230 Sediment input Q_{s0} was decreased by a factor two from one run to another but the water
 231 discharge Q_0 remained unchanged (Table 1). The downstream fining in this simulation is not only
 232 due to selective transport but also due to selective deposition (Paola & Seal, 1995; Seal et al., 1995;
 233 Ferguson & Wathen, 1998). Toro-Escobar et al. (1997) argued that infiltration may have occurred
 234 during the experiment and quantified it using a calibration parameter. As the width to depth ratio
 235 was less than 0.2, the bed form effect is neglected (Colombini, Seminara, & Tubino, 1987), and in
 236 principle 1D modelling can reproduce bed evolution.

Table 1 Parameters for the three runs of the Seal et al. (1997) experiments used as boundary conditions for the modelling (Q_0 : water discharge, Q_{s0} : sediment input rate, $z_{w,tail}$: water elevation at the tailgate, and T_f : run duration)

Run	Q_0 (ls^{-1})	Q_{s0} (kgs^{-1})	$z_{w,tail}$ (m)	T_f (h)
1	49	0.1900	0.40	16h 50mn
2	49	0.0950	0.45	32h 24mn
3	49	0.0475	0.50	65h 00mn

237 3.3 Porosity of the deposit

238 The porosity of the deposit has been estimated based on the calculation of the cumulated sediment
 239 input, reduced by 4% since the mass of fine sediments found downstream of the channel equalled
 240 to 4% (Seal et al., 1995), and the volume of the deposit V . The sediment input rate used hereafter
 241 will be $Q_{s0,adj} = 0.96 \times Q_{s,in}$. As a consequence, we obtained $\phi = (V - Q_{s0,adj}/\rho_s)/V$. One can

242 observe in Figure 5 a slight reduction of the porosity with time, at least for run 2 and 3, indicating
 243 that infiltration occurred during the experiment as discussed by Toro-Escobar et al. (1997).

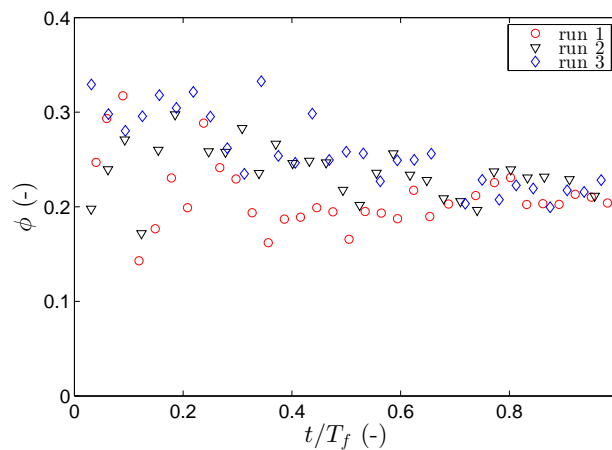


Figure 5 Estimated evolution of the deposit porosity for the three runs based on the cumulated mass introduced and the volume of the deposit.

244 When having a mixture of sediments of various grain sizes, one can observe some infiltration of
 245 the finest class within the pores of the coarsest class leading to a lower overall porosity following
 246 this simple linear relationship: $\phi = \phi_c - c_f(1 - \phi_f)$, where ϕ_c and ϕ_f are the porosity of the coarse
 247 and fine sediments respectively and c_f the volume concentration of fine sediments. A minimum
 248 is obtained for $c_f = \phi_c$ and $\phi_{min} = \phi_c\phi_f$. Actually, this simple model is valid only if $d_f \ll d_c$.
 249 Yu, Standish, and McLean (1993) suggested a model for the porosity of bimodal mixture taking
 250 into account the ratio $r_d = d_f/d_c$ through some empirical relationship. Following Parker and Cui
 251 (1998) and Cui and Parker (1998), and assuming the sediment mixture is bimodal (with 35% of
 252 fine sediments, $d_f \approx 0.4$ mm and 65% of coarse sediments, $d_c \approx 20$ mm) with a porosity $\phi = 0.4$
 253 for each single class, the Yu et al. (1993) model gives a theoretical overall porosity of a bimodal
 254 mixture with $r_d \approx 0.02$ equal to $\phi \approx 0.2$. This is consistent with the estimated final porosity for
 255 the deposit for the three runs (Figure 5) assuming the mass of fine sediments found downstream
 256 of the channel equal to 4% (Seal et al., 1995). For our calculations, we used a constant porosity
 257 $\phi = 0.2$ assuming there is no delay in the mixture equilibrium and so no infiltration during the
 258 experiment.

259 3.4 Position of the front

260 The deposit front x_F can be defined either as the position, for which 95% of the total deposit is
 261 located upstream ($V_{95\%}$) or as the position for which the local slope is maximum (S_F). The choice
 262 of 95% of the volume was made in order to avoid effects of uncertainties in the calculation of the
 263 volume. It can be observed in Figure 6 that both methods yield similar results for the time-averaged
 264 front velocity C_F (with the initial time as a reference), i.e. the front velocity decreases from 3×10^{-3}
 265 to 6×10^{-4} ms^{-1} . On the other hand, these methods differ significantly for the estimation of the
 266 instantaneous front velocity (with the previous time step as a reference), although both methods
 267 confirm the observation made by Seal et al. (1997) that the front moves irregularly. The first
 268 method ($V_{95\%}$) is more sensitive to the overall bed description whereas the second method (S_F) is
 269 more sensitive to the exact position of the front, and therefore often yields a zero value. The first
 270 method ($V_{95\%}$) will be used hereafter to estimate the front velocity.

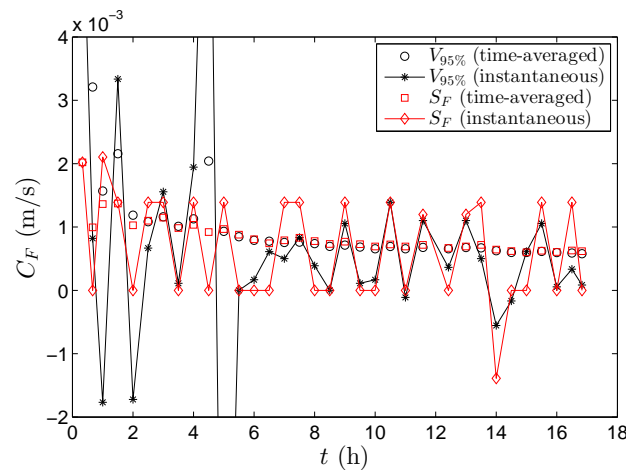


Figure 6 Front velocity estimation for the run 1 as a function of time (front position defined either as the position for which 95% of the total volume of deposit is located upstream ($V_{95\%}$) or the position for which the local slope is maximum (S_F)).

271 4 Numerical results and discussion

272 RubarBE code was run using a relatively loose mesh size ($\Delta x = 1$ m) to show its robustness.
 273 Nevertheless, a discussion on the sensitivity to the mesh size is presented. For all the computations
 274 presented here, the bed was represented by a single layer of sediments and a constant water depth
 275 downstream. The transport capacity equation from Meyer-Peter and Müller (1948) is used together
 276 with the critical bed shear stress calculated according to the Shields diagram. The model results
 277 are obviously sensitive to the choice of the transport capacity formula. It is yet not the purpose of
 278 this paper to discuss this point (El kadi Abderrezak & Paquier, 2011).

279 4.1 Bed evolution

280 Numerical results of the bed evolution are presented in Figure 7 for the run 1 for four distinct time
 281 steps from $T_f/4$ to T_f where T_f is the final time of the run. The Strickler coefficient was estimated
 282 based on the sediment characteristics (Strickler, 1923), i.e. $K_s \approx 23/d_{50}^{1/6} = 52 \text{ m}^{1/3}\text{s}^{-1}$. Results
 283 presented here were obtained using the Meyer-Peter and Müller (1948) bedload formula with no
 284 additional fit. A non-equilibrium adaptation length $L_a = 2 \text{ m} = 2\Delta x \approx 7B$ yields results in good
 285 agreement with experimental data (Figure 7) (Béraud, Le Coz, Camenen, & Paquier, 2011). It
 286 however appears with time that the slope of the deposit is slightly too mild; a stronger tilt would
 287 be needed to match the measured deposit elevation at both ends. The small mismatch at both ends
 288 may be linked to the choice of the bedload formula and critical bed shear stress (estimated from
 289 the Shields curve here).

290 The Froude number along the flume was calculated for the three runs at the final time-step (cf.
 291 Figure 8). Significant uncertainties exist in the estimation of the Froude number since small water
 292 depths are difficult to measure. It appeared that supercritical flow conditions always occurred in
 293 the first part of the channel especially for the run 1 as large sediment inputs generate large deposits
 294 and a sharp decrease in the water depth. For the run 1, one can observe that the Froude number
 295 is above 1 all along the deposit and reaches $F \approx 1.4$ at its upstream part. The RubarBE model
 296 reproduces properly the estimation of the Froude number with however a slight underestimation
 297 for the run 1 but not as pronounced as with the model by Cui et al. (1996). Their model using
 298 a coupled or decoupled scheme underestimates the Froude number for run 1 but yields consistent
 299 results. The models used by Wu and Wang (2008); Langendoen and Alonso (2008) were shown to
 300 handle both sediment transport and channel evolution under transcritical flow conditions. However,

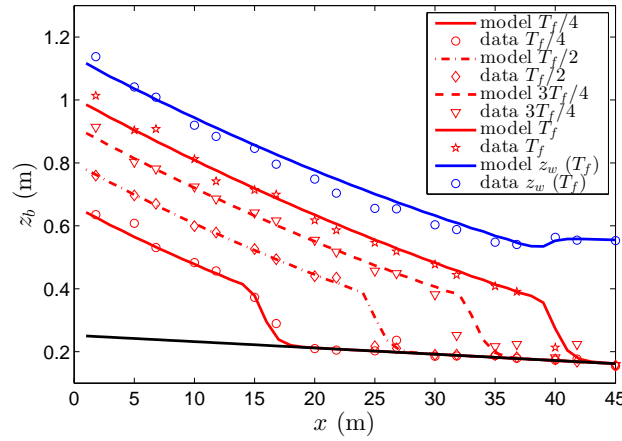


Figure 7 Experimental and simulated bed elevation at different times and water elevation at the final time-step of run 1 of the Seal et al. (1997) experiments using $\Delta x = 1$ m, $L_a = 2$ m, $L_d = 10$ m and $L_\sigma = 100$ m.

301 there is no clear discussion on their capability to capture supercritical flows as observed in run 1.
 302 The model used by Belleudy (2001) was not able to model run 1 and 2 in which supercritical flows
 303 were significant.

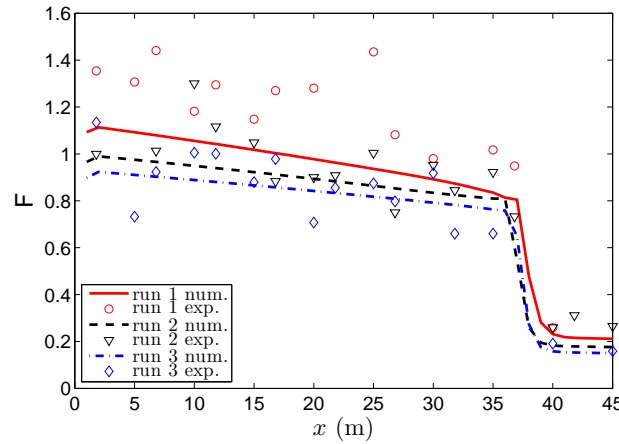


Figure 8 Experimental and simulated Froude number calculated at $t = T_f$ for the 3 runs ($\Delta x = 1$ m, $L_a = 2$ m, $L_d = 10$ m and $L_\sigma = 100$ m).

304 Figure 9 presents the experimental and simulated front velocity for the three runs. Again, the
 305 RubarBE code yields good results. The front velocity appears to be controlled by the sediment
 306 injection rate and by the height of the front. Indeed, assuming that the deposit can be described
 307 with a constant front height ($H_F \approx 0.15$ m) and a slope S constant over the deposit, the length
 308 of the deposit equals $L_d = (H_d - H_F)/(S - S_0)$ with H_d the height of the deposit at the injection
 309 point, and the front velocity can be estimated with:

$$C_F = \frac{1}{[H_F + (H_d - H_F)/2](1 - \phi)} \min(q_{sb}, q_{s,in}) \quad (11)$$

310 where q_{sb} is the bedload transport per unit width over the deposit calculated using some transport
 311 capacity formula (in this specific case, we used the Meyer-Peter & Müller, 1948, formula), and
 312 $q_{s,in} = Q_{s,in}/(\rho_s B)$ is the volume input of sediment per unit width. The Manning-Strickler equation
 313 is used to calculate the water depth and bed shear stress. If $q_{sb} < q_{s,in}$, the excess volume of sediment
 314 is laid linearly over the deposit such as $\Delta z = 0$ at the front (modification of the deposit slope).

315 This simple analytical formulation yields an average front velocity in agreement with experimental
 316 data (cf. Figure 9). There is however a slight underestimation of the front velocity at the beginning
 317 of the experiments because of the choice of a constant front height. It clearly confirms that the
 318 front velocity is mainly governed by the sediment injection rate and by the front height.

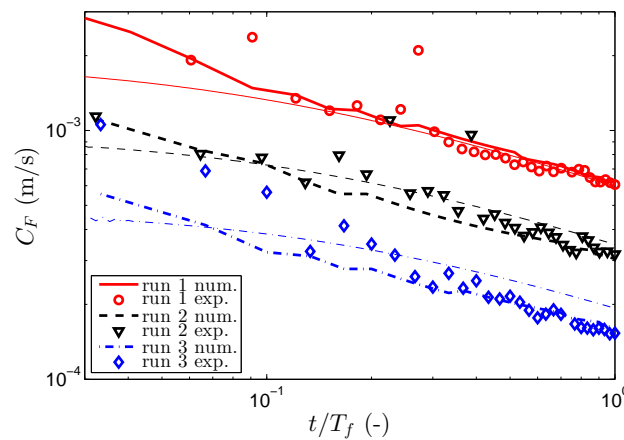


Figure 9 Experimental and simulated front velocity for the 3 runs ($\Delta x = 1$ m, $L_a = 2$ m $L_d = 10$ m and $L_\sigma = 100$ m, thin lines correspond to Eq. 11).

319 4.2 Sensitivity of the model on the mesh size and adaptation length

320 Figure 10a presents the temporal evolution of the front slope S_F for the experimental results of Seal
 321 et al. (1997). One should yet be careful when studying the slope of the front based on experimental
 322 results. Indeed, results are sensitive to the position of the measurements, which are not regular and
 323 maybe too scarce. When $\Delta x = 3.2$ m, the slope generally appears much smaller as the two points
 324 defining this slope do not necessarily correspond to the edges of the front slope. Even if a large
 325 scatter can be observed on Figure 10a, the front slope appears not to vary with time and $S_F \approx 0.1$.

326 Figure 10b presents some sensitivity analysis on the mesh size and adaptation length. These two
 327 parameters do not affect significantly the general trend of the results in terms of bed evolution.
 328 Their main impact is on the front slope, which decreases exponentially with larger values of Δx or
 329 L_a . Indeed, larger values for both Δx and L_a lead to larger diffusion in the system. When $L_a > \Delta x$,
 330 results are less sensitive to the size of the mesh. It is difficult here to assess which values for L_a
 331 are the more accurate to reproduce the experiment due to the lack of data and also because of
 332 the sensitivity of the front slope to the mesh size. However, $L_a \approx 1$ m for loose meshes to 2 m for
 333 finer meshes appears to be the best trade-off to reproduce both the front and the overall deposit
 334 shape (Béraud et al., 2011). Eq. 8 yields L_a values ranging from 1 to 3 m depending on the bed
 335 shear stress. Results obtained using Eq. 9 with $\alpha_{Las} = 5$ or 10 are similar to those obtained with
 336 a constant value ($L_a = 1$ or 2 m, respectively, see Figure 10) since it yields a value close to 1 m
 337 (respectively 2 m) at the downstream part of the deposit. As discussed previously, the width of the
 338 river B appears to be the best parameter to scale L_a for a 1D model although there needs
 339 additional experiments to confirm it.

340 4.3 Grain size evolution

341 Sub-surface was sampled for different time-periods by Seal et al. (1997) using the Klingeman, Cha-
 342 quette, and Hammond (1979) method. In the experiments, the sub-surface grain size appears to be
 343 finer than the surface grain size. This was explained by Toro-Escobar et al. (1997) as some vertical
 344 fining also occurred during the experiments due to infiltration. In Figure 11, the measured d_{50} and

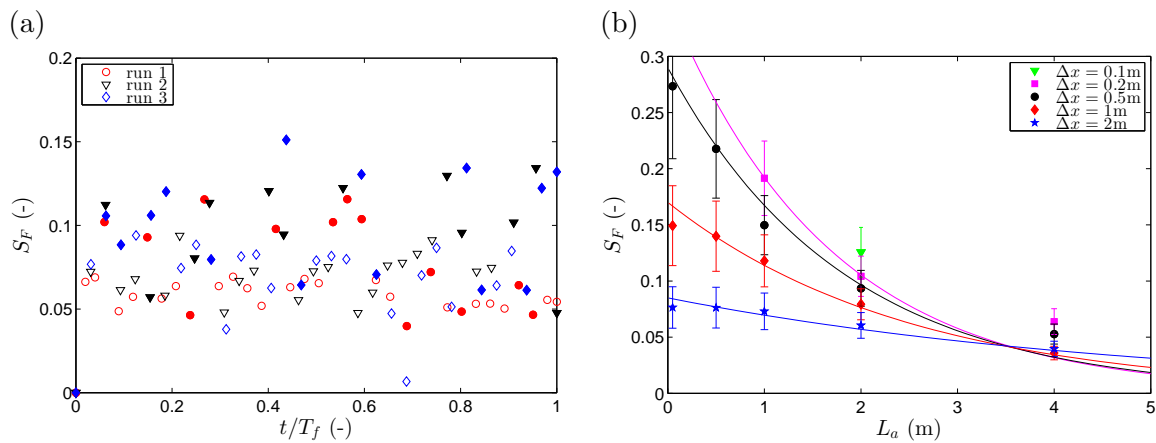


Figure 10 Evolution of the front slope S_F of the Seal et al. (1997) experiments for the three runs (filled symbols are used when $\Delta x = 1.8$ m, empty symbols when $\Delta x = 3.2$ m) (a) and averaged front slope obtained thanks to the numerical model for the run 1 using different values for the mesh size and adaptation length (b) (error bars correspond to the fluctuation throughout time, thin lines correspond to curve fitting using an exponential function).

345 σ in the sub-surface are plotted as a function of the downstream position made dimensionless with
 346 the front position x_F . Despite the significant scatter, one can observe a clear downstream fining
 347 resulting in a decrease of the median grain size from $d_{50} \approx 7.5$ mm to $d_{50} \approx 4.5$ mm, and a decrease
 348 of the standard deviation from $\sigma \approx 6.5$ to $\sigma \approx 5.5$. One can observe also a larger scatter close to
 349 the front with, for some of the measurements, a sharper decrease in the median grain size. Such
 350 behaviour in d_{50} and σ was also observed from the modelling by Cui et al. (1996); Langendoen
 351 and Alonso (2008) and typically corresponds to the selective transport with the finest fraction
 352 transported easier downstream.

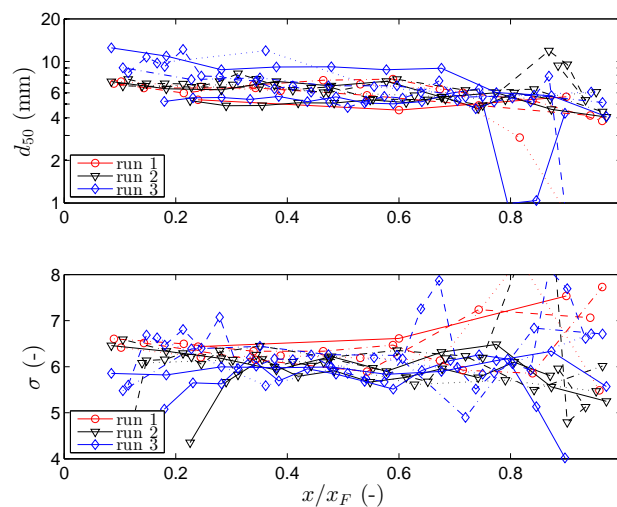


Figure 11 Evolution of the grain size characteristics of the Seal et al. (1997) experiments for the three runs as a function of the downstream position made dimensionless with the front position x_F .

353 In Figure 12, the experimental and simulated grain size characteristics are plotted as a function
 354 of the position along the deposit for the run 1. It is important here to distinguish the active layer (Figure 12a),
 355 corresponding to the surface, and the bed layer (Figure 12b), corresponding to the sub-layers. The active layer
 356 characteristics are consistent with measurements with a relatively low decrease of the grain size upstream
 357 and a sharper decrease close to the front. The bed layer characteristics appear more sensitive to L_d and L_σ .
 358 Median grain size (d_{50}) decreases rapidly in the first 10 m before reaching an asymptote (Cui et al., 1996),
 359 which is sensitive to L_d . This

360 effect is emphasized since a single layer is used for the modelling. Apart for the front, the active
 361 layer presents coarser sediments than the injection (fine sediments being transported downstream)
 362 whereas the bed layer is finer (sediment from the active layer mixed with finer sediments deposited
 363 previously). The use of small L_d and L_σ values ($L_d = 5$ m or $L_\sigma = 20$ m) yields a too strong
 364 decrease of d_{50} and σ , respectively. Also, $L_d \approx L_\sigma$ tends to affect results for σ leading to a sharper
 365 decrease. The model is not as sensitive to L_σ as to L_d .

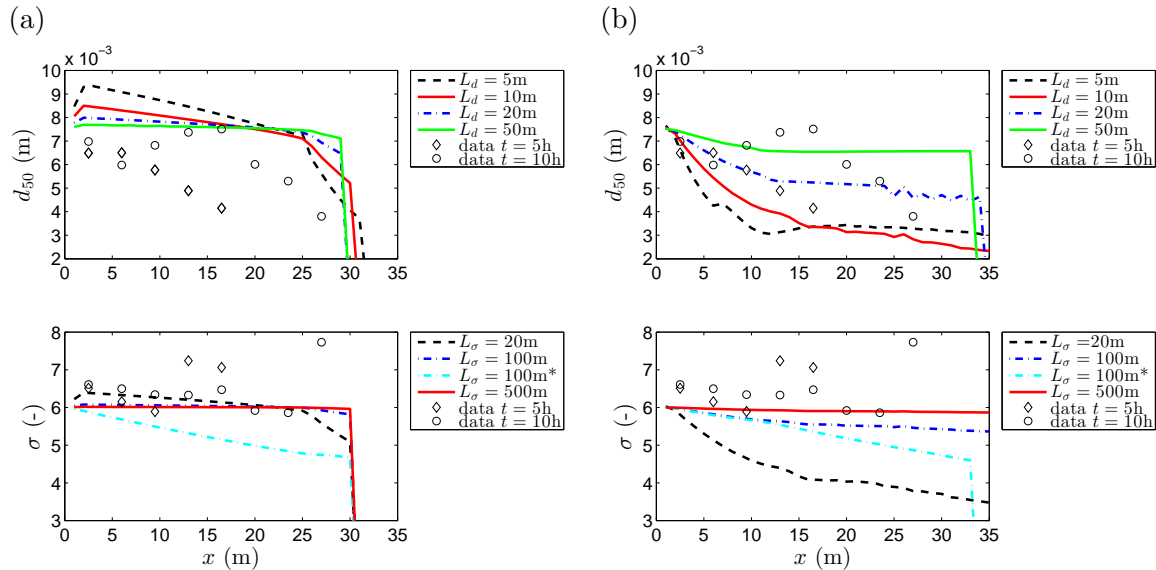


Figure 12 Experimental d_{50} and σ values along the deposit of Seal et al. (1997) experiments for the run 1 at $t = 5-6.5$ h and $t = 6.5-10$ h together with the numerical results for the active layer (a) and bed layer (b) at $t = 10$ h (The sensitivity analysis on L_d is made with $L_\sigma = 100$ m whereas the sensitivity analysis on L_σ is made with $L_d = 10$ m except for * for which $L_d = 100$ m).

366 As discussed previously, L_d and L_σ were first scaled with the length of the equilibrium reach
 367 $L \approx 45$ m (length of the flume). $L_d = 20$ m was chosen here since $L_d = 45$ m would have yielded
 368 a less pronounced downstream fining than measured as shown on Figure 12. On the contrary, for
 369 the sorting coefficient, a larger value was used ($L_\sigma = 100$ m) since nearly no longitudinal evolution
 370 was observed in the experiments. The use of too small values for L_d and L_σ would yield unrealistic
 371 results with too strong downstream fining and sorting. Although a single bed layer was assumed
 372 for these simulations, our results appear to be more accurate than those of Qian et al. (2015) and
 373 as good as those from Langendoen and Alonso (2008). The proposed model is robust and very
 374 parsimonious since only two parameters are needed to describe a sediment mixture (d_{50} and σ)
 375 and only two coefficients are needed to be calibrated (L_d and L_σ) and they can be scaled using
 376 the length of the reach in equilibrium. As a comparison, simulating downstream fining using a
 377 multi-class model may be difficult to calibrate since results are sensitive to several parameters that
 378 are still not perfectly understood and function of the grain size (critical bed shear stress, hiding
 379 and exposure functions, adaptation length).

380 5 Conclusion

381 A 1D numerical code (RubarBE) using an original simple grain size representation for minimizing
 382 the number of calibration constants was used to simulate the test case of bed aggradation by
 383 Seal et al. (1997). A discussion on the experimental test case (porosity, front velocity) is provided
 384 together with the validation of the RubarBE code. Good agreement was found between simulated
 385 and experimental results on morphological evolution. Trends of grain size downstream fining are

386 also properly reproduced by the model. RubarBE was shown to be a robust and parsimonious code,
387 well suited for natural cases where the description of the bed is generally limited, and complex flows
388 (supercritical, interaction with structures, etc.) can occur. It successively reproduced bed evolution
389 for natural test cases in case of short-term (event) or mid-term (several months or years) modelling
390 (El kadi Abderrezzak et al., 2008; El kadi Abderrezzak & Paquier, 2009; Camenen et al., 2015).
391 The present paper discussed more specifically three parameters that are inherent to this model to
392 simulate bed and sediment characteristic evolutions: the non-equilibrium adaptation length L_a and
393 the grain-size related adaptation lengths L_d and L_σ .

- 394 • The adaptation length for sediment transport L_a appears to be in the order of one metre for
395 the Seal et al. (1997) experiments. It affects mainly the slope of the front in a similar way
396 as the mesh size since it adds some diffusion in the model. As pointed out by Wu and Wang
397 (2008), for a 1D model, it should be larger than the mesh size and can typically be scaled by
398 the width of the river for river modelling.
- 399 • The sediment size and sorting adaptation lengths L_d and L_σ are one or two orders of magni-
400 tude larger than L_a and $L_\sigma > L_d$. Both adaptation lengths should be scaled by the equilibrium
401 reach length L .

402 One limit of the model is the discrete representation of the bed layers (Blom, 2008). Proper
403 criteria are needed to fix or adapt sub-layer thickness in order to better describe the possible
404 vertical evolution of the bed characteristics in case of aggrading beds and improve the present
405 results. Such criteria should be function of both the active and sub-layer characteristics. The
406 estimation of the critical bed shear stress may also be an issue when using excess-bed shear stress
407 formulas such as the Meyer-Peter and Müller (1948) formula since partial transport may occur.
408 A first improvement would be to use a formula that allows a weak sediment transport for very
409 low bed shear stress (Camenen & Larson, 2005; Recking, 2013). An additional routine would be
410 necessary to modify grain size characteristics of the sediments transported in this specific case. It
411 should be noted that results are also sensitive to the porosity, which is an important factor but
412 difficult to estimate in the case of a sediment mixture (Wu & Wang, 2007). In order to reproduce
413 the process of vertical sorting reported by Seal et al. (1997) and quantified by Toro-Escobar et al.
414 (1997) with an empirical function, a new transfer function could be added to RubarBE code in
415 order to mix a sediment deposit with the upper bed layer or create a new bed layer depending on
416 the grain size characteristics of both masses of sediments. A more physical thickness of the active
417 layer ($\delta_{AL} \approx 2D_{90}$) could also be used (van Niekerk et al., 1992). Such a function could also be
418 useful to reproduce bed armouring and sediment infiltration in an armoured bed (Béraud, 2012)
419 but also needs an adaptive porosity calculation for each layers.

420 Experiments by Seal et al. (1997) are of great interest for calibrating morphodynamic models.
421 It appears however that uncertainties in the estimation of the grain size characteristics remain
422 too large and local variations that are not fully understood remain present. Additional experimen-
423 tal data presenting detailed measurements of a grain fining would be of great interest for a full
424 validation of morphodynamic models.

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430 **References**

- 431 Armanini, A., & di Silvio, G. (1986). Discussion of "A depth-integrated model for suspended
432 sediment transport" by G. Galappatti and C. B. Vreugdenhil. *Journal of Hydraulic Research*,
433 24(5), 437-442.
- 434 Armanini, A., & di Silvio, G. (1988). A one-dimensional model for the transport of a sediment
435 mixture in non-equilibrium conditions. *Journal of Hydraulic Research*, 26(3), 275-292.
- 436 Ashida, K., & Michiue, M. (1971). An investigation of river bed degradation downstream of a
437 dam. In *Proc. 14th IAHR Congress* (p. 1-9). Paris, France.
- 438 Balayn, P. (2001). *Contribution à la modélisation numérique de l'évolution morphologique des*
439 *cours d'eau aménagés lors de crues [A contribution to numerical modelling of bed evolution*
440 *of channelized stream during floods]*. (Unpublished doctoral dissertation). Claude Bernard
441 University, Lyon 1, Lyon, France. (138p., in French)
- 442 Bell, R. G., & Sutherland, A. J. (1983). Nonequilibrium bed-load transport by steady flow. *Journal*
443 *of Hydraulic Engineering*, 109(3), 351-367.
- 444 Belleudy, P. (2000). Numerical simulation of sediment mixture deposition, part 1: analysis of a
445 flume experiment. *Journal of Hydraulic Research*, 38(6), 417-426.
- 446 Belleudy, P. (2001). Numerical simulation of sediment mixture deposition, part 2: a sensitivity
447 analysis. *Journal of Hydraulic Research*, 39(1), 25-31.
- 448 Béraud, C. (2012). *Modélisation numérique des impacts de recharges sédimentaires en rivière*
449 *aménagée. Cas du Vieux-Rhin entre Kembs et Breisach [Numerical modelling of the sediment*
450 *feeding impacts in engineered rivers. The case of the Old Rhine, between Kembs and Brisach]*
451 (Unpublished doctoral dissertation). C. Bernard University, Lyon 1, Lyon, France. (252 p.,
452 in French)
- 453 Béraud, C., Le Coz, J., Camenen, B., & Paquier, A. (2011). 1D numerical simulation of sediment
454 downstream fining during bed aggradation using a simplified grain size description. In *Proc.*
455 *7th IAHR symposium on River, Coastal and Estuarine Morphodynamics* (p. 1-10). Beijing,
456 China. (CD Rom)
- 457 Blom, A. (2008). Different approaches to handling vertical and streamwise sorting in modeling
458 river morphodynamics. *Water Resources Research*, 44(W03415), 1-16.
- 459 Camenen, B., Grabowski, R., Latapie, A., Paquier, A., Solari, L., & Rodrigues, S. (2015). On the
460 estimation of the bed-material transport and budget along a river segment: application to
461 the Middle Loire River, France. *Aquatic Sciences*, 78, 71-81.
- 462 Camenen, B., Holubová, K., Lukač, M., Le Coz, J., & Paquier, A. (2011). Assessment of methods
463 used in 1D models for computing bedload transport in a large river: the Danube River in
464 Slovakia. *Journal of Hydraulic Engineering*, 137(10), 1190-1199.
- 465 Camenen, B., & Larson, M. (2005). A bedload sediment transport formula for the nearshore.
466 *Estuarine, Coastal & Shelf Science*, 63, 249-260.
- 467 Colombini, M., Seminara, G., & Tubino, M. (1987). Finite-amplitude alternate bars. *Journal of*
468 *Fluid Mechanics*, 181, 213-232.
- 469 Cui, Y., & Parker, G. (1998). The arrested gravel front: stable gravel-sand transitions in rivers;
470 Part 2: General numerical solution. *Journal of Hydraulic Research*, 36(2), 159-182.
- 471 Cui, Y., Parker, G., & Paola, C. (1996). Numerical simulation of aggradation and downstream
472 fining. *Journal of Hydraulic Research*, 34(2), 185-204.
- 473 Daubert, A., & Lebreton, J. C. (1967). Etude expérimentale et sur modèle mathématique de
474 quelques aspects des processus d'érosion des lits alluvionnaires, en régime permanent et non-
475 permanent [Study with a mathematic model on some erosion processes in alluvial rivers for
476 steady and unsteady regimes]. In *Proc. 12th IAHR Congress* (Vol. 3, p. 26-37). Fort Collins,
477 Colorado, USA. (in French)
- 478 Deigaard, R. (1980). *Longitudinal and transverse sorting of grain sizes in alluvial rivers* (series
479 paper No. 26). Denmark: Institute for Hydrodynamic and Hydraulic Engineering, Technical

- 480 Univiversity of Denmark.
- 481 di Silvio, G. (1992). Modelling sediment transport under different hydrological and morphological
482 circumstances. In P. Billi, R. D. Hey, C. R. Thorne, & P. Tacconi (Eds.), *Gravel bed river*
483 *III: Dynamics of gravel-bed rivers* (p. 363-372). John Wiley and Sons Ltd. (Chapter 18)
- 484 Egiazaroff, I. V. (1955). Calculation of nonuniform sediment concentrations. *Journal of Hydraulic*
485 *Division*, 91(HY4), 225-253.
- 486 El kadi Abderrezzak, K., & Paquier, A. (2009). One-dimensional numerical modeling of sediment
487 transport and bed deformation in open channels. *Water Resources Research*, 45(W05404),
488 1-20.
- 489 El kadi Abderrezzak, K., & Paquier, A. (2011). Applicability of sediment transport capacity
490 formulas to dam-break flows over movable beds. *Journal of Hydraulic Engineering*, 137(2),
491 209-221.
- 492 El kadi Abderrezzak, K., Paquier, A., & Gay, B. (2008). One-dimensional numerical modelling of
493 dam-break waves over movable beds: application to experimental and field cases. *Environ-*
494 *mental Fluid Mechanics*, 8(2), 169-198.
- 495 Ferguson, R. I., Church, M., & Weatherly, H. (2001). Fluvial aggradation in vedder river: Testing
496 a one-dimensional sedimentation model. *Water Resources Research*, 37(12), 3331-3347.
- 497 Ferguson, R. I., & Wathen, S. (1998). Tracer-pebble movement along a concave river profile :
498 Virtual velocity in relation to grain size and shear stress. *Water Resources Research*, 34(8),
499 2031-2038.
- 500 Gomez, B., Rosser, B. J., Peacock, D. H., Murray Hicks, D., & Palmer, J. A. (2001). Downstream
501 fining in a rapidly aggrading gravel bed river. *Water Resources Research*, 37(6), 1813-1823,.
- 502 Hirano, M. (1971). River bed degradation with armouring. *Transaction of the Japanese Society*
503 *of Civil Engineering*, 3(2), 55-65. (in Japanese)
- 504 Hoey, T. B., & Ferguson, R. (1994). Numerical simulation of downstream fining by selective trans-
505 port in gravel bed rivers : model development and illustration. *Water Resources Research*,
506 30(7), 2251-2260.
- 507 Klingeman, P. C., Chaquette, C. J., & Hammond, S. B. (1979). *Bed material characteristics*
508 *near Oak Creek sediment research facilities* (Tech. Rep.). Corvallis, Oregon, USA: Water
509 Resources Research Institute, Oregon State University.
- 510 Langendoen, E. J., & Alonso, C. V. (2008). Modeling the evolution of incised streams: I. Model
511 formulation and validation of flow and streambed evolution components. *Journal of Hydraulic*
512 *Engineering*, 134(6), 749-762.
- 513 Meyer-Peter, E., & Müller, R. (1948). Formulas for bed-load transport. In *Proc. 2nd IAHR*
514 *Congress* (p. 39-64). Stockholm, Sweden.
- 515 Mikoš, M. (1993). *Fluvial abrasion of gravel sediments* (Tech. Rep. No. 123). Zürich, Switzerland:
516 Miteilungen der Versuchsanstalt für Wasserbau, Hydrologie und Glaziologie der ETH Zürich.
517 (322p.)
- 518 Morris, P. H., & Williams, D. J. (1999). A worldwide correlation for exponential bed particle size
519 variation in subaerial aqueous flows. *Earth Surface Processes & Landforms*, 24, 835-847.
- 520 Paola, C., & Seal, R. (1995). Grain size patchiness as a cause of selective deposition and downstream
521 fining. *Water Resources Research*, 31(5), 1395-1407.
- 522 Paquier, A., & El Kadi Abderrezzak, K. (2008). A model for bedload transport and morphological
523 evolution in rivers : description and pertinence. In S. Benzoni Gavage & D. Serre (Eds.),
524 *Hyperbolic problems: Theory, numerics, applications*. (p. 285-296). Springer.
- 525 Parker, G. (1990). Surface-based bedload transport relation for gravel rivers. *Journal of Hydraulic*
526 *Research*, 28(4), 417-436.
- 527 Parker, G. (1991). Selective sorting and abrasion of river gravel. I: Theory. *Journal of Hydraulic*
528 *Engineering*, 117(2), 131-147.
- 529 Parker, G., & Cui, Y. (1998). The arrested gravel front: stable gravel-sand transitions in rivers;
530 Part 2: Simplified analytical solution. *Journal of Hydraulic Research*, 36(1), 75-100.

- 531 Phillips, B. C., & Sutherland, A. J. (1989). Spatial lag effects in bed load sediment transport.
532 *Journal of Hydraulic Research*, 27(1), 115-133.
- 533 Qian, H., Cao, Z., Pender, G., Liu, H., & Hu, P. (2015). Well-balanced numerical modelling of non-
534 uniform sediment transport in alluvial rivers. *International Journal on Sediment Research*,
535 30, 117-130.
- 536 Rahuel, J. L., Holly, F. M., Chollet, J. P., Belleudy, P. J., & Yang, G. (1989). Modeling of
537 riverbed evolution for bedload sediment mixtures. *Journal of Hydraulic Engineering*, 115(11),
538 1521-1542.
- 539 Recking, A. (2013). A simple method for calculating reach-averaged bedload transport. *Journal*
540 *of Hydraulic Engineering*, 139(1), 70-75.
- 541 Seal, R., Paola, C., Parker, G., Southard, J. B., & Wilcock, P. R. (1997). Experiments on down-
542 stream fining of gravels: I. Narrow-channel runs. *Journal of Hydraulic Engineering*, 123(10),
543 874-884.
- 544 Seal, R., Parker, G., & Mullenbach, C. P. B. (1995). *Laboratory experiments on downstream*
545 *fining of gravel, narrow channel runs 1 through 3: supplemental methods and data.* (External
546 Memorandum No. 239). University of Minnesota, USA: St Anthony Falls Laboratory.
- 547 Stecca, G., Siviglia, A., & Blom, A. (2014). Mathematical analysis of the saint-venant-hirano
548 model for mixed-sediment morphodynamics. *Water Resources Research*, 50, 7563-7589.
- 549 Sternberg, H. (1875). Untersuchungen über längen- und querprofil geschiebeführender flüsse [Study
550 on the longitudinal and transversal bed-load transport in rivers]. *Zeitschrift Bauwesen*, 25,
551 483-506. (in German)
- 552 Strickler, A. (1923). *Beiträge zur frage der geschwindigkeitsformel und der rauheitszahlen*
553 *für ströme, kanäle und geschlossene leitungen [Contributions to the questions of velocity*
554 *formulations and roughness values for rivers, canals, and closed ducts]* (Mitteilung No. 16).
555 Bern, Switzerland: Amt für Wasserwirtschaft. (in German)
- 556 Surian, N. (2002). Downstream variation in grain size along an alpine river: analysis of controls
557 and processes. *Geomorphology*, 43, 137-149.
- 558 Toro-Escobar, C. M., Parker, G., & Paola, C. (1997). Transfer function for the deposition of poorly
559 sorted gravel in response to streambed aggradation. *Journal of Hydraulic Research*, 35(4),
560 563-566.
- 561 Toro-Escobar, C. M., Parker, G., & Paola, C. (2000). Experiments on downstream fining of gravel.
562 ii wide and sandy runs. *Journal of Hydraulic Engineering*, 126(3), 198-208.
- 563 van Niekerk, A., Vogel, K. R., Slingerland, R. L., & Bridge, J. S. (1992). Routing of heterogeneous
564 sediments over movable bed : Model development. *Journal of Hydraulic Engineering*, 118(2),
565 246-262.
- 566 Viparelli, E., Sequeiros, O. E., Cantelli, A., Wilcock, P. R., & Parker, G. (2010). River mor-
567 phodynamics with creation/consumption of grain size stratexperiments, 2: numerical model.
568 *Journal of Hydraulic Research*, 48(6), 727-741.
- 569 Vogel, K. R., van Niekerk, A., Slingerland, R. L., & Bridge, J. S. (1992). Routing of heterogeneous
570 sediments over movable bed : Model verification. *Journal of Hydraulic Engineering*, 118(2),
571 263-279.
- 572 Wilcock, P. R., & Crowe, J. C. (2003). Surface-based transport model for mixed-size sediment.
573 *Journal of Hydraulic Engineering*, 129(2), 120-128.
- 574 Wright, S., & Parker, G. (2005). Modeling downstream fining in sand-bed rivers. II: Application.
575 *Journal of Hydraulic Research*, 43(6), 621-631.
- 576 Wu, W., Viera, D. A., & Wang, S. S. Y. (2004). One-dimensional numerical model for nonuni-
577 form sediment transport under unsteady flows in channel networks. *Journal of Hydraulic*
578 *Engineering*, 130(9), 914-923.
- 579 Wu, W., & Wang, S. S. Y. (2007). One-dimensional modeling of dam-break flow over movable
580 beds. *Journal of Hydraulic Engineering*, 133(1), 48-58.

- 581 Wu, W., & Wang, S. S. Y. (2008). One-dimensional explicit finite-volume model for sediment
582 transport. *Journal of Hydraulic Research*, 46(1), 87-98.
- 583 Wu, W., Wang, S. S. Y., & Jia, Y. F. (2000). Nonuniform sediment transport in alluvial rivers.
584 *Journal of Hydraulic Research*, 38(6), 427-434.
- 585 Yu, A.-B., Standish, N., & McLean, A. (1993). Porosity calculation of binary mixtures of non-
586 spherical particles. *Journal of the American Ceramic Society*, 76(11), 2813-2816.

587 *Notation*

Latin and Greek variables

Ab	=	cross-sectional area of the bed above a reference datum
B	=	width of the river
c_f	=	volume concentration of fine sediments
C_F	=	time-averaged front velocity
d_{50}	=	median diameter
d_x	=	grain diameter at which x percent of the distribution in mass lies below
F	=	Froude number
h	=	water depth
H_d	=	height of the deposit at the injection point
H_F	=	front height
H_{tail}	=	height of the tailgate
L	=	length of the reach in morphological equilibrium
L_a	=	non-equilibrium adaptation length
L_d	=	adaptation length related to the median diameter evolution
L_σ	=	adaptation length related to the sorting coefficient evolution
M	=	sediment mass
q_{sb}	=	volumetric bedload transport per unit width
Q_0	=	water discharge
Q_s	=	volumetric sediment transport
Q_{s*}	=	equilibrium sediment transport or sediment transport capacity
Q_{s0}	=	sediment input
t	=	time
T_f	=	final time of the runs
u_*	=	shear velocity
U	=	depth-averaged velocity of the flow
r_d	=	ratio between fine and coarse sediment diameters
S_b	=	slope of the bed
V	=	volume of the deposit
W_s	=	settling velocity
x	=	longitudinal direction
x_0	=	position of the sediment input
x_F	=	position of the front
$z_{w,tail}$	=	water level at the tailgate
α_d	=	constant for Sternberg equation for d
α_σ	=	constant for Sternberg equation for σ
α_L	=	constant for adaptation length equations
δ_{AL}	=	active layer thickness
ϕ	=	porosity
ρ_s	=	sediment density
σ	=	sorting coefficient

θ = Shields parameter

θ_{cr} = critical Shields parameter for the inception of movement

Subscripts and exponents

dn = upstream cell

up = downstream cell

dep = deposit

ero = erosion

tra = transit

f = fine

c = coarse

adj = adjusted

F = front